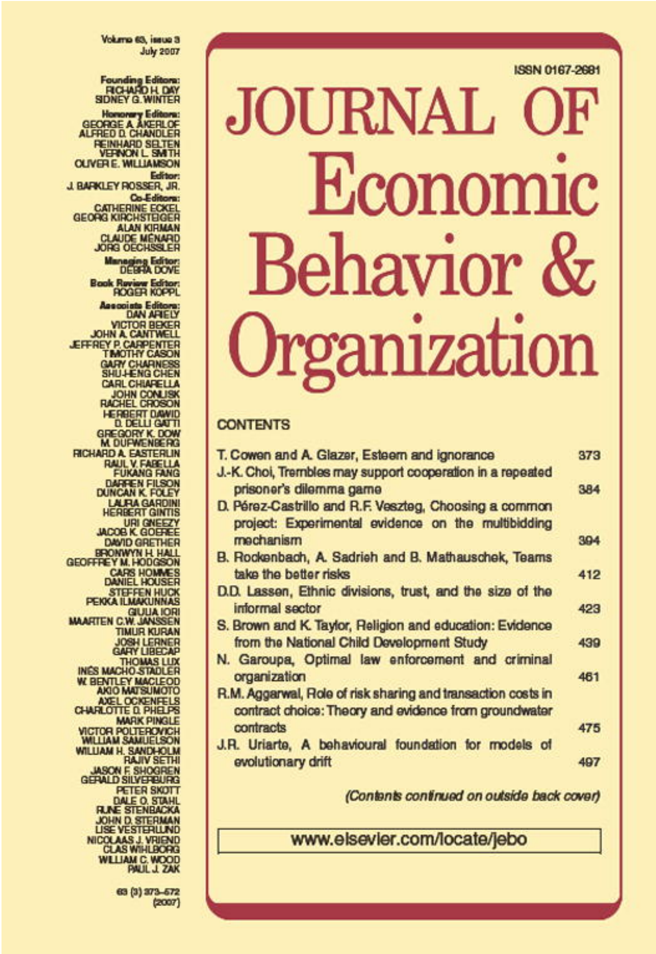

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A behavioural foundation for models of evolutionary drift

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Abstract

Binmore and Samuelson [Binmore, K., Samuelson, L., 1999. Evolutionary drift and equilibrium selection. Review of Economic Studies 66, 363–393] have shown that perturbations (drift) are crucial to study the stability properties of Nash equilibria. We contribute to this literature by providing a behavioural foundation for models of evolutionary drift based on the similarity theory introduced by Tversky [Tversky, A., 1977. Features of similarity. Psychological Review 84, 327–352]. The innovation is that we derive the similarity relations from the perception that each agent has about how well he is playing the game. We obtain different models of drift depending on how we model that perception and show the conditions for each model to stabilize elements in components of Nash equilibria that are not subgame-perfect.

JEL classification: C70; C72

Keywords: Similarity relations; Drift; Nash equilibrium; Selection dynamics; Learning

1. Introduction

It is common place to observe that the Nash equilibrium selected by a theory depends on the manner in which perturbations are handled (see, for example, Selten's (1975) perfect equilibrium, Myerson's (1978) proper equilibrium and, more recently, McKelvey and Palfrey's (1995) quantal response equilibrium). Binmore et al. (1995) and Binmore and Samuelson (1999), B&S henceforth, also emphasize the importance of perturbations, but they place these in the selection

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or learning process that takes the players to equilibrium, rather than perturbing the game itself. This paper follows the methodology used by the latter authors.

B&S studied the stability properties of Nash equilibria and dealt, most notably, with the issue of equilibrium selection, particularly in the ultimatum game (or the chain-store game). This game was chosen because it is often used to justify subgame-perfect equilibria, even though the choices observed in the laboratory experiments on this game are other than the subgame-perfect (too many experiments have been carried out on the ultimatum game to quote them here; see, for instance, Güth et al. (2001) and Camerer (2003) and the references quoted there). B&S show that states near equilibria that are not subgame-perfect can be stabilized by drift. Although their result is not particularly close to the experimental data, B&S have shown that evolutionary drift (which accounts for the perturbations that affect the selection or learning process through which equilibrium might be achieved) is a relevant element in game theory.

The difficulty of matching the experimental data of the ultimatum game is that the player's behaviour in this game seems to be led by fairness considerations. When the perception of a game is influenced by individual values or by social norms and conventions, it seems that different strategies in that game are, a priori, valued differently, and therefore the tendencies to abandon them might differ from one individual to another. If this is true, then how do we model this behaviour? B&S argue that drift could be a key tool to deal with this issue. Drift may capture some of the real world imperfections left aside by the learning process embedded in the selection dynamic model, thus adding more realism to the equilibrating process. But the model of drift proposed by B&S lacks psychological foundations, and little insight is given into what perturbations one should expect.

It is in modelling such perturbations realistically that the present paper is concerned. The approach we take is based on the similarity theory developed first by Tversky in psychology, and later applied to choice theory by Kahneman and Tversky (1979) and Rubinstein (1988, 1998) to explain observed choice behaviour (such as the one leading to the Allais Paradox). To do so, the present work develops a similarity theory valid for a dynamic setting to model the behaviour of the perturbing agents (whom we call the $\sum$ agents).

The most relevant assumption in the article is that each $\sum$ agent is endowed with a threshold function that measures the vagueness or ambiguity felt by the agent about how well he is playing the game. We assume that the level of vagueness felt be sensitive to the proportions of agents in his population playing the same strategy he is currently using. The threshold function plays the role of a primitive concept in modelling the $\sum$ agents' choice behaviour.

In essence, what a $\sum$ agent does is the following: he uses the threshold function to build a pair of similarity relations, one defined in the space of all possible expected payoffs to his current strategy and the other in the space of proportions of agents in his population playing that strategy. Then, with this pair of similarity relations, the $\sum$ agent builds a preference relation on the (product) space of payoffs-strategy proportions that tells him, at each period, how satisfied he is with his current strategy. This preference relation and the agent's aspiration set, which is assumed to be the preferred set, change over time. Dissatisfied $\sum$ agents will switch strategy and, as a rule of thumb, choose every strategy with the same probability. Hence, drift ensures that small fractions of all strategies, including those that are not currently played in the population, will be continuously injected and perturb the selection dynamic system.

Depending on the type of threshold function we deal with, we may build two classes of similarity relations. If the vagueness felt by the agent decreases when he observes that the proportion of agents playing his current strategy increases, then we would obtain the socially induced similarity relations (which are related to those introduced by Uriarte (1999) for the space of simple lotteries).

But when the level of vagueness felt is not sensitive to what the others are doing, then we would obtain the AINU similarity relations (which are related to those introduced by Aizpurua et al. (1993)) or, in short, AINU). Both classes of similarity relations are assumed to satisfy the axioms introduced by Rubinstein (1988).

From the socially induced similarity relations we derive three models of drift. In one model the $\sum$ agents are endowed with extremely sensitive threshold functions (or playing modes). This model seems to capture well the influence of social norms and convention on the agents' behaviour, but on the negative side, the model presents, in some cases, a certain degree of "adhocery". We avoid this issue, with two additional models of drift that use the data about payoffs and strategy proportions to determine endogenously the threshold function of each agent. Finally, from the AINU similarity relations we obtain a model of drift that is sensitive only to payoffs. We show that each model has different capabilities to stabilize Nash equilibria that are not subgame-perfect.

The article is organized as follows. Section 2 introduces the notation and explains the methodology used in the paper. Section 3 presents a detailed account of how the socially induced similarity relations are obtained and how the model of drift is built. This is the main section of the paper. Its extension is well justified as it facilitates the understanding of Sections 4 and 5. Section 4 presents the AINU similarity relations and the drift model based on them. Section 5 presents the results obtained with the four models of evolutionary drift. That section uses the example of the ultimatum minigame (or the chain-store game) to show the sufficient conditions for each model to create stationary states with different stability properties in the component of Nash equilibria that are not subgame-perfect. Section 6 relates the results with those obtained, in particular, with the QRE model of McKelvey and Palfrey (1998) and with the B&S model. Section 7 concludes the article.

2. Notation

Let G be a non-cooperative finite game in normal form, with $K=\{1, 2, \ldots, n\}$ as the set of players. We assume that there are n large player-populations. Randomly drawn members of the n player-populations, one from each population, are repeatedly matched to play the game. For each player $k \in K$, let $S_k = \{1, 2, \ldots, m_k\}$ be his finite set of pure strategies, for some integer $m_k \geq 2$. Throughout the paper, we shall refer to agent ki, a member of player-population $k \in K$ playing strategy $i \in S_k$. Thus, f_{ki} will denote the proportion of agents in player population $k \in K$ who play strategy $i \in S_k$ at time t, with f_k being the vector collecting such proportions in population k and $f=(f_1, \ldots, f_n)$ the population state at time t. Hence, $f \in \Delta = \times_{k=1}^{n} \Delta_k$, where Δ_k is the simplex of mixed strategies for player $k \in K$. $F_{ki} = [0, 1]$ is the space of proportions of agents in player-population k playing strategy i. Let $\pi_{ki}(f)$ denote the (expected) payoff to agent ki given the population state f at time t. The term $\bar{\pi}_k(f) = \sum_{i=1}^{m_k} f_{ki}\pi_{ki}(f)$ denotes the average expected payoff to player population $k \in K$. More specifications about payoffs are given in Section 3.2, below.

Like B&S, we shall start with a selection dynamic model, which one can find in biological models of natural selection such as the Replicator Dynamics (RD). Then, for a better approximation to the underlying stochastic strategy-adjustment process, we add the drift term to the selection dynamics. The relevance of evolutionary analysis to experimental data is emphasized by Binmore et al. (1995, 2002), Binmore and Samuelson (1999) and Samuelson (1998) in relation to the influence of social norms in the (short run) laboratory behaviour. In particular, the ultimatum game is relevant in this respect because the theoretical prediction of subgame-perfection is contradicted by experimental data. Börgers and Sarin (1997) have shown that the RD may serve as long-run approximations to simple learning rules related to that used by Roth and Erev (1995), the reinforcement learning rule, to explain experimental data.

3. Drift based on socially induced similarity relations

It is natural to observe different behaviours when different agents face the same decision problem. For this reason, diversity of tastes and values are central to economic analysis. Hence, in the society that evolves according to the agents whose behaviour leads to the selection dynamic model, the so-called SD agents (for instance, Binmore et al., 1995; Börgers and Sarin, 1997; Schlag, 1998; Cabrales, 2000 show how different individual behaviours may lead to the most popular selection dynamic model, the replicator dynamics), we shall add a new type of agents, the so-called $\sum$ agents. Thus, the perturbed selection dynamic will have large player-populations, and inside each population we assume there are two types of agents, the SD agents and the $\sum$ agents. Both population types are assumed to be equally large. We proceed now to describe the features of the $\sum$ agent *ki*.

3.1. The threshold function

We introduce first a function that plays the role of a primitive concept in the model.

It seems natural to assume that the participant in a continuously repeated interaction builds experience-based conjectures about how good or bad is playing the underlying game and that he may relate that evaluation, for instance, to the proportion of individuals who are playing exactly like him. We will assume that the $\sum$ agent *ki* has, at each stage, information about those proportions and thinks as follows: "the higher is the proportion of agents in my player population who are currently using the same strategy as mine, the less ambiguity (or insecurity or uncertainty or vagueness) I should feel about how well I am playing the game". Thus, we are relating the idea of "how well I am playing the game" with society (i.e. the rest of agents in my player population) and, therefore, with the social experience, conventions and norms that might exist therein. We assume that it is a subjective measure. It will become clear that we are not proposing a herding model. For some agents the measure depends exactly on playing according to what the rest of the society is doing. But for other agents, it depends less on what the others are doing and more on playing according to certain moral judgements or social norms or, simply, on what the experience is telling him (this would be the alert mode of playing described in Remark 1, below). Formally,

Assumption 1. Each $\sum$ agent *ki* is endowed with a differentiable function d_{ki}, called threshold function, in the set

$$D = \{d_{ki} : F_{ki} \to [0, 1] : \text{with}\, d_{ki}(0) = 1,\, d_{ki}(1) = 0 \,\text{and}\, \hat{f}_{ki} > \tilde{f}_{ki} \Rightarrow d_{ki}(\hat{f}_{ki}) < d_{ki}(\tilde{f}_{ki})\}. \quad (1)$$

Given a proportion $f_{ki} \in F_{ki}$ and any $d_{ki} \in D$, $d_{ki}(f_{ki})$ measures the ambiguity (about how well he is playing the game) felt by the $\sum$ agent *ki* when the proportion of agents in player population k playing strategy $i \in S_k$ at time t is f_{ki}. The ambiguity gradually decreases when he observes that more and more agents from his population come to play the same strategy as his. For a different use of the strategy proportions information, see Young (1993a,b, 1996).

Remark 1 *(The playing modes).* For any d_{ki}, $\hat{d}_{ki}$ in D, if for all $f_{ki} \in (0, 1)$, $d_{ki}(f_{ki}) < \hat{d}_{ki}(f_{ki})$, then we say that d_{ki} is sharper than $\hat{d}_{ki}$. Two important cases should be considered: for all $f_{ki} \in (0, 1)$, the extremely sharp threshold function, $\bar{d} \in D$ for which $\bar{d}(f_{ki})$ takes values that are "very close" to 0 (i.e. $\bar{d}(f_{ki}) \cong 0$) and the extremely unsharp threshold function, $\underline{d} \in D$, for which $\underline{d}(f_{ki})$ takes values that are "very close" to 1 (i.e. $\underline{d}(f_{ki} \cong 1)$). When $d_{ki} = \bar{d}$, we would say that the $\sum$ agent *ki* is in the alert mode of play and when $d_{ki} = \underline{d}$ then, we say the agent is in the absent mode of play.

3.2. Vagueness modelled by socially induced similarity relations

We assume that the level of vagueness felt by the $\sum$ agent ki develops intervals (in both the payoff and strategy frequency spaces) inside which events are not distinguishable. To model these intervals we use the similarity theory axiomatized by Rubinstein (1988, 1998). The main innovation with respect Rubinstein's similarities is that those developed in this section are built with the help of the threshold functions in D, and therefore it can be said that they are socially induced.

In essence, a similarity relation serves to capture the capacity of an individual to discriminate between events. Correlated similarity relations, a concept introduced by Aizpurua et al. (1993), describe how that discrimination capacity changes depending on the values of some relevant parameter. For instance, we shall define here correlated similarity relations on F_{ki} to capture the idea that the efforts dedicated to discriminate on F_{ki} increase if the payoffs at stake increase. Similarly, we define correlated similarity relations on the set of all expected payoffs to pure strategy $i \in S_k$ to formalize the idea that discrimination between different payoffs increases when the proportion f_{ki} increases (i.e. a finer discrimination is obtained if the accumulated experience is increased and this is assumed to occur when more agents from population k come to play strategy i).

Without loss of generality, we may assume that the payoffs to the $\sum$ agents are strictly positive and do not exceed 1 (we might imagine that to put in practice their similarity-based decision procedure, the $\sum$ agents would perform suitable positive affine transformations of the payoff functions of the underlying game G combined with local shifts of such functions; these operations, assumed to be the same for all $\sum$ agents, would leave invariant the best reply correspondences and, hence, the set of Nash equilibria of G. Thus, the game G', obtained by means of those transformations, and G are equivalent). Then, $\pi_{ki}(f)$ denotes the expected payoff to SD agent ki and $p_{ki}(f)$ the expected payoff to $\sum$ agent ki, with $p_{ki}(f) \in \Pi_{ki} = (0, 1]$.

Let $(p_{ki}(f), f_{ki})$ be the vector of expected payoff-proportion of agents of player population k attached to strategy $i \in S_k$ at time t. The d_{ki} function serves two purposes:

(a) To define on Π_{ki} correlated similarities of the difference-type: each $\sum$ agent ki calculates expected payoffs correctly, but due to the ambiguity about his play, he develops a similarity interval for $p_{ki}(f)$: given f_{ki}, the interval is $[p_{ki}(f) - d_{ki}(f_{ki}), p_{ki}(f) + d_{ki}(f_{ki})]$. Thus, given f_{ki}, $d_{ki}(f_{ki})$ defines the threshold level on Π_{ki}. By Assumption 1, if f_{ki} increases, the threshold decreases, so the similarity interval of $p_{ki}(f)$ shrinks.

(b) To build the λ_{ki} function which will be used to define on F_{ki} correlated similarity relations of the ratio-type (see Rubinstein, 1988). Assuming that each $\sum$ agent ki is endowed with a $d_{ki} \in D$, the function λ_{ki} is defined as follows:

$$\lambda_{ki}(p_{ki}(f)) = \frac{p_{ki}(f)}{p_{ki}(f) - d_{ki}(f_{ki})} > 1. \tag{2}$$

Thus, given d_{ki} and a proportion $f_{ki} \in (0, 1)$, the domain of λ_{ki} will be the payoffs satisfying $p_{ki}(f) > d_{ki}(f_{ki})$. Hence, there is a family of λ_{ki} functions parameterized by $f_{ki} \in (0, 1)$. If $p_{ki}(f) \leqq d_{ki}(f_{ki})$, then λ_{ki} is not defined and we would have the degenerate similarity relation (i.e. a relation for which the similarity interval of any point in Π_{ki} is the whole space Π_{ki}; see Rubinstein, 1988).

The λ_{ki} function is a threshold function that measures the discrimination capacity on F_{ki}. Hence, given $p_{ki}(f)$ the similarity interval for f_{ki} is $[f_{ki}/\lambda_{ki}(p_{ki}(f)), f_{ki}\lambda_{ki}(p_{ki}(f))]$.

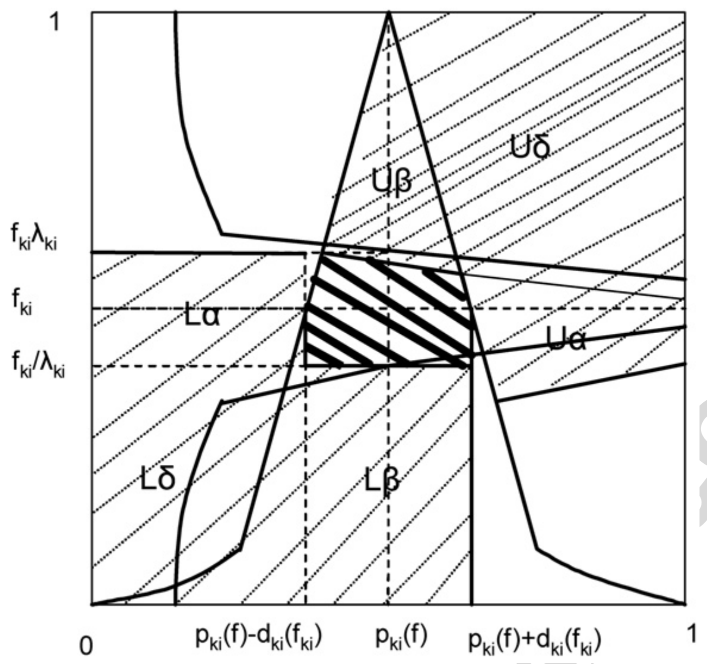


Fig. 1. The $\sum$ preference $\succsim_{ki}$. Given the vector $(p_{ki}(f), f_{ki})$ its upper and lower contour sets, obtained by a procedure based on a pair of socially induced similarity relations, are denoted by U and L, respectively. The indifference set is the darkest area.

3.3. *Satisficing $\sum$ preferences and endogenous aspiration sets*

For any vector $(p_{ki}(f), f_{ki}) \in \Pi_{ki} \times F_{ki}$ attached to strategy $i \in S_k$ at time t, the pair of similarity relations are used to define a preference relation that would determine the upper and lower contour sets relative to that vector as well as its indifference set (the procedure is described in Appendix I; the appendices are available in the JEBO website). Fig. 1 depicts the resulting (non-complete and non-transitive) $\sum$ preference relation, $\succsim_{ki}$ defined on $\Pi_{ki} \times F_{ki}$, when the $\sum$ agent ki is outside the two playing modes mentioned in Remark 1. We assume that the preferred set, denoted by $U = U\alpha \cup U\beta \cup U\delta$, represents the $\sum$ agent ki's aspiration set at time t. By definition, as $(p_{ki}(f), f_{ki})$ changes the corresponding aspiration set, obviously, changes.

We assume that a $\sum$ agent ki is a $\sum$ preference satisficer in the sense that he chooses a strategy just to minimize the distance from $(p_{ki}(f), f_{ki})$ to his aspiration set U. We should note that

(i) Given f_{ki} and d_{ki}, $\frac{\partial \lambda_{ki}}{\partial p_{ki}(f)} = \frac{-d_{ki}(f_{ki})}{(p_{ki}(f)-d_{ki}(f_{ki}))^2} < 0$. Hence, the size of the similarity interval of f_{ki}, $[f_{ki}/\lambda_{ki}(p_{ki}(f)), f_{ki}\lambda_{ki}(p_{ki}(f))]$, decreases if $p_{ki}(f)$ increases. This is the shrinking property of the correlated similarity relation defined by λ_{ki} on F_{ki} (see Uriarte (1999)). It means that if the expected payoff at stake increases, the $\sum$ agent ki's perception on the space F_{ki} increases. The horizontal wedge-shaped form of Fig. 1 shows this property.

(ii) An increase in f_{ki}, (say, to $\bar{f}_{ki}$) has more subtle implications: given $p_{ki}(f)$ and d_{ki}, since $d_{ki}(\bar{f}_{ki}) < d_{ki}(f_{ki})$, the similarity interval of p_{ki}, $[p_{ki}(f) - d_{ki}(\bar{f}_{ki}), p_{ki}(f) + d_{ki}(\bar{f}_{ki})]$, will shrink. This property is shown by the vertical triangle-like form of Fig. 1. Furthermore, the change in f_{ki} creates a new function, $\bar{\lambda}_{ki}$, such that $\lambda_{ki}(p_{ki}(f)) = \frac{p_{ki}(f)}{p_{ki}(f)-d_{ki}(\bar{f}_{ki})} < \lambda_{ki}(p_{ki}(f))$ for every $p_{ki}(f) > d_{ki}(f_{ki})$.

Therefore, both in (i) and (ii) we get a thinner indifference set $\sim_{ki}[(p_{ki}(f), f_{ki})]$ – the dark area of Fig. 1 – and a smaller value of λ_{ki}. A thinner indifference set implies a smaller distance from $(p_{ki}(f), f_{ki})$ to the aspiration set and hence, the agent will feel more satisfied with his current strategy. Since in this event we would have a smaller value of λ_{ki}, the function λ_{ki} could be thought of as an indicator of the degree of satisfaction of $\sum$ agent ki. The smaller the value of λ_{ki}, the happier the agent would feel with his current strategy

3.4. The dynamic of drift

We take the ratio $\frac{1}{\lambda_{ki}}$ as the probability that $\sum$ agent ki will retain his current strategy $i \in S_k$ in the next period; $1 - \frac{1}{\lambda_{ki}}$ will then be the probability of switching to a different strategy in S_k.

The $\sum$ agent ki only has information about the strategy he is currently using (i.e. the payoffs and frequencies attached to strategy i). The next assumption describes his behaviour when he feels dissatisfied with his current strategy.

Assumption 2. When a $\sum$ agent ki is dissatisfied with his current strategy, he will choose the $m_k - 1$ available strategies $j \in S_k$, $j \neq i$, with the same probability $\frac{1}{m_k-1}\left(1 - \frac{1}{\lambda_{ki}}\right)$.

Thus, the $\sum$ agents follow the rule "try every other action if you feel dissatisfied with your current strategy". Given Assumption 2, the $\sum$ agents perturb the SD system by injecting strategies that are not currently played. We assume that when a $\sum$ agent switches strategy, he learns, by imitation or education, to measure how well he is playing with the newly adopted threshold function. Inside the $\sum$ agents of population k, strategy $i \in S_k = \{1, 2, \ldots, m_k\}$ will be played by those dissatisfied $\sum$ agents kj, $j \neq i$, coming from the rest of $m_k - 1$ strategies (the inflow): $\frac{1}{m_k-1}\sum_{j\neq i}^{m_k} f_{kj}\left(1 - \frac{1}{\lambda_{kj}}\right)$. The outflow is the proportion of dissatisfied $\sum$ agents ki who abandon the strategy i : $f_{ki}\left(1 - \frac{1}{\lambda_{ki}}\right)$. We shall assume that the drift term added to the ki-th selection dynamics equation (see Eq. (5) below) is the difference between these two flows. Hence, those who retain their current strategy are not included in the drift term. Therefore, we would have

$$\frac{1}{m_k - 1}\sum_{j\neq i}^{m_k} f_{kj}\left(1 - \frac{1}{\lambda_{kj}}\right) - f_{ki}\left(1 - \frac{1}{\lambda_{ki}}\right).$$

If we simplify the notation by denoting $\theta_{ki}(f) = \frac{1}{m_k-1}\sum_{j\neq i}^{m_k} f_{kj}\left(1 - \frac{1}{\lambda_{kj}}\right) + f_{ki}\frac{1}{\lambda_{ki}}$ then, the drift term takes the following form:

$$[\theta_{ki}(f) - f_{ki}]. \tag{3}$$

It is worth mentioning that θ_{ki} in Eq. (3) is not exogenously given as in B&S and the noise models of Hopkins (2002). B&S interpret $\theta_{ki}(f)$ as mistake probabilities that might reflect rules of thumb and favour the assumption that each $\theta_{ki}(f)$ is fixed and positive. For instance, the specification of $\theta_{ki}(f)$ could reflect the rule of calculations based on a uniform distribution over strategies. The assumption implies that drift will point into the state space and this could become problematic because it prevents exact convergence on the equilibria observed in the laboratory. In the present model, $\theta_{ki}(f) = \frac{1}{m_k-1}\sum_{j\neq i}^{m_k} f_{kj}\left(1 - \frac{1}{\lambda_{ki}}\right) + f_{ki}\frac{1}{\lambda_{ki}}$ is endogenous, so drift is not necessarily inward-pointing, therefore the problem of approximate convergence and quantitative matching with laboratory data can be avoided.

3.5. The connection between threshold function, socially induced similarity relations and drift

We have two limiting values of drift. If the $\sum$ agent ki is almost completely certain that he is playing well (alert mode) then no matter what the values of $p_{ki}(f)$ and f_{ki} are, the similarity intervals and the indifference sets will have almost an empty interior; thus $(p_{ki}(f), f_{ki})$ is "near" the agent's aspiration set. In other words, the agent is very satisfied with his current strategy and λ_{ki} will have very small values. Hence, the probability $1-\frac{1}{\lambda_{ki}}$ of switching, which is the source of drift, will be negligible. If the $\sum$ agent ki were in the absent mode then, the above implications will be reversed; his indifference sets would cover almost the entire space $\Pi_{ki}\times F_{ki}$ and the agent could be said to be "highly dissatisfied" with his current strategy.

Payoffs and strategy proportions will play an active role in determining the value of λ_{ki} and therefore the level of drift when the agent is outside the two playing modes (Fig. 1 depicts this case). In this case, increases in f_{ki} and $p_{ki}(f)$ imply a decrease in drift.

4. Drift based on AINU's similarity relations

Now we shall deal with similarity relations that are not socially induced. To this end, we use the correlated similarity relations introduced by AINU for the space of simple lotteries. It is assumed that each $\sum$ agent feels a constant level of vagueness about how well he is playing. That is, we assume that the $\sum$ agents of each player role $k\in K$ is endowed with a threshold function g_k, such that, for every $i\in S_k$,

$$g_k : F_{ki} \rightarrow [0,1], \quad \text{with } g_k(f_{ki}) = \varepsilon_\kappa, \quad \text{for all } f_{ki}\in[0,1], \quad \text{being } \varepsilon_\kappa\in[0,1).$$

Hence, the threshold function that measures the ambiguity felt by the $\sum$ agent ki is not sensitive to the proportions f_{ki} (thus, the agent is not influenced by the social norms and conventions that those proportions might convey). Assuming that each $\sum$ agent ki is endowed with the constant function g_k, a new function ψ_{ki} is defined as follows: for all $p_{ki}(f)>\epsilon_k>0$,

$$\psi_{ki}(p_{ki}(f)) = \frac{p_{ki}(f)}{p_{ki}(f)-\varepsilon_k} > 1. \tag{4}$$

Notice that the λ_{ki} function defined in (2) was parameterized by f_{ki}, so that there is one λ_{ki} for each $f_{ki}\in(0,1)$. This is not the case for the ψ_{ki} function because ϵ_k is constant. With the aid of g_k and ψ_{ki}, we can define similarity relations on the payoff and the strategy proportion spaces respectively. On Π_{ki} we define similarity relations of the difference type, for which the similarity intervals are $[p_{ki}(f)-\epsilon_k, p_{ki}(f)+\epsilon_k]$. On F_{ki} we define correlated similarities of the ratio type. With a choice procedure (described in Appendix I), we can build a preference relation on $\Pi_{ki}\times F_{ki}$ depicted in Fig. 2. Note that for a given payoff $p_{ki}(f)$, the size of the indifference set and, therefore, the drift level introduced by the $\sum$ agent ki depends on ϵ_k. Since $\partial\psi_{ki}/\partial p_{ki}(f)<0$, then if the expected payoff increases, the size of the indifference set shrinks and the level of drift decreases.

We proceed as in the previous case to build the drift term and we find

$$\frac{1}{m_k-1}\sum_{j\neq 1}^{m_k} f_{kj}\left(1-\frac{1}{\psi_{kj}}\right) - f_{ki}\left(1-\frac{1}{\psi_{ki}}\right) = \varepsilon_k\left[\frac{1}{m_k-1}\sum_{j\neq 1}^{m_k}\left(\frac{f_{kj}}{p_{kj}(f)}\right) - \frac{f_{ki}}{p_{ki}(f)}\right].$$

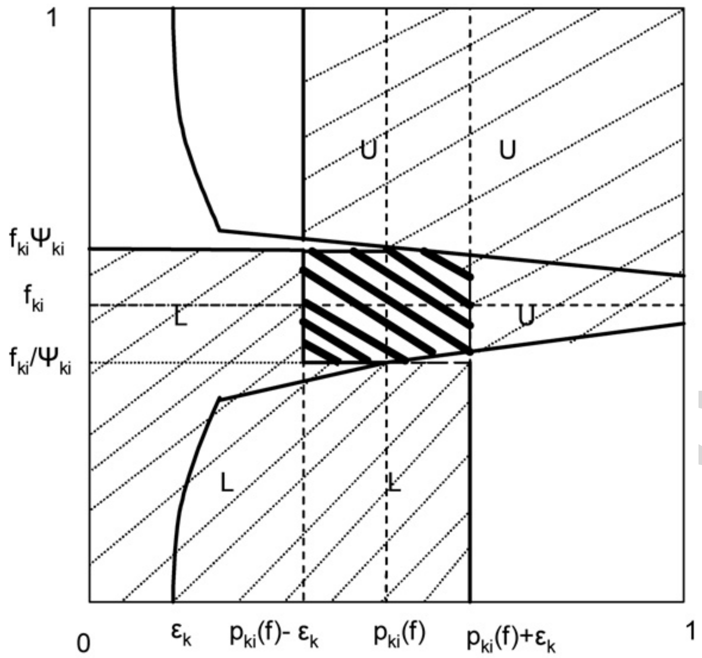
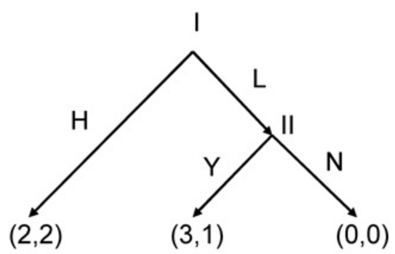


Fig. 2. The AINU $\sum$ preference $\succsim_{ki}$. Given the parameters ϵ_{I}, ϵ_{II} and the vector $(p_{ki}(f), f_{ki})$, the upper and lower contour sets, obtained by a procedure based on a pair of AINU similarity relations, are denoted by U and L, respectively. The indifference set is the darkest area.

5. Results

We present now the results that may be obtained with the two models of drift described in the previous sections. The results refer to the drift specifications needed to stabilize Nash equilibria, in particular, those that are not subgame-perfect.

To this end, we shall consider first a simplified version of the ultimatum game (UG) (see Binmore et al. (1995)) whose strategic form is described in Fig. 3. Player I is the population of Proposers with two available strategies: to propose a high division (H) or a low division (L) of a cake of size 4. Player population II, the responders, when they are offered the low division, may choose between accepting it (Y) or rejecting it (N). The chain-store game has the same structure as the ultimatum minigame but is of a different economic nature. The empirical results



	(y) Y	N
(x) H	2,2	2 ,2
L	3,1	0,0

Fig. 3. The extensive and strategic forms of the ultimatum minigame.

of these two games suggest that different interpretations of (formally) the same game, a game of entry-deterrence or a bargaining game, may have different observed behaviours.

5.1. Drift produced by socially induced similarity relations with $\sum$ agents in playing modes

When the perception of a game is influenced by individual values or by social norms and conventions, it might happen that different strategies in that game are a priori valued differently (i.e. before the game and during the play); therefore the tendencies to abandon them might differ from one individual to another. To capture this situation, we assume that each $\sum$ agent plays his current strategy in a given playing mode. Note that given a game, the existence of a playing mode attached to a strategy must be deduced, in our opinion, from the data obtained in the laboratory about that game as well as from the knowledge of society's modal tastes and values. In other words, the specification of the playing modes must be empirically determined.

For concreteness, the selection dynamics of this section are assumed to be the standard replicator dynamics (RD). The resulting perturbed deterministic RD, derived by the joint behaviour of the SD – *ki* agents (whose behaviour leads to the RD) and the $\sum$ – *ki* agents, will therefore be

$$\dot{f}_{ki} = f_{ki}(\pi_{ki}(f) - \bar{\pi}_k(f)) + [\theta_{ki}(f) - f_{ki}]. \tag{5}$$

Note that for each player-population k

$$\sum_{i=1}^{m_k} \dot{f}_{ki} = \sum_{i=1}^{m_k} f_{ki}(\pi_{ki}(f) - \bar{\pi}_k(f)) + \sum_{i=1}^{m_k} [\theta_{ki}(f) - f_{ki}] = 0.$$

Example 1 *(The ultimatum minigame (UM)).* Let the probabilities of playing H and Y be denoted as x and y, respectively. This game has a unique subgame-perfect equilibrium $(x, y) = (0, 1)$ and a component of Nash equilibria, denoted NC, the segment joining (1, 0) and (1, 2/3). Let λ_H, λ_L, λ_Y, λ_N, d_H, d_L, d_Y and d_N denote the threshold functions of the $\sum$ agents playing strategy High, Low, Yes and No, respectively. Then, the perturbed system (5) for the UM is the following (time index suppressed):

$$\begin{aligned}\dot{x} &= x(1-x)(2-3y) - x\left(1 - \frac{1}{\lambda_H}\right) + (1-x)\left(1 - \frac{1}{\lambda_L}\right)\\ &= x(1-x)(2-3y) - \frac{x d_H(x)}{p_H(y)} + \frac{(1-x)d_L(1-x)}{p_L(y)}.\end{aligned} \tag{6}$$

$$\begin{aligned}\dot{y} &= y(1-y)(1-x) - y\left(1 - \frac{1}{\lambda_Y}\right) + (1-y)\left(1 - \frac{1}{\lambda_N}\right)\\ &= y(1-y)(1-x) - \frac{y d_Y(y)}{p_Y(x)} + \frac{(1-y)d_N(1-y)}{p_N(x)}.\end{aligned} \tag{7}$$

The experimental findings about the UG are very robust (however, see also the findings of Güth et al., 2001) and show that people share a common notion about what is a fair, reasonable or acceptable offer and that their play is largely guided by those notions. How is this result interpreted in terms of our model? Let us suppose that someone's behaviour is guided by the following norm: "Be magnanimous and learn to say no to injustice". Then, our model would capture this (pregame) attitude by saying that this person would very likely play High (in the role of proposer) and No (in the role of responder) in the alert mode. The robustness of the experimental findings about the UM seems to suggest that a high percentage of people are inequity averse and have an priory idea of what is the right way of playing the UG. This would mean, in terms of the present model, that they would probably play the fair strategies in the alert mode, but we may think as well that some agents, mainly proposers, might initially experiment with strategies that are not fair, just to see how the opponent reacts. We conjecture that those agents will play those strategies knowing, in advance, that sooner or later they must abandon them. In other words, they will play in the absent mode.

Case I of Proposition 1 shows that, even if initially there is a very small percentage of $\sum$ agents playing H and N in the alert mode and the rest of agents in both populations are in the absent mode, $(1, 0) \in$ NC will be the only asymptotically stable outcome. In other words, if the initial play for the perturbed system (6) and (7) is arbitrarily near, say, the subgame-perfect equilibrium, (0, 1), where there is only a small percentage of highly fairness-motivated $\sum$ agents in both player populations, the theorem shows that both the SD agents (now the replicator dynamic agents) and the $\sum$ agents learn to coordinate in the non-perfect equilibrium where all proposers choose H and all responders N.

Example 2 *(The chain-store game (CH-S)).* The UM and CH-S games describe different economic situations and therefore the drift terms need not be the same. We shall assume, for simplicity, that both games have the payoffs of Fig. 3. Player I is now the potential entrant and player II the incumbent (Monopolist). Thus, change in Fig. 3, H and L for NE (Not Enter) and E (Enter), respectively; Y (Yield) and F (Fight) are now the strategies for player II. Hence, the CH-S game would correspond to the (only) weak monopolist game of Jung et al. (1994) in which the incumbent would prefer to share the market if entry occurred. We may conjecture two different situations modelled by two different specifications of drift. For instance, let us consider the case when potential entrant $\sum$ agents playing NE are in the alert mode, those playing E are in the absent mode (that is being action E riskier, agents playing E have a high uncertainty about how well they are playing) and all $\sum$ incumbent agents, i.e. those playing Y and F, are in the absent mode or almost in that mode. Thus, $\sum$ incumbents think to know the trade well, overestimate their power and do not care about their play. Let x denote the proportion of potential entrants playing NE and y the proportion of incumbents playing Y. Then, in Proposition 1, *Case II*, below, we get (1, 1/2) as a global asymptotic attractor.

The next situation would approach the case of experienced players with sufficient time and learning without the experimenter-induced strong monopolist of Jung et al. The appropriate specification of drift for this situation could be that both potential entrants playing E and incumbents playing Y are in the alert mode, while the rest of agents in both populations are in the absent mode. Then, in *Case III* of Proposition 1, we show that the subgame-perfect equilibrium is a global asymptotic attractor (and elements of NC are not local attractors). The next result is a full stability study of these two games.

Proposition 1. *Case I. Suppose in the UM game that the* $\sum$ *H agents (i.e.* $\sum$ *proposers offering H) are in the alert mode (so* $d_{\mathrm{H}} = \bar{d}$*) and the* $\sum$ *L agents are in the absent mode (*$d_{\mathrm{L}} = \underline{d}$*). Then, if responders playing Y are in the absent mode (*$d_{\mathrm{Y}} = \underline{d}$*) and those playing N are in the alert mode (*$d_{\mathrm{N}} = \bar{d}$*), the only asymptotic attractor is the equal-split Nash equilibrium* (1, 0).

Case II. Suppose in the CH-S game that the $\sum$ *NE agents are in the alert mode (*$d_{\mathrm{NE}} = \bar{d}$*) and the* $\sum$ *E agents are in the absent mode (*$d_{\mathrm{E}} = \underline{d}$*). Then, if both* $\sum$ *agents Y and F are almost in the absent mode and* $d_{\mathrm{Y}}(\cdot) = d_{\mathrm{F}}(\cdot)$*, the only asymptotic attractor is* (1, 1/2).

Case III. Suppose in the CH-S game that the $\sum$ *NE agents are in the absent mode (so* $d_{\mathrm{NE}} = \underline{d}$*) and the* $\sum$ *E agents are in the alert mode (*$d_{\mathrm{E}} = \bar{d}$*). Then, if* $d_{\mathrm{Y}} = \bar{d}$ *and* $d_{\mathrm{F}} = \underline{d}$*, the only asymptotic attractor is the subgame-perfect equilibrium* (0, 1).

Proof. In Appendix II (available in the JEBO website). □

Remark 2. It is easy to see that the playing modes model of drift can result in stabilized Nash equilibria in components with empty interior (see, for instance, the game shown in p. 382 of B&S). Hence, contrary to the B&S model, predictions could be based on arguments that do not depend on the size of the Nash component (for the notion of "size of a component", see B&S, Section 6, p. 378).

Remark 3. The assumption of *Case I* that the profile of strategies that are a Nash equilibrium are played in the alert mode and those outside the profile are played in the absent mode can be weakened. If we only assume that the strategies that are a Nash equilibrium are played in the alert mode, then we would get two asymptotic attractors (1, 0) and the subgame-perfect equilibrium (0, 1).

5.2. Drift produced when the $\sum$ *agents are outside the playing modes and the use of laboratory data*

It could be said about the previous model that, in some cases, it uses ad hoc assumptions about how the playing modes are assigned to strategies. To avoid this issue, we propose now two methods (of course, they are not the only ones) to determine the d_{ki} functions in D by using laboratory data. We could say that in this way the $\sum$ agents are endowed with an endogenously determined threshold function d_{ki}, as the result of a process of interactive learning.

- *Method I*: We should note first that in the set D, for any $r_{ki} \in [0,1]$, $d_{ki} = r_{ki}\bar{d} + (1 - r_{ki})\underline{d} \in D$. Then, we can use the laboratory data about f_{ki} at each round to give values to r_{ki} and estimate the d_{ki} of each $\sum$ agent ki. Thus, for instance in the UM Game, d_{H} and d_{L} could be defined as: $d_{\mathrm{H}}(x) = x\bar{d}(x) + (1 - x)\underline{d}(x)$ and $d_{\mathrm{L}}(1 - x) = (1 - x)\bar{d}(1 - x) + x\underline{d}(1 - x)$; d_{Y} and d_{N} are defined in a similar manner. An alternative definition posits that $d_{ki}(f_{ki}) = f_{ki}\bar{d}(s) + (1 - f_{ki})\underline{d}(s)$, for some fixed $s \in (0, 1)$, but this would not imply any change in Proposition 2 below.
- *Method II*: A more interesting approach seems to be when we use both f_{ki} and the data about the expected payoffs $p_{ki}(f)$ of each round to determine the d_{ki} functions (in the lab we would use the realized payoffs). We define the threshold functions as $d_{ki} = [f_{ki}\bar{d} + (1 - f_{ki})\underline{d}]^{\beta(p_{ki}(f))}$ and assume that the degree of alertness increases with payoffs. That is, $\partial\beta(p_{ki}(f))/\partial p_{ki}(f) > 0$, so that when $p_{ki}(f)$ approaches 1, $d_{ki}(f_{ki})$ approaches $\bar{d}$ and the $\sum$ agent ki would be near the alert mode, and when $p_{ki}(f)$ tends to 0 the agent would be near the absent mode.

The equation system for the UM and CH-S games is now

$$\dot{x} = x(1 - x)(2 - 3y) - \frac{x d_{\mathrm{H}}(x)}{p_{\mathrm{H}}(y)} + \frac{(1 - x)d_{\mathrm{L}}(1 - x)}{p_{\mathrm{L}}(y)}. \tag{8}$$

$$\dot{y} = y(1 - y)(1 - x) - \frac{y d_{\mathrm{Y}}(y)}{p_{\mathrm{Y}}(x)} + \frac{(1 - y)d_{\mathrm{N}}(1 - y)}{p_{\mathrm{N}}(x)}. \tag{9}$$

The next result shows that the perturbed system resulting from Method I has the same properties as the unperturbed replicator dynamics.

Proposition 2. *If each* $\sum$ *agent ki is endowed with a threshold function defined as* $d_{ki}(f_{ki}) = f_{ki}\bar{d} + (1 - f_{ki})\underline{d}$ *then the subgame-perfect equilibrium* (0, 1) *is the only asymptotic attractor of the system* (8) *and* (9) *and each element of the interior of NC is a local attractor*.

Proof. In Appendix II. □

Proposition 3. *If each* $\sum$ *agent ki is endowed with a threshold function defined as* $d_{ki} = [f_{ki}\bar{d} + (1 - f_{ki})\underline{d}]^{\beta(p_{ki}(f))}$ *with* $\partial\beta(p_{ki}(f))/\partial p_{ki}(f) > 0$ *then*

(a) *if* d_{H}, d_{L}, d_{Y} *and* d_{N} *are convex functions*, *the asymptotic attractors are* (0, 1) *and* (1, 0), *and*
(b) *if* d_{H} *and* d_{L}, *are convex and* d_{Y} *and* d_{N} *are concave functions*, *the asymptotic attractors are* (0, 1) *and* (1, 1/2).

Proof. In Appendix II. □

5.3. Drift based on AINU's type of similarity relations

Let $\epsilon_{\mathrm{I}}, \epsilon_{\mathrm{II}} \in [0, 1)$ denote the constant levels of vagueness or uncertainty about how well they are playing felt by proposers and responders, respectively. Then, the system would be

$$\dot{x} = x(1 - x)(2 - 3y) + \varepsilon_{\mathrm{I}} \left[\frac{-x}{p_{\mathrm{H}}(y)} + \frac{1 - x}{p_{\mathrm{L}}(y)} \right]. \tag{10}$$

$$\dot{y} = y(1 - y)(1 - x) + \varepsilon_{\mathrm{II}} \left[\frac{-y}{p_{\mathrm{Y}}(x)} + \frac{1 - y}{p_{\mathrm{N}}(x)} \right]. \tag{11}$$

Note that the noise introduced by player k increases with ϵ_k. We have the following result, described by Fig. 4.

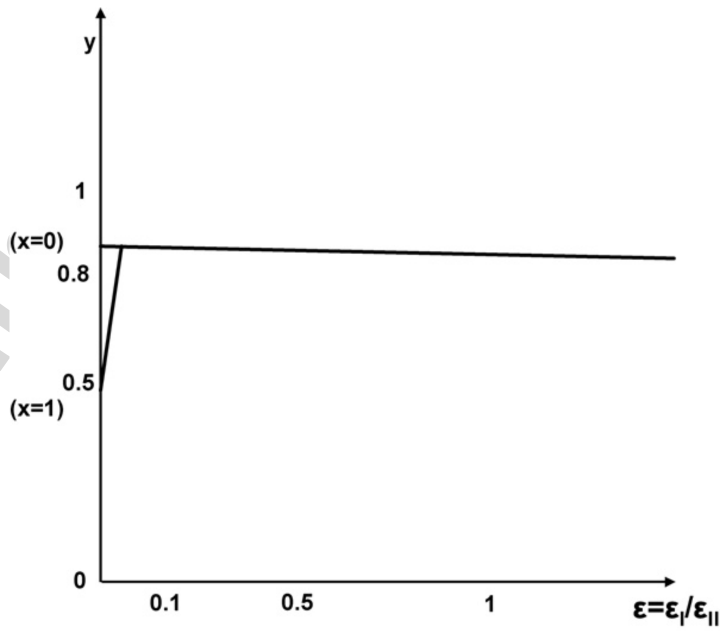


Fig. 4. The ϵ-correspondence shows the asymptotic attractors for each value of $\epsilon_{\mathrm{I}}/\epsilon_{\mathrm{II}}$. The figure assumes $\epsilon_{\mathrm{II}} = 0.1$ and ϵ_{I} taking values from 0 onwards.

Proposition 4. *For values of* $\epsilon_{\mathrm{I}}/\epsilon_{\mathrm{II}}$ *greater than* 0, *the system* (10) and (11) *has a unique asymptotic attractor located in the vicinity of the subgame-perfect equilibrium* $(x, y) = (0, 1)$. *As the ratio* $\epsilon_{\mathrm{I}}/\epsilon_{\mathrm{II}}$ *approaches* 0, *there is an additional asymptotic attractor, which in the limit, when* $\epsilon_{\mathrm{I}}/\epsilon_{\mathrm{II}} = 0$, *is the element* $(x, y) = (1, 1/2)$.

Proof. In Appendix II. □

6. Relation with the literature

One might find some resemblances between the theory of this paper and that of the quantal response equilibrium (QRE). In the latter, players make correct estimates of expected payoffs but have an additive payoff disturbance (or error). Here, the $\sum$ agents too make correct estimates of the expected payoffs, but since they have doubts about how well they are playing the game, the estimated value is not distinguished from those on a similarity interval. In the QRE, experience implies a decrease in the errors. The similarity theory used in the present paper assumes as well that an increase in the number of $\sum$ agents ki is equivalent to an increase in experience (and therefore the size of the similarity interval of expected payoff $P_{ki}(f)$ decreases) and that if the payoffs at stake increase, perturbations decreases. We find more coincidences between the QRE and the model of drift based on AINU's type of similarity relations that imply the above system (10) and (11). Recall that ϵ_k, $(k = \mathrm{I, II})$, is a parameter that measures the vagueness felt by player k about how well he is playing; this parameter determines the size of the similarity interval of the expected payoffs to each strategy i available to k: $[p_{ki}(f) - \epsilon_k, p_{ki}(f) + \epsilon_k]$. This is a "noisy interval" for both the QRE theory and the present paper's theory. Hence, the ratio $\epsilon = \epsilon_{\mathrm{I}}/\epsilon_{\mathrm{II}}$ would be the relative noise between proposers and responders in the payoff space. In the system (10) and (11) we also see that the value of ϵ_k, $(k = \mathrm{I, II})$, determines the degree of influence of drift upon the replicator dynamic equation for each pure strategy of player k. In other words, ϵ_k is also a parameter measuring the sensitivity of player k's replicator equations to the noise produced by each $\sum$ agent ki. This feature has some vague resemblance with the parameter λ of the logit QRE that measures the sensitivity of the response function to the level of error.

Needless to say that the main difference between the two models is the dynamic approach taken here, but if we concentrate on the results, we find, under some conditions, coincidences between the logit equilibria and the limit points of some of the models of drift presented above. McKelvey and Palfrey (1998) study the quantal response equilibrium of two extensive versions of the chain-store paradox game, the extensive (i.e. sequential) version and, what they call, the "strategy" version, having both the same normal form with the payoffs of the above Fig. 3. The purpose is to test the invariance property. When studying the features of the Agent QRE correspondence (for the probabilities of each action) as a function of λ, they find that it is the AQRE that converges to the subgame-perfect equilibrium and, as opposed to the extensive version, in the strategy version for a large λ, there is an additional QRE that converges on the imperfect equilibrium $x = 1$, $y = 1/2$ (recall that x is the probability of NE and y is the probability of Y). The present paper assumes, as usual, that the invariance property is satisfied. Let us turn our attention to Proposition 4 and to the ϵ-correspondence of Fig. 4 that summarizes this result. The ϵ-correspondence shows, for each ϵ, the asymptotic attractors of the system (10) and (11). For high values of ϵ, there is only one limit point which is near $(x, y) = (0, 1)$, depending on the values of ϵ_{I} and ϵ_{II}, but as ϵ goes to 0, the system converges in the limit, (i.e. when $\epsilon_{\mathrm{I}} = 0$), to $x = 1$, $y = 1/2$, as well as to (an approximation of) the subgame-perfect equilibrium. When $\epsilon_{\mathrm{I}} = \epsilon_{\mathrm{II}} = 0$ the system (10) and (11) is reduced to the replicator dynamic equations and hence the limit point will be (0, 1).

Let us look now to the model of drift produced by socially induced similarity relations with $\sum$ agents in playing modes. In *Case II* of Proposition 1, $x = 1$, $y = 1/2$ appears again as an asymptotic attractor (of the system (6) and (7)). How is this result explained? McKelvey and Palfrey (1998) find empirical support to their result in Schotter et al. (1994) and rationalize it in terms of plausibility, in the sense that it is more likely to observe that player 2 perceives the suboptimality of F only when player 1 has chosen E (see p. 19 of McKelvey and Palfrey, 1998). We instead explain the result in terms of relative drift. As we said above, in *Case II* we assume that incumbents overestimate their market power and play without taking too much care, while potential entrants take a lot of care and pay much more attention to their decisions. In other words, they are more alert (and hence less noisy) than incumbents. Therefore, under the assumptions of *Case II*, perturbations in the entrant population introduce NE much more frequently than E, while in the incumbent population the $\sum$ agents are assumed to be equally noisy so that perturbations introduce both Y and F with the same frequency. Hence, as the frequency of NE increases, drift approaches the component NC, where Y and F get equal payoff. Then, the state of NC which will be stabilized depends on the relative sharpness of d_{Y} and d_{F} and, since in *Case II* it is assumed that they are equally sharp, the stabilized state will be (1, 1/2). We have seen in Proposition 4 that responders should be noisier than proposers to stabilize $x = 1$, $y = 1/2$. This happen when the relative noise, measured by $\epsilon_{\mathrm{I}} = \epsilon_{\mathrm{I}}/\epsilon_{\mathrm{II}}$, is 0.

Therefore, we can conclude that the equilibria obtained in the chain-store game with both the model of drift based on AINU's type of similarity relations and the model of drift with $\sum$ agents in playing modes might coincide, under some conditions, with those of the logit equilibria.

It is also worth noting that Binmore et al. (1995), assuming endogenous drift and uniform mistake probabilities for both populations, i.e. $\theta_{ki}(f) = 1/2$ for each k and each i, show, in their Proposition 3, that the asymptotically stable outcomes are the subgame-perfect equilibrium, (0, 1) and, due to the assumption of fixed and uniform mistakes (which implies inward pointing drift, an approximation to (1, 1/2). A key element in Binmore et al.'s (1995) Proposition 3 is that responders should drift more than proposers, and this can happen only if drift is sufficiently sensitive to payoffs. Similarly, Proposition 4 shows that to stabilize (1, 1/2), the ratio of noise $\epsilon = \epsilon_{\mathrm{I}}/\epsilon_{\mathrm{II}}$ should be 0.

Finally, *Case I* of Proposition 1 shows the conditions to stabilize the state (1, 0) and be obtained as the only asymptotically stable outcome. This case is related to the work of Abbink et al. (2001). We may say, in their words, that we are dealing with fairness motivated agents, loyal to the strategy that would implement the equal split equilibrium. Thus, in our model $\sum$ responders playing No would be "programmed" to punish unfair offers. The result of *Case I* predicts learning in both populations, whereas in Abbink et al. there is only evidence for first movers learning.

In Uriarte (2005) we present numerical simulations to observe the degree of quantitative matching of the above models with the experimental results about the (full) ultimatum game of Roth et al. (1991) and Roth and Erev (1995).

7. Conclusions

We have completed a (biologically based) selection dynamic model by adding different models of drift originated by agents whose behaviour is based on decision procedures compatible with similarity relations. We have shown that the addition of this type of behaviour, studied by authors like Kahneman and Tver-sky, has positive implications. With a threshold function that measures the ambiguity felt by each perturbing agent about how well he is playing, we built three models of drift based on socially induced similarity relations. The playing modes model of drift combined with the replicator dynamics (RD) seems to fit well to explain the influence of norms and conventions, such as fairness, but it could be said that the model makes a certain use of ad hoc assumptions. To avoid this issue we endogenized the threshold functions by using the data about payoffs and strategy proportions and obtained two additional models. We showed that one of them is capable to stabilize equilibria that are not subgame-perfect. If similarity relations are not socially induced, we get the AINU model of drift. This last model shows resemblances with the QRE model of McKelvey and Palfrey (1995) and stabilizes the same equilibria as McKelvey and Palfrey (1998) in the chain-store game. In Uriarte (2005) it is shown how the different models of drift perform with the (full) ultimatum game of Roth et al. (1991) and Roth and Erev (1995).

We deduce that the failure of Binmore and Samuelson's model to match the observed data could be the sensitivity of drift to payoffs only and to being inward pointing (see Section 6 above). In a different terrain, the results mitigate somehow Cheung and Friedman's (1998) disappointing tests with the (unperturbed) RD.

There are things to be done in future works. One, in particular, is that the knowledge of "how many subjects are playing like me" influences an agent's decisions should be tested (after each round in the experiment, every subject should be given information about the proportion f_{ki} of people in his population who have used the same strategy as his current one). This may help us to understand the building of new conventions or the robustness (i.e. survival) of "old" individual and social values to evolutionary pressures. More work left for future research could use the two classes of similarity relations mentioned here to build, instead of drift, different models of selection dynamics.

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Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at doi:10.1016/j.jebo.2005.06.009.

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