On the Informational Role of Term Structure in the U.S. Monetary Policy Rule*

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Abstract

This paper uses a structural approach based on the indirect inference principle to estimate a standard version of the new Keynesian monetary (NKM) model augmented with term structure using both revised and real-time data. The estimation results show that the term spread and policy inertia are both important determinants of the U.S. estimated monetary policy rule whereas the persistence of shocks plays a small but significant role when revised and real-time data of output and inflation are both considered. More importantly, the relative importance of term spread and persistent shocks in the policy rule and the shock transmission mechanism drastically change when it is taken into account that real-time data are not well behaved.

JEL classification numbers: C32, E30, E52

Key words: NKM model, term structure, monetary policy rule, indirect inference, real-time and revised data.

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1 INTRODUCTION

There is currently a fast-growing body of literature (see, for instance, Hördahl, Tristani and Vestin (2006), Rudebusch and Wu (2008) and references cited therein) that aims to link the New Keynesian Monetary (NKM) model dynamics with the term structure of interest rates.[1] Most papers in this literature assume a sort of dichotomy where the three-equation NKM model is solved first and independently from term structure, i.e. they consider no feedback from term structure to the macroeconomy. An exception is Nimark (2008) which considers that policy makers may take into account the information revealed by the term structure about the expectations of bond market participants on the future of the economy. In a similar vein and using little macroeconomic structure, Ang, Dong and Piazzesi (2005) consider a single latent factor interpreted as a transformation of Fed policy actions on the short-term rate. In their model, persistent policy shocks are allowed but policy inertia is not.

Another branch of literature (see, for instance, Clarida, Galí and Gertler (2000)) has found empirical evidence that the lagged interest rate is a key component in estimated monetary policy rules. Two alternative interpretations have been proposed in the relevant literature. On the one hand, the significant role of the lagged interest rate may reflect the existence of a traditional concern of central banks for the stability of financial markets (see Goodfriend (1991)). On the other hand, Rudebusch (2002) argues that the significance of the lagged rate in estimated rules is due to the existence of relevant omitted variables. This is because it is hard to reconcile the lack of evidence on the predictive power of the term structure for future values of the short-term interest rate with the existence of policy inertia. Moreover, the existence of omitted variables may result in persistent monetary shocks in estimated rules.[2]

The aim of this paper is to analyze the role of the term spread in the U.S. estimated policy rule by bridging the gap between these two branches of literature. We build on the first branch by estimating the policy rule of an NKM model augmented with term structure using a classical structural approach based on the indirect inference principle suggested by Smith (1993).

Considering term structure in an otherwise standard NKM model introduces

[1] There is also a related body of literature (see, for instance, Ang and Piazzesi (2003), and Diebold, Rudebusch and Aruoba (2003)) linking macro variables to the yield curve using little or no macroeconomic structure.

[2] By using reduced-form estimation approaches some empirical studies, such as English, Nelson, and Sack (2003) and Gerlach-Kristen (2004), have shown that both policy inertia and persistent shocks enter into the estimated monetary policy rule.

two types of feature. On the one hand, it allows us to consider the term spread, in addition to output and inflation, as a potential candidate for explaining the highly persistent dynamics of the short-term policy rate. A pure informational argument to motivate the inclusion of the term spread in the policy rule is the following: a central bank may consider that real-time data on inflation and output available at the time of implementing policy are not a rational forecast of revised data. Thus, a monetary authority may consider that the term spread, which is observed in real-time, may contain relevant information about true, revised data on inflation and output that real-time data do not provide.[3] Another related argument works as follows. In practice, the Fed uses much more information than that available on output and inflation as announced by statistical agencies. This additional information may contain information on revised output and inflation (and possibly information on market expectations about these variables). This additional information is also likely to be observable by private market participants and taken into account in pricing government bonds. By acknowledging the asymmetric problems faced by both private agents and policy makers, the Fed may then view the term spread as a noisy indicator of the additional information observed in real time on revised output and inflation which is not included in its initial announcements.On the other hand, in a similar vein to Nimark (2008), the model departs from the relevant literature by allowing a dynamic feedback from the term structure of interest rates to the macroeconomy. However, our paper differs from Nimark's in many aspects. Among others, we consider both revised and real time data available at the time of implementing monetary policy whereas Nimark (2008) uses only revised data.[4]

The timing and availability of data used in the empirical evaluation of monetary policy rules have now become important issues (see, among others, Orphanides (2001), and Ghyssels, Swanson and Callan (2002)).[5] A general conclusion reached from the estimation of monetary policy rules based on real-time data is that it allows for the potential reduction of the effects of parameter uncertainty in actual policy setting, which is relevant when real-time announcements of macroeconomic variables are biased.[6]

[3] Empirical evidence found by many researchers (see, for instance, Estrella and Mishkin (1997) and Ang, Dong and Piazzesi (2005)) points out that the term spread contains useful information concerning market expectations of both future real economic activity and inflation.

[4] Moreover, as discussed below, we estimate a Taylor-type policy rule instead of the parameters involved in a central bank loss function since one of our main concerns is the analysis of the importance of persistent shocks in the policy rule once the term spread is included in the rule.

[5] A pioneering study is that of Mankiw, Runkle and Shapiro (1984), who develop a theoretical framework for analyzing preliminary announcements of economic data and apply that framework to the money stock.

[6] Arouba (2008) documents the empirical properties of revisions to major macroeconomic variables in the U.S. and points out that they are not white noise. That is, they do not satisfy simple desirable properties such as zero mean, which indicates that the revisions of initial announcements made by statistical agencies are biased, and that they are predictable using the information set at

The use of real-time data in the estimation of a structural DSGE model may look tricky because it is the decisions of private agents (households and firms) that determine the true (revised) values of macroeconomic variables, such as output and inflation, and they are not observable without error by policy makers in real time. This paper extends the NKM model to include revision processes of output and inflation data, and thus to analyze revised and real-time data together. This extension allows for (i) a joint estimation procedure of both monetary policy rule and revision process parameters, and (ii) an assessment of the interaction between these two sets of parameters.

The empirical results based on revised data for output and inflation show that the term spread plays an important and statistically significant role in the monetary policy rule. Moreover, the policy rule is characterized by both strong policy inertia and persistent policy shocks. Policy inertia and the term spread remain important when using both revised and real-time data. However, the persistence of policy shocks becomes less important, but remains significant. Furthermore, the estimates of the revision process parameters show that the initial announcements of output and inflation are not rational forecasts of revised data for output and inflation. For instance, a 1% increase in the initial announcement of inflation leads to a downward revision in inflation of 0.58%. More important, the relative importance of term spread and persistent shocks in the policy rule and the shock transmission mechanism drastically change when it is taken into account that real-time data are not well behaved.

The rest of the paper is organized as follows. Section 2 introduces the log-linearized approximation of a standard version of the NKM model augmented with term structure. Section 3 describes the structural estimation method used in this paper, motivates its use and discusses how it relates to other estimation methods, such as the Bayesian estimation strategies used in recent literature. Section 4 presents and discusses the estimation results using revised data. Section 5 extends the NKM model to consider revision processes of output and inflation data, and discusses the empirical evidence found when using revised and real-time data together in the estimation process. Section 6 provides some robustness exercises and diagnostic tests of the model. Section 7 concludes.

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the time of the initial announcement.

2 AN AUGMENTED NEW KEYNESIAN MONETARY MODEL

The model analyzed in this paper is a now-standard version of the NKM model augmented with term structure which is given by the following set of equations:

$$x_t = E_t x_{t+1} - \tau(i_t^{\{1\}} - E_t \pi_{t+1}) - \phi(1-\rho_\chi)\chi_t, \tag{1}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + z_t, \tag{2}$$

$$i_t^{\{1\}} = \rho i_{t-1}^{\{1\}} + (1-\rho)[\psi_1 \pi_t + \psi_2 x_t + \psi_3 (i_t^{\{n\}} - i_t^{\{1\}})] + v_t, \tag{3}$$

$$i_t^{\{n\}} = \frac{1}{n} \sum_{k=0}^{n-1} E_t i_{t+k}^{\{1\}} + \xi_t^{\{n\}}, \tag{4}$$

where x denotes the output gap (that is, the log-deviation of output with respect to the level of output under flexible prices) and π and $i^{\{n\}}$ denote the deviations from the steady states of inflation and nominal interest rate associated with an n-period maturity bond, respectively. E_t denotes the conditional expectation based on the agents' information set at time t. χ, z, v and $\xi^{\{n\}}$ denote aggregate productivity, inflation, monetary policy and risk premia shocks, respectively. Each of these shocks is further assumed to follow a first-order autoregressive process. $\epsilon_{\chi t}$, ϵ_{zt}, ϵ_{vt} and $\epsilon_{\xi t}^{\{n\}}$denote i.i.d. random innovations associated with these shocks, respectively.

Equation (1) is the log-linearized first-order condition obtained from the representative agents' optimization plan. Appendix 1 shows that there is a set of redundant IS curves associated with the alternative maturity bonds of the economy and, altogether, they imply that the rational expectations of the term structure in a log-linear form, given by equation (4), holds. We have introduced a risk premium shock, $\xi_t^{\{n\}}$, into the term structure, which is well justified empirically and has different impacts depending on the horizon considered.[7]

Equation (2) is the new Phillips curve that is obtained in a sticky price à la Calvo (1983) model where monopolistically competitive firms produce (a continuum of) differentiated goods and each firm faces a downward sloping demand curve for its

[7] As discussed by Ireland (2004), there is a long-standing tradition of introducing additional disturbances into DSGE models until the number of shocks equals the number of data series used in estimation. The reason is that models of this type are quite stylized and introduce fewer shocks than observable variables, which implies that models are stochastically singular. That is, the model implies that certain combinations of endogenous variables are deterministic. If these combinations do not hold in the data, any approach that attempts to estimate the complete model will fail.

produced good. The parameter $\beta \in (0,1)$ is the agent discount factor and κ measures the slope of the New Phillips curve, which is related to other structural parameters as follows

$$\kappa = \frac{[(1/\tau) + \eta](1-\omega)(1-\omega\beta)}{\omega},$$

where τ, η and ω denote the consumption intertemporal elasticity, the Frisch elasticity and Calvo's probability, respectively. In particular, κ is a decreasing function of ω. The parameter ω is a measure of the degree of nominal rigidity; a larger ω implies that fewer firms adjust prices in each period and that the expected time between price changes is longer.[8]

Equation (3) is a standard Taylor-type monetary rule where the nominal interest rate exhibits inertial behavior, captured by parameter ρ, for which two alternative interpretations are proposed in the relevant literature. On the one hand, there are several arguments suggesting that the significant role of the lagged interest rate may reflect the existence of an optimal policy inertia. These arguments range from the traditional concern of central banks for the stability of financial markets (see Goodfriend (1991)) to the more psychological one posed by Lowe and Ellis (1997), who argue that policy makers are likely to be embarrassed by reversals in the direction of interest-rate changes. On the other hand, Rudebusch (2002) argues that the significance of the lagged rate in estimated rules is due to the existence of relevant omitted variables.

The monetary policy rule (3) further assumes that the nominal interest rate responds on the one hand to output gap and inflation, and on the other hand to term spread, $i_t^{\{n\}} - i_t^{\{1\}}$. The inclusion of the term spread is motivated in this paper by acknowledging that the term spread, observed in real-time, may contain relevant information about revised data on inflation and output that real-time data on these variables do not provide.[9] Moreover, from an econometric perspective, if one accepts Rudebusch's (2002) argument that the significance of the lagged interest rate in estimated policy rules is due to the existence of relevant omitted variables, one may wonder whether the term spread is one of them.

[8] See, for instance, Walsh (2003, chapter 5.4) for detailed analytical derivations of the IS and New Phillips curves, and the definition of flexible-price equilibrium level of output, y_t^f, considered below.

[9] The inclusion of the term spread in the policy rule has been well motivated in the relevant literature (as in Laurent (1988) and McCallum (1994)). In particular, the term spread is viewed as an indicator of monetary policy looseness, so a high value of the term spread calls for corrective actions. Related to this argument for including the term spread in the policy rule is the central bank's aim of monitoring the transmission channel of monetary policy itself by trying to affect the slope of the yield curve. A look at speeches by former Fed Chairman Greenspan reveals that central banks do not seem to be able to affect the slope of the yield curve, and are frustrated by this.

Since the structural econometric approach implemented is computationally quite demanding, we consider an economy with only two bonds: a 4-period bond as the long-term bond and a 1-period bond as the short-term bond. The system of equations (1)-(4) (together with five extra identities involving forecast errors) can be written in matrix form as follows (for the sake of simplicity we further assume that the 1-period bond and the policy interest rate are the same):

$$\Gamma_0 Y_t = \Gamma_1 Y_{t-1} + \Psi \epsilon_t + \Pi \eta_t, \tag{5}$$

where

$$Y_t = (x_t, \pi_t, i_t^{\{1\}}, i_t^{\{4\}}, E_t x_{t+1}, E_t \pi_{t+1}, \chi_t, z_t, \xi_t^{\{4\}}, v_t, E_t i_{t+1}^{\{1\}}, E_t i_{t+2}^{\{1\}}, E_t i_{t+3}^{\{1\}})',$$

$$\epsilon_t = (\epsilon_{\chi t}, \epsilon_{zt}, \epsilon_{\xi t}^{\{4\}}, \epsilon_{vt})',$$

$$\begin{aligned} \eta_t \quad = \quad & (x_t - E_{t-1}[x_t], \pi_t - E_{t-1}[\pi_t], i_t^{\{1\}} - E_{t-1}[i_t^{\{1\}}], E_t[i_{t+1}^{\{1\}}] - E_{t-1}[i_{t+1}^{\{1\}}], \\ & E_t[i_{t+2}^{\{1\}}] - E_{t-1}[i_{t+2}^{\{1\}}])'. \end{aligned}$$

Equation (5) represents a linear rational expectations system that can be solved using standard routines.[10] The model's solution yields the output gap, x_t. This measure is not observable. In order to estimate the model by simulation, we have to transform the output gap into a measure that has an observable counterpart. This is quite a straightforward exercise since the log-deviation of output from its steady state can be defined as the output gap plus the (log of the) flexible-price equilibrium level of output, y_t^f, and the latter can be expressed as a linear function of the productivity shock:

$$y_t^f = \phi \chi_t,$$

where $\phi = \left[\frac{1+\eta}{(1/\tau)+\eta}\right]$. The log-deviation of output from its steady state is also unobservable. However, the growth rate of output is observable and its model counterpart is obtained from the first-difference of the log-deviation of output from its steady state.

Similarly, the solution of the model yields the deviations of inflation and the two interest rates from their respective steady states. In order to obtain the levels of inflation and nominal interest rates, we first calibrate the steady-state value of inflation as the sample mean of the inflation rate. Second, using the calibrated value of steady-state inflation and the definition of the steady-state value of the

[10]We use the solution algorithm suggested by Lubik and Schorfheide (2003).

real interest rate, we can easily compute the steady-state value of nominal interest rates. Notice that the long- and short-term interest rates are equal in the steady state. Third, the level of each nominal rate is obtained by adding the deviation (from its steady-state value) of the nominal rate to its steady-state value computed in the previous step. Finally, since a period is identified with a quarter and the two interest rates are thus measured in quarterlized values, the quarterlized interest rates are transformed into annualized values as in actual data.

3 ESTIMATION PROCEDURE

In order to estimate the structural and policy parameters of the NKM model with term structure we follow the *indirect inference* principle proposed by Smith (1993), which considers a VAR representation as the auxiliary model. María-Dolores and Vázquez (2006, 2008) are recent applications of this estimation strategy in the context of NKM models. More precisely, we first estimate an unrestricted VAR with four lags in order to summarize the joint dynamics exhibited by U.S. quarterly data on output growth, inflation, the Fed funds rate and the 1-year Treasury rate. The lag length considered is quite reasonable when using quarterly data. Second, we apply the simulated moments estimator (SME) suggested by Lee and Ingram (1991) and Duffie and Singleton (1993) to estimate the underlying structural and policy parameters of the NKM model. In this vein, Rotemberg and Woodford (1997), and Christiano, Eichenbaum and Evans (2005), among many others, use a minimum distance estimator based on impulse-response functions instead of VAR coefficients.

This estimation procedure starts by constructing a $p \times 1$ vector with the coefficients of the VAR representation and the standard deviations of inflation and output growth obtained from actual data, denoted by $H_T(\theta_0)$, where p in this application is 80. We have 68 coefficients from a four-lag, four-variable system, 10 coefficients from the non-redundant elements of the variance-covariance matrix of the VAR residuals and two extra moments capturing the volatility of output growth and inflation. We have added these two volatility statistics in order to take into account the units of measurement. T denotes the length of the time series data, and θ is a $k \times 1$ vector whose components are the model parameters. The true parameter values are denoted by θ_0.

Since our main goal is to estimate policy rule parameters, prior to estimation we split the model parameters into two groups. The first group is formed by the pre-assigned structural parameters β, τ, η and ω. We set $\beta = 0.99$, $\tau = 0.5$, $\eta = 2.0$ and

$\omega = 0.75$, corresponding to standard values assumed in the relevant literature for the discount factor, consumption intertemporal elasticity, the Frisch elasticity and Calvo's probability, respectively. Moreover, preliminary attempts to estimate the remaining parameters result in very large estimates of both the inflation and the term spread coefficients in the monetary policy rule (i.e. ψ_1 and ψ_3, respectively). One possible interpretation is that it is hard to identify these two parameters separately. That is, it is difficult to distinguish the responses of the Fed funds rate to term spread changes from those of its responses which are driven by inflation movements.[11] This identification issue is particularly important when trying to quantify the scale of term spread changes in Fed rate movements. For this reason, we decided to set $\psi_1 = 1.5$, i.e. the value suggested by Taylor (1993), and investigate whether the term spread still played a role in the policy rule. Below, we analyze the robustness of the empirical results by fixing $\psi_1 = 2$.[12]

The second group, formed by the remaining policy and shock parameters, is the one being estimated. In the NKM model with term structure, the estimated parameters are $\theta = (\rho, \psi_2, \psi_3, \rho_\chi, \rho_\xi^{\{4\}}, \rho_z, \rho_v, \sigma_\chi, \sigma_\xi^{\{4\}}, \sigma_z, \sigma_v)$ and then $k = 11$.[13] In order to obtain reasonable parameter estimates, we further imposed the following support intervals. The inertia and shock persistence parameters ρ, $\rho_\chi, \rho_\xi^{\{4\}}, \rho_z$ and ρ_v are all restricted to belonging to the $(0, 0.99)$ interval so that we have to deal only with a stationary solution of the model. The remaining six parameters $(\psi_2, \psi_3, \sigma_\chi, \sigma_\xi^{\{4\}}, \sigma_z, \sigma_v)$ are restricted to being positive.

As pointed out by Lee and Ingram (1991), the randomness in the estimator is derived from two sources: the randomness in the actual data and the simulation. The extent of the randomness in the simulation to the covariance matrix of the estimator is decreased by simulating the model a large number of times. For each simulation a $p \times 1$ vector of VAR coefficients, denoted by $H_{N,i}(\theta)$, is obtained from the simulated time series of output growth, inflation and the two interest rates generated from the NKM model, where $N = sT$ is the length of the simulated data. By averaging

[11] As pointed out by Canova and Sala (2009), identification problems in the estimation of DSGE models are quite widespread.

[12] We have also run our estimation procedure using other parameter values such as $\tau = 1/3$, $\eta = 3.0$ and $\omega = 0.5$. As discussed in Section 6, the qualitative conclusions of the paper are robust to alternative parameterizations. Nevertheless, the size of the term spread coefficient in the policy rule and its standard deviation change substantially depending on the parameterization chosen.

[13] We consider a six-variable VAR with four lags when estimating the extended NKM model using both revised and real-time data below. We also consider the standard deviations of real-time inflation and output growth. In this case, $p = 175$, that is, we have 150 coefficients from a four-lag, six-variable system, 21 coefficients from the non-redundant elements of the covariance matrix of the VAR residuals and 4 extra coefficients capturing the volatilities of output growth and inflation for revised and real-time data. k is 19 in the extended model (i.e. the eleven parameters of the NKM model plus eight parameters from the revisions processes of output and inflation described below).

the m realizations of the simulated coefficients, i.e., $H_N(\theta) = \frac{1}{m}\sum_{i=1}^{m} H_{Ni}(\theta)$, we obtain a measure of the expected value of these coefficients, $E(H_{Ni}(\theta))$. The choice of values for s and m deserves some attention. Gouriéroux, Renault and Touzi (2000) suggest that is important for the sample size of synthetic data to be identical to T (that is, $s = 1$) to get an identical size of finite sample bias in estimators of the auxiliary parameters computed from actual and synthetic data. We make $s = 1$ and $m = 500$ in this application. To generate simulated values of the output growth, inflation and interest rate time series we need the starting values of these variables. For the SME to be consistent, the initial values must have been drawn from a stationary distribution. In practice, to avoid the influence of starting values we generate a realization from the stochastic processes of the four variables of length $200 + T$, discard the first 200 simulated observations and use only the remaining T observations to carry out the estimation. After two hundred observations have been simulated, the influence of the initial conditions must have disappeared.

The SME of θ_0 is obtained from the minimization of a distance function of VAR coefficients from actual and synthetic data. Formally,

$$\min_{\theta}\ J_T = [H_T(\theta_0) - H_N(\theta)]' W [H_T(\theta_0) - H_N(\theta)],$$

where W is a block-diagonal weighting matrix. The first two blocks contain the inverse of the covariance matrix associated with the VAR coefficients and the non-redundant elements of the covariance matrix of the VAR residuals, respectively. The third block of W, associated with the volatilities of output growth and inflation for revised and real-time data, is the identity matrix.

Denoting the solution of the minimization problem by $\hat{\theta}$, Lee and Ingram (1991) and Duffie and Singleton (1993) prove the following results:

$$\sqrt{T}(\hat{\theta} - \theta_0) \to \mathrm{N}\left[0, \left(1 + \frac{1}{m}\right)(B'WB)^{-1}\right],$$

$$\left(1 + \frac{1}{m}\right) T J_T \to \chi^2(p - k), \tag{6}$$

where B is a full rank matrix given by $B = E(\frac{\partial H_{Ni}(\theta)}{\partial \theta})$.

The objective function J_T is minimized using the OPTMUM optimization package programmed in GAUSS language. We apply the Broyden-Fletcher-Glodfard-Shanno algorithm. To compute the covariance matrix we need to obtain B. Computation of B requires two steps: first, obtaining the numerical first derivatives of the coefficients of the VAR representation with respect to the estimates of the structural parameters θ for each of the m simulations; second, averaging the m-numerical first derivatives to get B.

At this point, the reader might be wondering: (i) why we do not estimate the NKM model directly by maximum likelihood (ML); and (ii) why we use an unrestricted VAR as the auxiliary model when implementing the indirect inference approach instead of matching structural impulse response functions as in Rotemberg and Woodford (1997). With reference to the first question, it must be stressed that the NKM model is a highly stylized model of a complex world. Therefore, ML estimation of the NKM model will impose strong restrictions which are not satisfied by the data and inferences will be misleading. We believe that one of the main virtues of the indirect inference approach is that in principle econometricians have the possibility of choosing an auxiliary model that imposes looser restrictions than ML. As regards the second question, the NKM model augmented with term structure could be approximated by a VAR. We consider an unrestricted VAR instead of matching the structural impulse responses because a reduced form VAR does not require the arbitrary identification of structural shocks. Moreover, some researchers include additional variables in order to derive 'sensible' impulse responses. For instance, to solve the so called price puzzle, a commodity price index is included in the impulse response analysis even though the NKM model says nothing about how the commodity price index is determined.

By following a classical approach, we obviously depart from papers that use a Bayesian approach. The Bayesian estimation approach operates in a different metric and under a different philosophy from frequentist estimators such as indirect inference. Fernández-Villaverde and Rubio-Ramírez (2004) claim that when Bayesian methods are used to estimate DSGE models, parameter estimates and model comparison are consistent even when models are misspecified. An important advantage of the Bayesian approach is the treatment of model uncertainty. Brock, Durlauf and West (2003) attempt to place theoretical and empirical evaluation exercises in a framework that properly accounts for different types of uncertainty and conclude that model uncertainty can be accounted for using standard Bayesian methods, making it useful for policy analysis. There are also papers that rely on the same VAR approximation as we do, but use a flexible Bayesian framework. Del Negro and Schorfheide (2004) and Del Negro, Schorfheide, Smets and Wouters (2007) derive priors from New Keynesian DSGE models for VARs and show that imposing restrictions from the DSGE model non-dogmatically on the VAR produces better results in terms of both forecastability and policymaking. The Bayesian approach suggested by Del Negro and Schorfheide (2004) and the indirect inference approach are two alternative ways (each with its pros and cons) of dealing with potential model misspecification. In this perspective, the indirect inference approach adopted in this paper can be viewed as a way of dealing with model misspecification within a classical rather than a Bayesian framework.

4 EMPIRICAL EVIDENCE

4.1 The data

We consider quarterly U.S. data for the growth rate of output, the inflation rate obtained for the implicit GDP deflator, the Fed funds rate and the 1-year Treasury constant maturity rate during the post-Volcker period (1983:1-2008:1). The choice of the 1-year rate is motivated by Nimark's (2008) finding that the information in the U.S. term structure about the state of the business cycle could be found in yields with maturities of less than one year at least when considering standard macroeconomic models. In addition, we also consider real-time data on output and inflation as reported by the Federal Reserve Bank of Philadelphia.[14] Figure 1 shows the six time series considered in the paper.

We focus on this sample period for two main reasons. First, the Taylor rule seems to fit better in this period than in the pre-Volcker era. Second, considering the pre-Volcker era opens the door to many more issues studied in the relevant literature, including the presence of macroeconomic switching regimes and the existence of breaks in monetary policy. These issues are beyond the scope of this paper.

4.2 Preliminary evidence using revised data

Table 1 shows the estimation results using revised data. The value of the goodness-of-fit statistic, $(1 + 1/m)TJ_T$, which is distributed as a $\chi^2(p-k)$,[15] confirms the hypothesis stated above that the NKM model with term structure is still too stylized to be supported by actual data. The estimation results also show that the policy rule is characterized by (i) high inertia ($\rho = 0.77$) and persistent policy shocks

[14] See Croushore and Stark (2001) for the details of the real-time data set.

[15] For the NKM model with term structure the goodness-of-fit statistic is distributed as a $\chi^2(69)$ since the number of VAR coefficients is $p = 80$ and the number of parameters being estimated is $k = 11$.

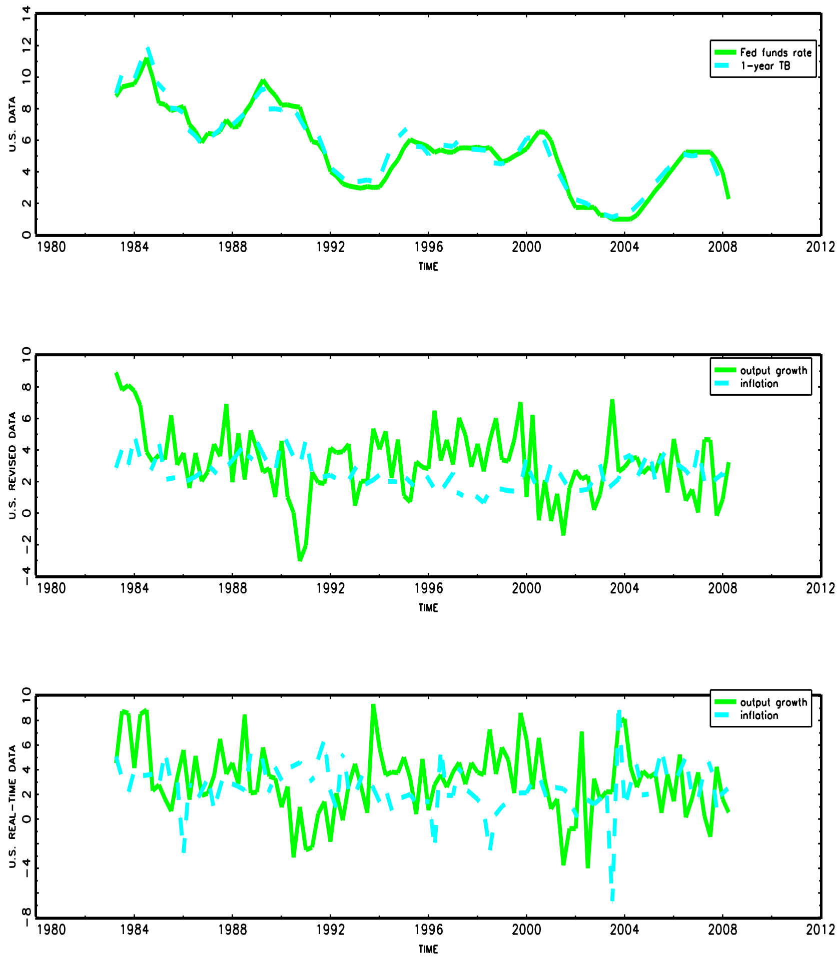

Figure 1: U.S. Time Series

$J_T(\theta)$	9.7738		
Policy parameter	Estimate	Shock parameter	Estimate
ρ	0.7693	ρ_χ	0.9411
	(0.0339)		(0.0055)
ψ_1	1.5	$\rho_\xi^{\{4\}}$	0.8736
	–		(0.0202)
ψ_2	0.0000	ρ_z	0.9886
	(0.0030)		(0.0267)
ψ_3	2.3691	σ_χ	$5.6e-03$
	(0.3670)		$(3.5e-04)$
ρ_v	0.5875	$\sigma_\xi^{\{4\}}$	$1.7e-04$
	(0.0358)		$(5.8e-05)$
σ_v	$6.7e-04$	σ_z	$4.6e-05$
	$(9.7e-05)$		$(3.8e-05)$

Table 1: NKM model with term structure using revised data.
Note: Standard errors in parentheses.

($\rho_v = 0.59$); (ii) a zero-coefficient associated with output gap and (iii) and a large and significant term spread coefficient ($\psi_3 = 2.37$). The estimates of the remaining shock parameters exhibit high persistence and low variance.

Based on a structural estimation approach, our empirical results qualitatively confirm the reduced-form estimation results obtained by English et al. (2003) and Gerlach-Kristen (2004) that policy inertia and persistent policy shocks play a role in the U.S. estimated policy rule. The next section considers real-time data on output and inflation in the estimation procedure instead of revised data.

5 ESTIMATION USING REAL-TIME DATA

We start this section by estimating the NKM model augmented with term structure using real-time data on output and inflation instead of revised data. If revisions of real-time data were rational forecast errors (i.e. zero mean, serially uncorrelated and uncorrelated with any variable belonging to the information set available at the time of the initial release of data), then the arrival of revised data would not be relevant for policy makers' decisions and policy rule estimates would be rather similar regardless of whether revised or real-time data were used. We motivate the inclusion of both term spread and real-time data in the NKM model in two steps. First, we analyze whether real-time data are rational forecasts of revised data. Second, we preliminarily support the argument that if policy makers have evidence that real-time data are not rational forecasts, they may consider that the term spread contains additional relevant information on revised data beyond that provided by real-time data of output and inflation.

Following Aruoba (2008), Table 2 shows a set of summary statistics and tests that allows us to analyze whether revision processes for output growth and inflation are 'well behaved' (i.e. are white noise processes as stated above). For both revision processes, we cannot reject the null hypothesis that the unconditional mean is null. However, on the one hand, the standard deviations for the two revision processes are quite large, especially when compared to revised data standard deviations (i.e. noise/signal parameter). On the other hand, revision processes are likely to show a first order autocorrelation pattern. The evidence that revisions are not rational forecast errors is further supported by the statistics displayed in the bottom panel of Table 2. For both output growth and inflation, revision processes are not orthogonal to their respective initial announcements, and the conditional mean is not null. Moreover, the term spread seems to play a role in explaining the revision process of output growth in addition to real-time output and inflation. This evidence is in line with the empirical evidence provided by Aruoba (2008), who finds that data revisions for these variables (and many others) are not white noise.

Before we discuss the estimation results a word of caution is in order. The estimation of the NKM model with only revised or real-time data is likely to be misspecified for two main reasons. First, IS and Phillips curves should be characterized by true (revised) inflation and output data because these two aggregate variables are the result of households' and firms' choices. Second, since real-time data are observable with a lag, the policy rule should be based on actual information available to monetary authorities at the time of implementing policy, i.e. lagged values of real-time data on output and inflation. We address these two shortcomings by considering an extended version of the NKM model below.

In spite of these shortcomings, it is nevertheless useful to estimate the NKM model using only real-time data because it is expected to deliver similar estimation results to those in Table 1 under the null hypothesis that real-time data are a rational forecast of revised data. Table 3 shows the estimation results using real-time data. Comparing the estimates of the policy rule in Tables 1 and 3, we observe

Summary Statistics

	r_t^y		r_t^π	
Mean	0.074		0.046	
Median	−0.176		0.033	
Min	−7.053		−7.273	
Max	6.343		8.940	
St. Dev	2.968		2.039	
Noise/signal	1.350		2.076	
Corr(y_t, y_t^r)	0.319		0.238	
AC(1)	−0.229	**	−0.316	**
$E(r_t) = 0$ (t-stat)	0.301		−0.302	

Conditional Mean

	r_t^y			r_t^π		
	coef.	t-stat		coef.	t-stat	
Expl. Variables						
Constant	2.702	6.186	***	2.092	11.938	***
$(y_t^r - y_{t-1}^r) * 400$	−0.798	−10.500	***	0.040	1.166	
$(\pi_t^r) * 400$	−0.091	−0.794		−0.879	−19.484	***
$(i_t^4 - i_t)$	0.937	2.366	**	−0.019	−0.132	
$F_{1(4,97)}$		38.652	***		131.753	***

Table 2: Revision process analysis. Actual data

Notes: Revisions are calculated over annual GDP growth and inflation respectively. Since revisions are likely to have a first-order autocorrelation pattern, t-statistics for testing whether unconditional means are null are calculated based on Newey-West corrected standard deviations. Noise/signal is calculated as the standard deviation of the revision over the standard deviation of the revised data. The null hypothesis for the F-test in the bottom panel (null conditional mean hypothesis) is that all coefficients for real-time information explaining the revision processes are null.

$J_T(\theta)$	7.8572		
Policy parameter	Estimate	Shock parameter	Estimate
ρ	0.7309 (0.0737)	ρ_χ	0.9479 (0.0095)
ψ_1	1.5 –	$\rho_\xi^{\{4\}}$	0.7717 (0.0540)
ψ_2	0.0000 (0.2399)	ρ_z	0.7788 (0.04925)
ψ_3	0.5402 (0.4994)	σ_χ	$7.0e-03$ $(5.5e-04)$
ρ_v	0.7183 (0.0206)	$\sigma_\xi^{\{4\}}$	$8.3e-04$ $(6.7e-05)$
σ_v	$1.3e-03$ $(3.0e-04)$	σ_z	$4.6e-04$ $(2.3e-04)$

Table 3: NKM model with term structure using real-time data

Note: Standard errors in parentheses.

two important differences that can be viewed as supporting the evidence displayed in Table 2 that data revisions are not white noise, and that this has an impact on estimated policy rules. First, the term spread coefficient is not significantly different from zero. Second, the size (σ_v) and persistence (ρ_v) of policy shocks are larger when real-time data are used.

The characteristics of the revision processes and the differences in estimated parameters when real-time and revised data are used suggest preliminary evidence that policy makers' decisions could be determined by the availability of data at the time of policy implementation. In order to account for this possibility, we modify the NKM model with term structure in three ways. First, we assume that the IS and Phillips-curve equations are described in terms of revised output and inflation data whereas the policy rule is determined by real-time data on output and inflation. Second, the initial announcement of quarterly (monthly) macroeconomic variables corresponding to a particular quarter (month) appears in the vintage of the next quarter (month), roughly 45 (at least 15) days after the end of the quarter (month). Then, a backward-looking Taylor rule that includes lagged values of real-time data on output and inflation would more accurately approximate the information set available to the Fed at the time of implementing the policy. Third, the model is extended to incorporate two ad-hoc relationships describing the revision processes of output and inflation data, respectively. Formally, the extended NKM model is described by the following set of equations

$$x_t = E_t x_{t+1} - \tau(i_t^{\{1\}} - E_t \pi_{t+1}) - \phi(1-\rho_\chi)\chi_t, \tag{7}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + z_t, \tag{8}$$

$$i_t^{\{1\}} = \rho i_{t-1}^{\{1\}} + (1-\rho)[\psi_1 \pi_{t-1}^r + \psi_2 x_{t-1}^r + \psi_3 (i_t^{\{4\}} - i_t^{\{1\}})] + v_t. \tag{9}$$

$$i_t^{\{4\}} = \frac{1}{4}\sum_{k=0}^{3} E_t i_{t+k}^{\{1\}} + \xi_t^{\{4\}}, \tag{10}$$

$$x_t \equiv x_t^r + r_t^x, \tag{11}$$

$$\pi_t \equiv \pi_t^r + r_t^\pi, \tag{12}$$

$$r_t^x = b_{xx} x_t^r + b_{x\pi} \pi_t^r + b_{xsp}(i_t^{\{4\}} - i_t^{\{1\}}) + \epsilon_{xt}^r, \tag{13}$$

$$r_t^\pi = b_{\pi x} x_t^r + b_{\pi\pi} \pi_t^r + b_{\pi sp}(i_t^{\{4\}} - i_t^{\{1\}}) + \epsilon_{\pi t}^r. \tag{14}$$

Equations (7), (8) and (10) are just the IS and Phillips curves and the rational expectations hypothesis of term structure, respectively (they are written out again here for the sake of completeness). Equation (9) describes the policy rule based on real-time data of output (x_t^r) and inflation (π_t^r) actually available at the time of implementing monetary policy.[16] Equation (11) ((12)) is an identity showing how revised data on output, x_t, (inflation, π_t) is related to real-time output, x_t^r, (inflation, π_t^r). Then, r_t^x (r_t^π) denotes the revision of output (inflation). By adding the log of potential output on both sides of (11), we have that r_t^x also denotes the revision of the log of output. Equations (13) and (14) describe the revision processes associated with output and inflation, respectively. These processes allow for the existence of a contemporaneous correlation between the revision of output and inflation and the initial announcements of these variables.[17] Moreover, we introduce the possibility that revision processes could be determined by the term spread, which is observable with no error and no delay, as preliminarily suggested by the evidence in Table 2. Only under the null hypothesis $H_0 : b_{xx} = b_{x\pi} = b_{\pi x} = b_{\pi\pi} = b_{xsp} = b_{\pi sp} = 0$, can r_t^x and r_t^π be viewed as rational forecast errors. That is, the two revision processes are characterized by white noise processes ϵ_{xt}^r and $\epsilon_{\pi t}^r$, with zero mean and variance σ_x^r and σ_π^r, respectively.

[16] Notice that a backward-looking policy rule does not necessarily exclude a forward-looking Taylor rule setting usually assumed for the monetary policy. After all, expectations of output and inflation can be expressed as a function of the information set available to the Fed in every period (in this case, real-time output and inflation and term spread).

[17] The two revision processes assumed do not seek to provide a structural characterization of the revision processes actually followed by statistical agencies, but to provide a simple framework for assessing whether the nature of the revision process might affect the estimated policy rule.

For the sake of completeness, we now display the system of equations (7)-(14) together with five extra identities involving forecast errors in matrix form

$$\Gamma_{R0}Y_{Rt} = \Gamma_{R1}Y_{Rt-1} + \Psi_R\epsilon_{Rt} + \Pi_R\eta_t, \tag{15}$$

where

$$Y_{Rt} = (x_t, \pi_t, i_t^{\{1\}}, i_t^{\{4\}}, E_t x_{t+1}, E_t\pi_{t+1}, \chi_t, z_t, \xi_t^{\{4\}}, v_t, x_t^r, \pi_t^r, r_t^x, r_t^\pi, E_t i_{t+1}^{\{1\}}, E_t i_{t+2}^{\{1\}}, E_t i_{t+3}^{\{1\}})',$$

$$\epsilon_{Rt} = (\epsilon_{\chi t}, \epsilon_{zt}, \epsilon_{\xi t}^{\{4\}}, \epsilon_{vt}, \epsilon_{xt}^r, \epsilon_{\pi t}^r)'.$$

In order to carry out a joint estimation of the NKM model with term structure and the revision processes using both revised and real-time data, we consider a six-variable VAR with four lags as an auxiliary model to summarize the joint dynamics exhibited by U.S. quarterly data on revised output growth, revised inflation, real-time output growth, real-time inflation, Fed funds rate and 1-year Treasury constant maturity rate.

Table 4 shows the estimation results obtained using both revised and real-time data. The policy inertia parameter is even larger than the estimates obtained above. In contrast with the results based on revised data, the term spread enters the estimated policy rule with an even larger coefficient ($\psi_3 = 4.92$) and the shock persistence parameter is much smaller, but still significant ($\rho_v = 0.35$). All the remaining model shocks reported in Table 4 display large persistence. Especially, the inflation shock exhibits high persistence ($\rho_z = 0.95$).

The estimation results also show that many revision process parameters are significant, suggesting that real-time data are not rational forecasts in line with the evidence provided in Table 2 for our sample. In particular, the coefficients of inflation in the output and inflation revision equations are large and significant ($b_{x\pi} = -0.54$, $b_{\pi\pi} = -0.58$).[18] Moreover, the term spread and the initial announcements of output help to predict inflation and output revisions, respectively ($b_{\pi sp} = -0.43$ and $b_{xx} = -0.18$). Finally, the innovations associated with output revision are larger than those associated with the inflation revision process. These estimation results based on a structural estimation approach are not directly comparable with those in Table 2, which are based on a reduced-form estimation approach. Below, we also implement the reduced-form estimation approach using simulated data from the model in order to facilitate comparison and assess the ability of the model to reproduce the features of the actual revision processes displayed in Table 2.

[18] Even when inflation is not significant in the preliminary analysis for the conditional mean of the revision of the output growth process, it becomes significant in the revision process of the output gap in the structural estimation approach.

$J_T(\theta_r)$	17.6433				
Policy parameter	Estimate	Shock parameter	Estimate	Revision parameter	Estimate
ρ	0.9290	ρ_χ	0.8683	b_{xx}	-0.1822
	(0.0066)		(0.0101)		(0.0666)
ψ_1	1.5	$\rho_\xi^{\{4\}}$	0.7081	$b_{x\pi}$	-0.5356
	$-$		(0.0405)		(0.1084)
ψ_2	0.0000	ρ_z	0.9486	b_{xsp}	-0.3577
	(0.0340)		(0.0286)		(0.3573)
ψ_3	4.9177	σ_χ	$5.5e-03$	$b_{\pi x}$	0.0085
	(0.9577)		$(3.1e-04)$		(0.0065)
ρ_v	0.3498	$\sigma_\xi^{\{4\}}$	$5.4e-04$	$b_{\pi\pi}$	-0.5828
	(0.0615)		$(5.7e-05)$		(0.0291)
σ_v	$2.4e-04$	σ_z	$2.5e-04$	$b_{\pi sp}$	-0.4263
	$(6.7e-05)$		$(1.1e-04)$		(0.1355)
				σ_x^r	$3.3e-03$
					$(3.3e-04)$
				σ_π^r	$3.0e-04$
					$(1.1e-04)$

Table 4: Joint estimation of the NKM model with term structure and the revision processes using both revised and real-time data

Note: Standard errors in parentheses.

In order to assess first the importance of revision processes and then the impact of the term spread information in the monetary policy rule, we perform three additional exercises in the next three subsections. First, we analyze a restricted model where revision processes are forced to be well-behaved. Second, we analyze a model where the role of the term spread is removed from the monetary policy rule. Finally, we consider expected inflation and output gap, in addition to the components already included in the policy rule (9), in order to distinguish the term spread's role of forecasting future inflation and output from that of forecasting data revision on current initial announcements of inflation and output.[19]

5.1 Well-behaved revision processes

To see if the characteristics of revision processes for both actual and simulated data have an effect on estimated policy rule results, and in particular on the role of the term spread, we estimate the system (15) under the null hypothesis that r_t^x and r_t^π are rational forecast errors, i.e. $H_0 : b_{xx} = b_{x\pi} = b_{\pi x} = b_{\pi\pi} = b_{xsp} = b_{\pi sp} = 0$. Table 5 shows the estimation results imposing H_0. By using the asymptotic result (6), we know that the null hypothesis H_0 can be tested using the following Wald statistic

$$F_1 = \left(1 + \frac{1}{m}\right) T \left[J_T(\theta') - J_T(\theta)\right] \to \chi^2(6).$$

The F_1-statistic takes the value 361.03. Therefore, we can reject the hypothesis that the revision processes of output and inflation are white noise at any standard significance level. Moreover, comparing the estimation results in Tables 4 and 5 it is interesting to observe that the term spread coefficient and policy shock persistence estimates are very different depending on whether or not the two revision processes are assumed to be white noise. Thus, the term spread coefficient becomes much larger whereas the persistence of policy shocks becomes much smaller, but remains significant, when revision processes are well behaved. In short, the relative importance of the term spread and persistent shocks in the policy rule changes drastically when it is taken into account that real-time data are not rational forecasts.

5.2 Policy rule without term spread

Next we estimate the extended NKM model by removing the term spread from the policy rule. Table 6 shows the estimation results in this case. A comparison of

[19]We thank an anonymous referee for suggesting the analysis of a forward-looking policy rule.

$J_T(\theta')$	21.3578				
Policy parameter	Estimate	Shock parameter	Estimate	Revision parameter	Estimate
ρ	0.9402 (0.0066)	ρ_χ	0.8908 (0.0104)	σ_x^r	$4.6e-03$ $(1.5e-04)$
ψ_1	1.5 –	$\rho_\xi^{\{4\}}$	0.6218 (0.0335)	σ_π^r	$7.7e-04$ $(3.0e-04)$
ψ_2	0.0000 (0.0121)	ρ_z	0.9436 (0.0207)		
ψ_3	10.1124 (1.4676)	σ_χ	$3.1e-03$ $(3.0e-04)$		
ρ_v	0.1510 (0.0342)	$\sigma_\xi^{\{4\}}$	$1.0e-04$ $(5.2e-05)$		
σ_v	$5.2e-04$ $(4.7e-05)$	σ_z	$1.9e-04$ $(1.1e-04)$		

Table 5: Joint estimation of the NKM model with term structure under Ho
Note: Standard errors in parentheses.

the impulse response functions obtained from the model with and without the term spread in the policy rule allows us to assess the extent to which considering the term spread in the policy rule affects the transmission mechanism of shocks. But before we do that, it is useful to compare Tables 4 and 6. We observe that by considering the term spread in the policy rule the estimates of the remaining parameters of the model do not change significantly, but the term spread parameters in the revision processes do.[20] Those parameters are smaller when the term spread enters into the policy rule and is only significant in the inflation revision process. Moreover, by using once again the asymptotic result (6), we know that the significance of the term spread coefficient in the policy rule can be tested using the following Wald statistic

$$F_2 = \left(1 + \frac{1}{m}\right) T \left[J_T(\theta'') - J_T(\theta)\right] \to \chi^2(1).$$

The F_2-statistic takes the value 70.51. Therefore, we can reject the hypothesis that the term spread does not enter into the policy rule.

[20]Notice that the fact that the output gap coefficient in the policy rule ψ_2 is zero in all kind of specifications is not related either to the inclusion of term spread in the policy rule as shown in Table 6 where the term spread is removed from the policy rule or to considering only revised data as shown in Table 1. Moreover, the estimate $\psi_2 = 0$ falls within a reasonable range. For instance, Smets and Wouters (2007), which consider a medium-scale New Keynesian model and a Bayesian econometric approach, have also obtained a rather small point estimate for this coefficient (0.08).

$J_T(\theta'')$	18.3687				
Policy parameter	Estimate	Shock parameter	Estimate	Revision parameter	Estimate
ρ	0.9239 (0.0050)	ρ_χ	0.8765 (0.0085)	b_{xx}	-0.1351 (0.0606)
ψ_1	1.5 –	$\rho_\xi^{\{4\}}$	0.7933 (0.0379)	$b_{x\pi}$	-0.6756 (0.1168)
ψ_2	0.0000 (0.0276)	ρ_z	0.9389 (0.0297)	b_{xsp}	1.0056 (0.3202)
ψ_3	0.0 –	σ_χ	$5.6e-03$ $(3.3e-04)$	$b_{\pi x}$	0.0085 (0.0056)
ρ_v	0.3029 (0.0490)	$\sigma_\xi^{\{4\}}$	$7.9e-04$ $(5.5e-05)$	$b_{\pi\pi}$	-0.5552 (0.0337)
σ_v	$2.9e-04$ $(4.3e-05)$	σ_z	$3.4e-04$ $(1.9e-04)$	$b_{\pi sp}$	-0.8836 (0.1344)
				σ_x^r	$3.4e-03$ $(3.1e-04)$
				σ_π^r	$2.7e-04$ $(8.6e-05)$

Table 6: Joint estimation of the NKM model without including term spread in the policy rule

Note: Standard errors in parentheses.

Figures 2-5 show the impulse-responses (annualized and in percentage terms)[21] of the endogenous variables of the extended NKM model (15) with (solid line) and without (dotted line with diamonds) considering the term spread in the policy rule to a productivity shock, an inflation shock, a monetary policy shock, and a risk premium shock using the estimates displayed in Tables 4 and 6, respectively. The short-dashed line with solid circles represents the impulse-responses using the estimates displayed in Table 5 assuming that the revision process are well behaved. In these figures the dashed lines are 5%-95% confidence bands. The confidence bands and the size of each shock are determined by the standard errors of parameters and the estimated standard deviation of the corresponding shock, respectively, displayed in Table 4. We can observe that the inclusion of the term spread mildly enhances the short-run effects of some shocks. However, most of the transmission mechanisms characterized by the impulse-response functions change significantly when the restriction is imposed that the revision processes must be well behaved.

Focusing on the impulse-response functions associated with the non-restricted model (solid line), Figure 2 shows that a positive productivity shock initially increases output growth as expected. This shock also reduces the output gap (this is not shown for brevity's sake) since, as expected, the flexible-price equilibrium level of output increases more than the actual one, in the short-run, but the output gap rapidly recovers.[22] This expansive shock also has a negative effect on inflation and interest rates. Figure 3 shows that a positive inflation shock increases inflation and interest rates whereas the output gap decreases. Figure 4 shows the responses to a positive monetary policy shock. The policy shock increases short- and long-term interest rates whereas output gap and inflation decrease. After these initial effects, all variables quickly reach the steady state. Finally, Figure 5 shows that a positive risk premium shock initially increases the long- and the short-term interest rates while slightly reducing the output gap and inflation. After the initial increase, the short-term interest decreases below the steady-state level due to the fall in output gap and inflation.

[21] The only exception is that we do not annualize the response of output growth to a productivity shock in Figure 2.

[22] In the remaining figures, we have decided to plot the output gap impulse response function instead of the impulse response function associated with output growth because they are qualitatively similar.

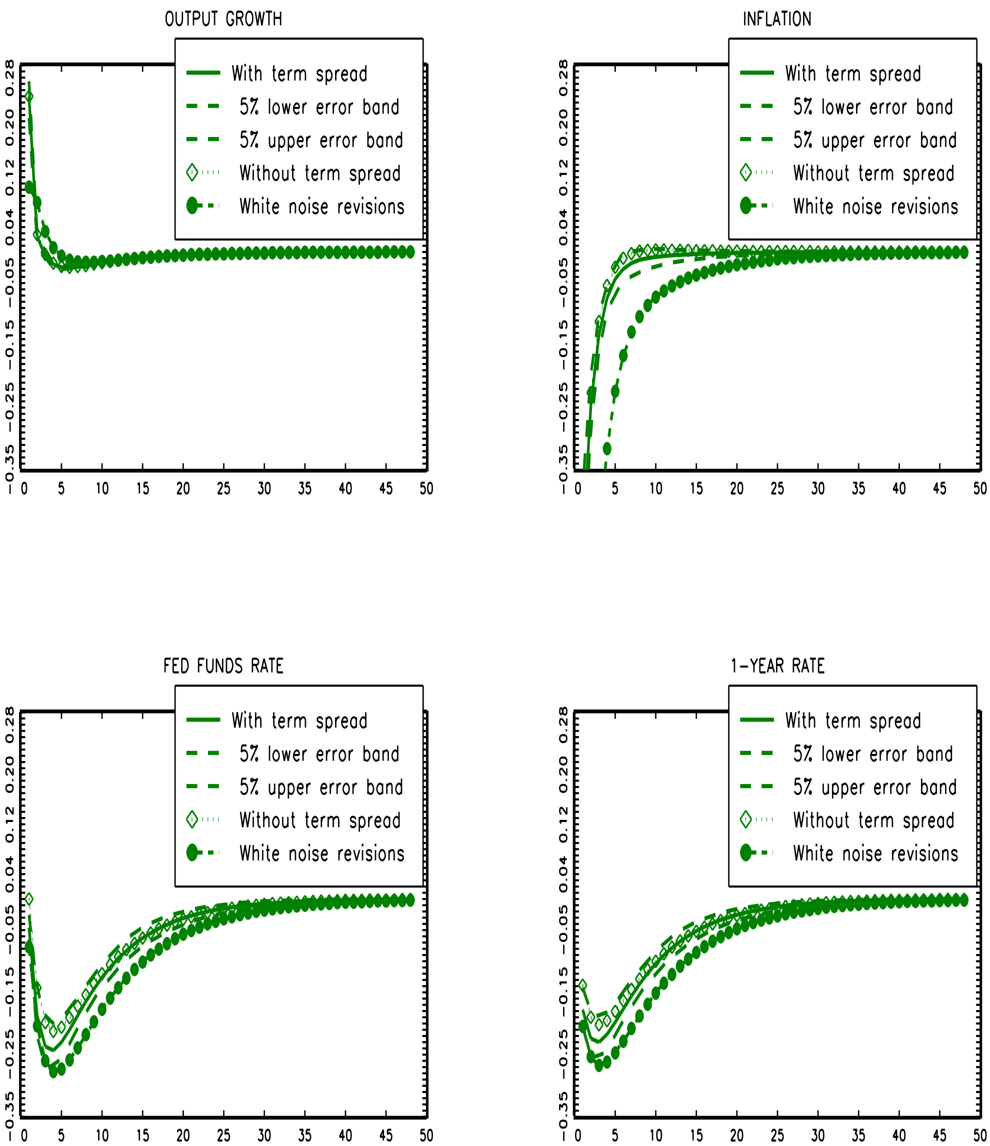

Figure 2: Impulse-responses to a productivity shock

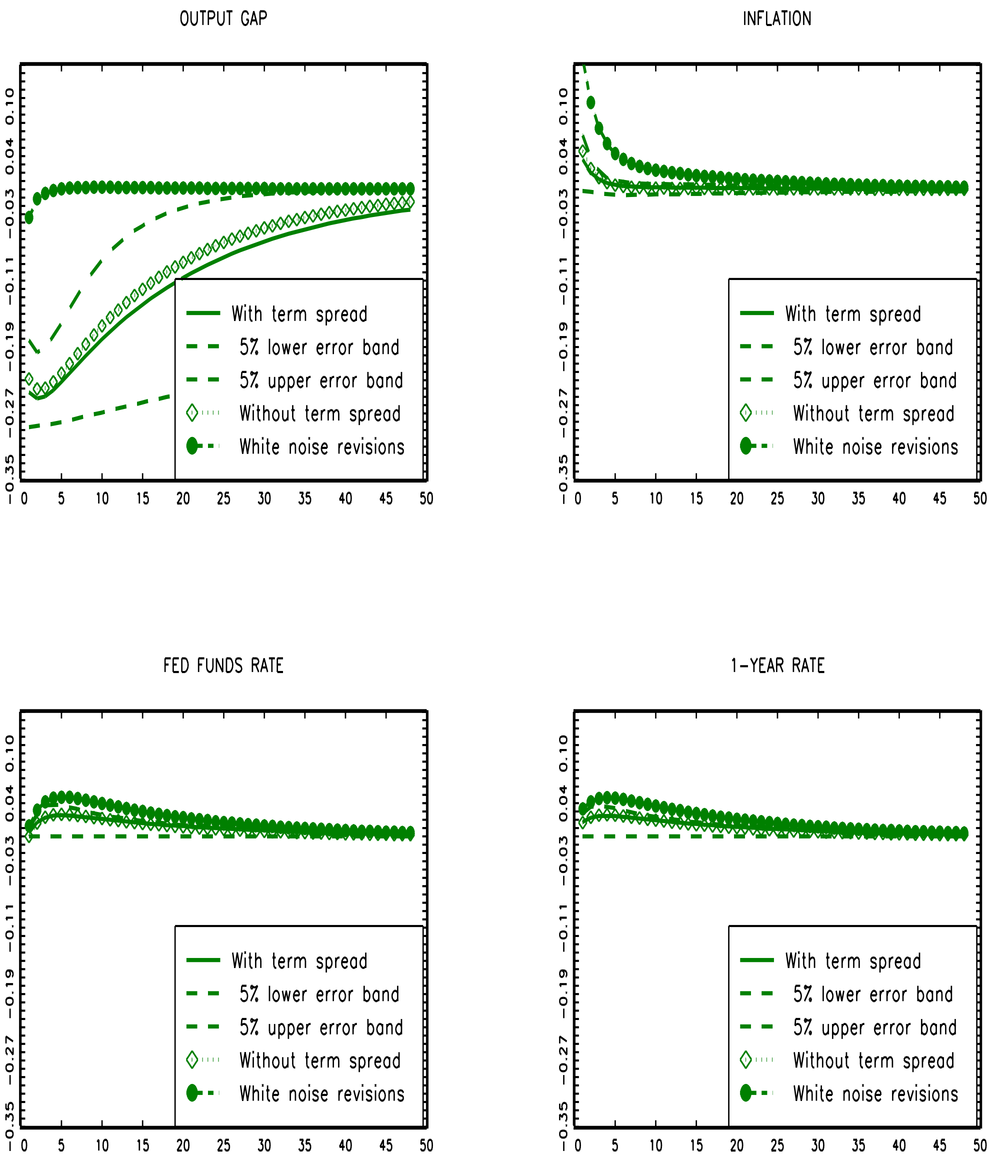

Figure 3: Impulse-responses to an inflation shock.

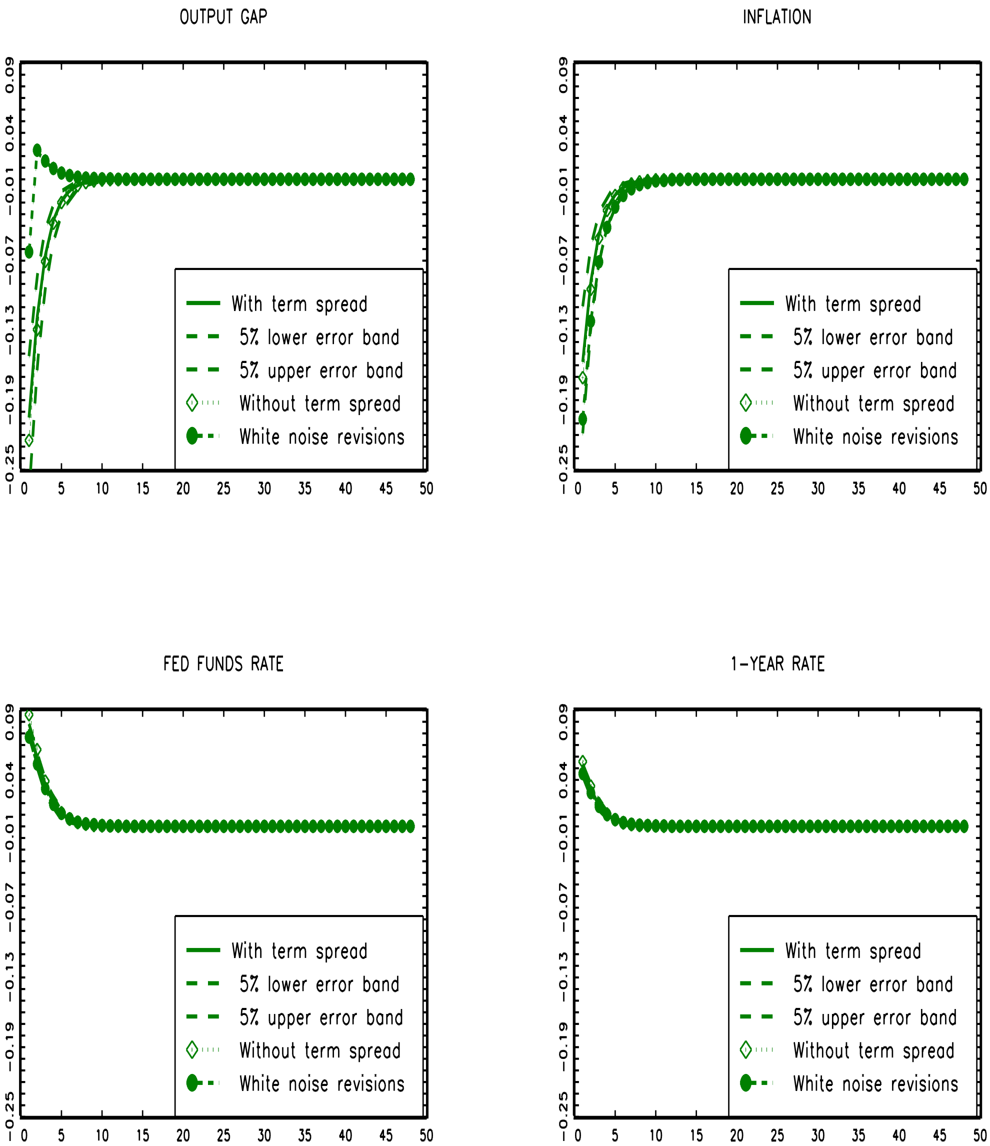

Figure 4: Impulse-responses to a monetary policy shock.

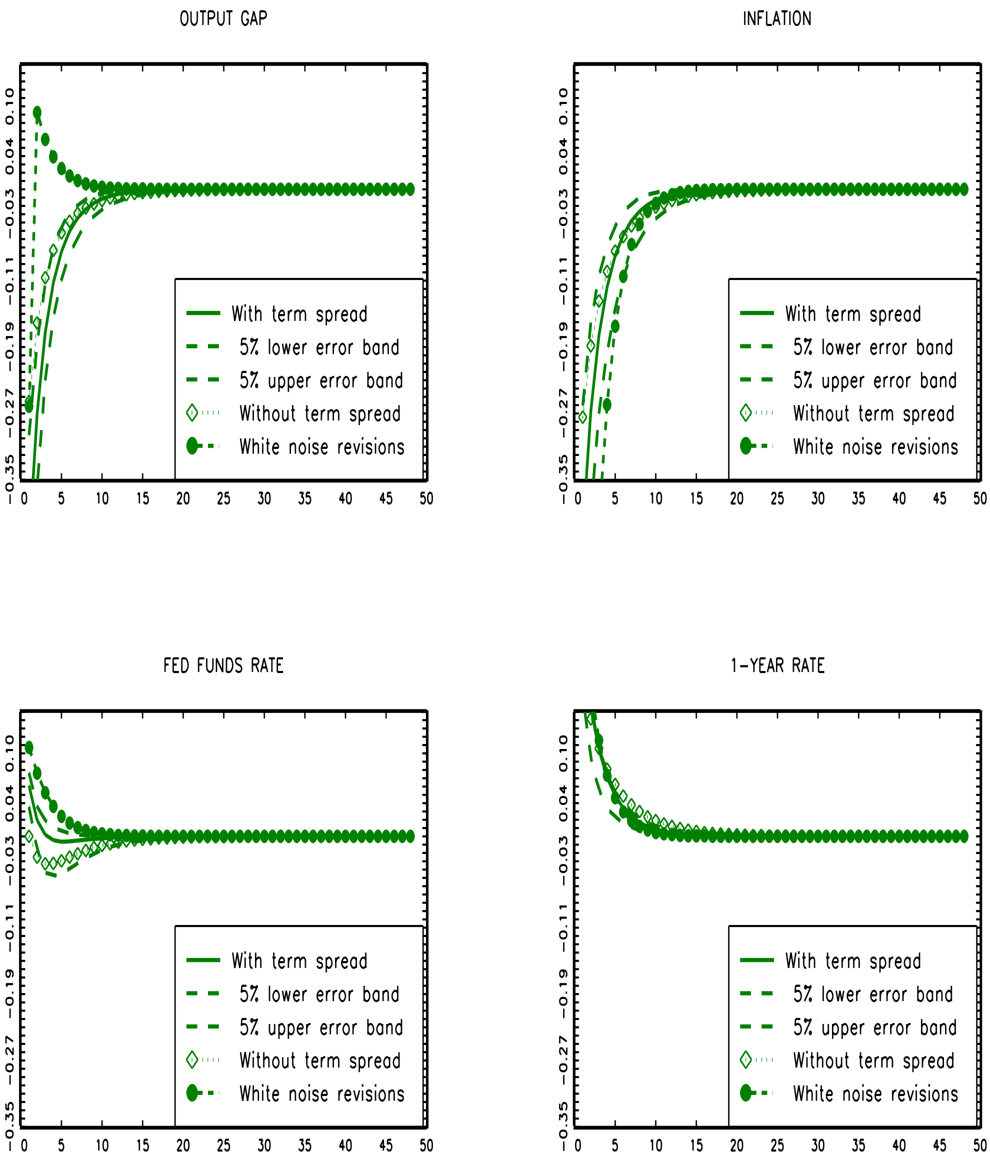

Figure 5: Impulse-responses to a risk premium shock.

5.3 Forward-looking policy rule

This subsection considers a forward-looking policy rule given by

$$\begin{aligned} i_t^{\{1\}} &= \rho i_{t-1}^{\{1\}} + (1-\rho)[\psi_1 \left(\alpha_\pi \pi_{t-1}^r + (1-\alpha_\pi) E_t \pi_{t+1}\right) \\ &\quad + \psi_2 \left(\alpha_x x_{t-1}^r + (1-\alpha_x) E_t x_{t+1}\right) + \psi_3 (i_t^{\{4\}} - i_t^{\{1\}})] + v_t, \end{aligned} \tag{16}$$

where the Fed is assumed to react to two indexes of inflation and output, respectively. The inflation (output) index is a convex linear combination of the initial announcement of inflation (output gap) and the expected value of future inflation (output gap). The new parameter α_π (α_x) captures the relative importance of real time inflation (output gap) data in the inflation (output) index. By including the expected values of inflation and output gap in the policy rule, we seek to isolate the role of the term spread in predicting future revisions of lagged and current inflation and output gap from its ability to forecast future inflation and output as well.

As discussed above in the context of the benchmark model, preliminary estimation attempts of the model under this policy rule also result in very large estimates of both the inflation and the term spread coefficients, whereas the estimate of the output gap coefficient is zero, which in turn implies that the weight of the initial announcement of output gap in the output index, α_x, is not identified. We then set $\psi_1 = 1.5$, $\alpha_x = 0$ and investigate the role of the term spread under this policy rule specification.[23] Table 7 shows the estimation results under policy rule (16). Most parameter estimates are not significantly different from those found under policy rule (9), with two exceptions. First, the policy inertia parameter, ρ, slightly decreases. Second, the term spread coefficient is much smaller ($\psi_3 = 1.29$) under (16). This is as expected, since the term spread no longer captures the reaction of the current Fed rate to expected movements in inflation under this policy rule, so ψ_3 measures the reaction of Fed rate to term spread movements induced by revisions of inflation and output. Moreover, the estimate of $\alpha_\pi = 0.29$ implies that the role of expected inflation in the inflation index of the policy rule is much more important than the role of real time inflation.

Considering policy rule (16) instead of (9) has two important advantages. First, the model fit is slightly, but significantly, improved. Second, by explicitly considering expected inflation in the policy rule we are able to distinguish the term spread's role of forecasting inflation data revision from that of forecasting future inflation. In spite of these advantages, policy rule (9) still has an appealing feature. Namely, like

[23] As expected, estimation results are identical if α_x is set to any other arbitrary value since the point estimate of ψ_2 is zero.

$J_T(\theta_r)$	17.5782				
Policy parameter	Estimate	Shock parameter	Estimate	Revision parameter	Estimate
ρ	0.8641 (0.0159)	ρ_χ	0.8734 (0.0121)	b_{xx}	-0.1197 (0.0452)
ψ_1	1.5 –	$\rho_\xi^{\{4\}}$	0.6492 (0.0441)	$b_{x\pi}$	-0.6981 (0.1058)
ψ_2	0.0000 (0.0411)	ρ_z	0.9525 (0.0331)	b_{xsp}	-0.3517 (0.3455)
ψ_3	1.2926 (0.3843)	σ_χ	$3.8e-03$ $(3.4e-04)$	$b_{\pi x}$	0.0087 (0.0068)
α_π	0.2438 (0.0560)	$\sigma_\xi^{\{4\}}$	$6.9e-04$ $(5.7e-05)$	$b_{\pi\pi}$	-0.4322 (0.0402)
ρ_v	0.4988 (0.0580)	σ_z	$2.8e-04$ $(1.4e-04)$	$b_{\pi sp}$	-0.9849 (0.1977)
σ_v	$2.5e-04$ $(4.4e-05)$			σ_x^r	$3.6e-03$ $(2.7e-04)$
				σ_π^r	$4.2e-04$ $(1.6e-04)$

Table 7: Joint estimation of the NKM model under forward-looking rule (16).
Note: Standard errors in parentheses.

the McCallum (1994) rule and in the spirit of the original Taylor rule, policy rule (9) depends only (up to an error term) on variables which are observed in real time and it is therefore directly verifiable by all economic agents, helping to determine the Fed's stance on monetary policy.[24]

[24] We have also explored another alternative policy rule specification where the term spread enters only into the inflation and output gap indexes and is not allowed an independent role as in (9) and (16). The model fit does not improve under this specification and parameter estimates are quite similar to those found under (9) and (16), so they are not discussed any further. These estimation results are available from the authors upon request.

6 SOME ROBUSTNESS EXERCISES AND MODEL DIAGNOSTICS

This section carries out some robustness exercises and provides a few model diagnostic tests of the model. We have run our estimation procedure by using alternative parameterizations in order to analyze the robustness of the estimation results found. The top panel in Table 8 shows the parameter estimates and the associated standard errors of the policy rule and revision process parameters.[25] The label in each column indicates the parameterization chosen for the pre-assigned structural parameters β, τ, η and ω. Thus, the first column labeled "Benchmark" uses the standard parameterization discussed above. The second column labeled "$\psi_1 = 2.0$" uses the benchmark parameterization with a difference, ψ_1 is 2.0 instead of 1.5. The next three columns are labeled similarly. Finally, the last column labelled "spread" shows the estimation results under the benchmark parameterization when the volatility of the term spread is considered to build the distance function in the estimation algorithm. We observe that the parameter estimates associated with ρ and ρ_v are similar across the parameterizations and distance functions. However, the parameter estimate associated with the term spread changes quite dramatically depending on the parameterization chosen. In particular, changing the inflation coefficient from 1.5 (benchmark parameterization) to 2.0 implies a large increase in the estimate of ψ_3 (i.e. from 4.92 to 8.20). This result points in the same direction mentioned above: that inflation and term spread parameters in the policy rule are not separately identified. Moreover, the large range of parameter estimates of ψ_3 and their large standard errors suggest that the term spread plays a significant role, but it is also hard to quantify. Furthermore, the revision process parameter estimates are mostly similar across parameterizations. Nevertheless, it should be noticed that some quantitative differences in the estimated parameters show up when Calvo's probability parameter, ω, is reduced from 0.75 to 0.5.

Next, we run some diagnostic tests. We start by assessing the ability of the model to reproduce some actual volatility moments. The bottom panel in Table 8 shows volatility statistics for revised and real-time data on output and inflation, and the term spread from actual and simulated data from the model. The first column shows the standard deviations of actual data. We see that real-time data are more volatile than revised data. The simulated volatilities of these variables obtained from the model under alternative parameterizations and parameter estimates are displayed in the remaining columns. They show that the simulated volatilities are quite similar

[25] We do not include the output gap coefficient because it is consistently zero in all estimation exercises. Moreover, the estimates of the remaining parameters are very similar in all the parameterizations considered. For the sake of brevity, they are not shown here, but they are available upon request from the authors.

across alternative parameterizations and estimates. In particular, the simulated volatilities are always lower than the actual volatilities. This is especially true for the standard deviation of the term spread and revised output growth. Moreover, the estimated model reproduces the qualitative feature that real-time data is more volatile than revised data. Similar conclusions are obtained when considering term spread volatility in the estimation algorithm.

Next, we analyze the ability of the model to capture the revision process features displayed in Table 2. Table 9 shows a set of summary statistics for the simulated revision processes of output and inflation, respectively. The simulated series are computed using the estimates shown in Table 4. Comparing Tables 2 and Table 9, we observe that for output growth, the model captures to a certain extent the main features of the actual revision process. Thus, the output growth revision process is not well behaved. Standard deviations are quite large for both actual and simulated data. We also find evidence of a negative autocorrelation pattern, and the conditional mean is clearly different from zero. Using simulated data, all real-time variables seem to play a role in explaining the revision process of output, which confirms the hypothesis that this revision process is not a rational forecast error, whereas only real-time output growth is significant when using actual data. For inflation, however, the model systematically underestimates the standard deviation of the revision process. This result is driven by the low estimate for the standard deviation of the innovation associated with the inflation revision process. With such a low standard deviation, for over 30% of the simulated series we could not reject the hypothesis that the unconditional mean is null at the 5% confidence level. Consistent with actual data, the conditional mean is also different from zero when the simulated series are analyzed. However, the term spread also adds additional relevant information for explaining the revision process of inflation in contrast with the insignificant role obtained with actual data. In short, we can conclude that the revision processes implied by the model reproduce to a certain extent most of the features characterizing the actual revision processes displayed in Table 2.

7 CONCLUSIONS

This paper follows a structural econometric approach based on the *indirect inference* principle to analyze the relative importance of policy inertia, term spread and persistent monetary policy shocks in the characterization of the estimated monetary policy rule for the U.S. using both revised and real-time data. The framework considered

Policy Rule and Revision Process Parameters

	Benchmark	Forward	ψ_1= 2.0	$\omega = 0.5$	$\tau = 1/3$	$\eta = 3.0$	Spread
$J_T(\theta)$	17.6463	17.5782	17.5844	17.6660	16.2562	17.6519	17.6822
ρ	0.9290	0.8641	0.9447	0.9192	0.9311	0.9274	0.9264
	(0.0066)	(0.0159)	(0.0054)	(0.0055)	(0.0063)	(0.0065)	(0.0064)
ψ_3	4.9177	1.2926	8.2037	3.3529	3.9301	4.6871	4.3100
	(0.9577)	(0.3843)	(1.3700)	(0.6305)	(0.9304)	(0.9065)	(0.8254)
ρ_v	0.3498	0.4988	0.3463	0.3081	0.3443	0.3500	0.3324
	(0.0615)	(0.0580)	(0.0631)	(0.0500)	(0.0604)	(0.0610)	(0.0600)
b_{xx}	−0.1822	−0.1197	−0.1927	−0.1989	−0.2121	−0.1891	−0.1808
	(0.0665)	(0.0452)	(0.0724)	(0.0859)	(0.0927)	(0.0719)	(0.0682)
$b_{x\pi}$	−0.5356	−0.6981	−0.5284	−0.4639	−0.4793	−0.5272	−0.5389
	(0.1084)	(0.1058)	(0.1146)	(0.0763)	(0.1196)	(0.1048)	(0.1078)
b_{xsp}	−0.3577	−0.3517	−0.3673	−0.8694	0.3212	−0.3616	−0.3554
	(0.3573)	(0.3455)	(0.3591)	(0.3525)	(0.3341)	(0.3489)	(0.3443)
$b_{\pi x}$	0.0085	0.0087	0.0088	0.0123	0.0130	0.0087	0.0080
	(0.0065)	(0.0068)	(0.0069)	(0.0100)	(0.0100)	(0.0067)	(0.0062)
$b_{\pi\pi}$	−0.5827	−0.4323	−0.5853	−0.7042	−0.5808	−0.5980	−0.5896
	(0.0291)	(0.0402)	(0.0292)	(0.0244)	(0.0295)	(0.0282)	(0.0284)
$b_{\pi sp}$	−0.4263	−0.9849	−0.4253	−0.7581	−0.3534	−0.4588	−0.3864
	(0.1355)	(0.1977)	(0.1362)	(0.1407)	(0.1312)	(0.1342)	(0.1296)

Inflation, Output Growth and Term Spread Volatilities

	Actual	Benchmark	Forward	ψ_1= 2	$\omega = 0.5$	$\tau = 1/3$	$\eta = 3$	Spread
σ_π	0.9824	0.7942	0.9440	0.7815	0.6182	0.8481	0.7732	0.7882
$\sigma_\pi(r)$	2.0361	1.8095	1.5748	1.8087	1.9566	1.8970	1.8210	1.8169
$\sigma_{\Delta y}$	2.1989	1.1924	0.8036	1.2650	1.2259	1.0845	1.1819	1.2141
$\sigma_{\Delta y}(r)$	2.8140	2.6771	2.6609	2.6774	2.6906	2.6558	2.6794	2.6757
σ_{sp}	0.5229	0.3103	0.3662	0.2795	0.3049	0.3667	0.3128	0.3406

Table 8: Joint estimation of the NKM model. Robustness exercise with alternative parameter values

Note: The label in each column indicates the parameterization chosen for the pre-assigned structural parameters. Thus, the first column labeled "Benchmark" uses the standard parameterization discussed in the text. The second column labeled "ψ_1= 2.0" uses the benchmark parameterization with a difference, ψ_1 is 2.0 instead of 1.5. The next three columns are labeled similarly. Finally, the last column labelled "Spread" shows the estimation results under the benchmark parameterization when the volatility of the term spread is considered to build the distance function in the estimation algorithm. Standard errors are in parentheses. σ_π, $\sigma_\pi(r)$, $\sigma_{\Delta y}$, $\sigma_{\Delta y}(r)$ and σ_{sp} denote the standard deviations of revised inflation, real-time inflation, revised output growth, real-time output growth and term spread, respectively.

Summary Statistics

	r_t^y		r_t^π	
Mean	0.000		−0.070	
Median	−0.002		−0.068	
Min	−6.610		−2.690	
Max	6.677		2.557	
St. Dev	2.633		1.052	
Noise/signal	2.219		1.328	
Corr (y_t, y_t^r)	0.258		0.973	
AC(1)	−0.421	**	0.469	**
$E(r_t) = 0$ (t.stat)	−0.002		−0.670	

Conditional Mean

	r_t^y			r_t^π		
	coef.	t.stat		coef.	t.stat	
Expl.Variables						
Constant	0.879	6.437	***	1.445	66.893	***
$(y_t^r - y_{t-1}^r) * 400$	−0.910	31.419	***	0.002	0.388	
$(\pi_t^r) * 400$	−0.345	8.034	***	−0.580	85.232	***
$(i_t^{\{4\}} - i_t^{\{1\}})$	−2.119	8.304	**	−0.434	10.862	***
$F_{1(4,97)}$		332.243	***		2384.506	***

Table 9: Simulated series revision process analysis

Note: The results in this table are comparable with those in Table 2. The results displayed here are actually averages over 500 simulated series for each variable when the system (15) is simulated using the estimated parameters in Table 4.

is a standard new Keynesian monetary model augmented with term structure where the monetary policy rule is one of the building blocks.

The empirical results based on revised data for inflation and output suggest that the monetary policy rule features policy inertia and persistent policy shocks and the term spread plays an important and statistically significant role. Policy inertia and the term spread increase their importance when an extended version of the model that includes both revised and real-time data is considered. However, the persistence of policy shocks becomes less important, but remains significant. Moreover, the empirical results show that the initial announcements of output and inflation are not rational forecasts and that the revision processes of these variables can be forecasted not only by the initial announcements of these macroeconomic variables but also by the term spread. We can then conclude that the term spread, observed in real time, contains useful information for the Fed about revised data on output and inflation which is not included in their respective initial announcements available at the time of implementing monetary policy.

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APPENDIX 1 (not intended for publication)

This appendix derives the set of IS equations (1). Consider that the representative consumer solves the problem of maximizing

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t),$$

subject to the condition that

$$C_t + \sum_{j=1}^{n} B_t^{\{j\}} \leq Y_t + \sum_{j=1}^{n} B_{t-j}^{\{j\}} R_{t-j}^{\{j\}},$$

where C, N, Y, $B^{\{j\}}$, $R^{\{j\}}$ denote consumption, labor, income, stock of j-period bonds and gross real return of j-period bonds, respectively. Under fairly general conditions this problem has a solution with a finite value of the objective function. The first-order necessary conditions are given by

$$U_C = \lambda_t,$$

$$\beta^j E_t(\lambda_{t+j} R_t^{\{j\}}) = \lambda_t, \text{ for } j = 1, ..., n,$$

where $\{\lambda_t\}$ is a sequence of Lagrange multipliers. Substituting the first equation into each of the j-conditions gives the familiar consumption-based asset pricing equations

$$E_t \left[\beta^j \frac{U_C(C_{t+j}, N_{t+j})}{U_C(C_t, N_t)} R_t^{\{j\}} \right] = 1, \text{ for } j = 1, ..., n.$$

Following Walsh (2003 chapter 5.4), by (i) assuming that the utility function is of the form

$$U(C_t, N_t) = \frac{C_t^{1-1/\tau}}{1 - 1/\tau} - \Psi \frac{N_t^{1+\eta}}{1+\eta};$$

(ii) taking a log-linear approximation for $j = 1$ and $j = n$; (iii) assuming that output is a linear function solely of labor input and an aggregate productivity shock, e^{χ_t}; (iv) substituting for the market clearing condition $Y_t = C_t$ for all t; and (v) using the definition of output gap (i.e. the gap between actual output and flexible-price equilibrium level of output); we then obtain

$$x_t = E_t x_{t+1} - \tau(i_t^{\{1\}} - E_t \pi_{t+1}) - \left[\frac{1+\eta}{(1/\tau) + \eta} \right] (1 - \rho_\chi) \chi_t, \quad (17)$$

$$\frac{x_t}{n} = \frac{E_t x_{t+n}}{n} - \tau(i_t^{\{n\}} - \frac{1}{n} \sum_{k=1}^{n} E_t \pi_{t+k}) - \frac{1}{n} \left[\frac{1+\eta}{(1/\tau) + \eta} \right] (1 - \rho_\chi^n) \chi_t, \quad (18)$$

where ρ_χ is the autoregressive coefficient of the productivity shock. Solving equation (17) in a forward recursive manner n-times ahead we get

$$\frac{x_t}{n} = \frac{E_t x_{t+n}}{n} - \tau(\frac{1}{n}\sum_{k=0}^{n-1} E_t i_{t+k}^{\{1\}} - \frac{1}{n}\sum_{k=1}^{n} E_t \pi_{t+k}) - \frac{1}{n}\left[\frac{1+\eta}{(1/\tau)+\eta}\right](1-\rho_\chi^n)\chi_t. \quad (19)$$

Since the two equations (18) and (19) must hold in equilibrium, they imply that the rational expectations of the term structure of interest rates must hold, i.e.

$$i_t^{\{n\}} = \frac{1}{n}\sum_{k=0}^{n-1} E_t i_{t+k}^{\{1\}}.$$

Finally, we introduce a risk premium shock into the term structure, $\xi_t^{\{n\}}$,

$$i_t^{\{n\}} = \frac{1}{n}\sum_{k=0}^{n-1} E_t i_{t+k}^{\{1\}} + \xi_t^{\{n\}},$$

where the notation clearly establishes that the impact of this shock differs depending on bond maturity.

APPENDIX 2 (not intended for publication)

This appendix shows the matrices involved in Equation (5) and (15). First, we show the matrices in Equation (5).

$$\Gamma_0 = \begin{pmatrix}
1 & 0 & \tau & 0 & -1 & -\tau & \Gamma_0^{1,7} & 0 & 0 & 0 & 0 & 0 & 0 \\
-\kappa & 1 & 0 & 0 & 0 & -\beta & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
\Gamma_0^{3,1} & \Gamma_0^{3,2} & \Gamma_0^{3,3} & \Gamma_0^{3,4} & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & -1/4 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & -1/4 & -1/4 & -1/4 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0
\end{pmatrix},$$

$$\Gamma_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \rho_\chi & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_z & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_\xi^{\{4\}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_v & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix},$$

$$\Pi = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix},$$

$$\Psi = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\Gamma_0^{1,7} = \phi(1-\rho_\chi),$$
$$\Gamma_0^{3,1} = -(1-\rho)\psi_2,$$
$$\Gamma_0^{3,2} = -(1-\rho)\psi_1,$$
$$\Gamma_0^{3,3} = 1+(1-\rho)\psi_3,$$
$$\Gamma_0^{3,4} = -(1-\rho)\psi_3.$$

Next, we show the matrices involved in Equation (15).

$$
\Gamma_{R0} = \begin{pmatrix}
1 & 0 & \tau & 0 & -1 & -\tau & \Gamma_0^{1,7} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\kappa & 1 & 0 & 0 & 0 & -\beta & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \Gamma_0^{3,3} & \Gamma_0^{3,4} & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1/4 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & -1/4 & -1/4 & -1/4 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & b_{xsp} & -b_{xsp} & 0 & 0 & 0 & 0 & 0 & 0 & -b_{xx} & -b_{x\pi} & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & b_{\pi sp} & -b_{\pi sp} & 0 & 0 & 0 & 0 & 0 & 0 & -b_{\pi x} & -b_{\pi\pi} & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0
\end{pmatrix},
$$

$$
\Gamma_{R1}=\begin{pmatrix}
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&\rho&0&0&0&0&0&0&0&-\Gamma_0^{3,1}&-\Gamma_0^{3,2}&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&\rho_\chi&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&\rho_z&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&\rho_\xi^{\{4\}}&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&\rho_v&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1
\end{pmatrix},
$$

$$
\Pi_R=\begin{pmatrix}
0&0&0&0&0\\
0&0&0&0&0\\
0&0&0&0&0\\
0&0&0&0&0\\
0&0&0&0&0\\
0&0&0&0&0\\
0&0&0&0&0\\
0&0&0&0&0\\
0&0&0&0&0\\
0&0&0&0&0\\
0&0&0&0&0\\
0&0&0&0&0\\
1&0&0&0&0\\
0&1&0&0&0\\
0&0&1&0&0\\
0&0&0&1&0\\
0&0&0&0&1
\end{pmatrix},
$$

$$\Psi_R = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$