

New Keynesian model features that can reproduce lead, lag and persistence patterns*

Steven P. Cassou[†]
Kansas State University

Jesús Vázquez[‡]
Universidad del País Vasco

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Abstract

This paper uses a new method for describing dynamic comovement and persistence in economic time series which builds on the contemporaneous forecast error method developed in den Haan (2000). This data description method is then used to address issues in New Keynesian model performance in two ways. First, well known data patterns, such as output and inflation leads and lags and inflation persistence, are decomposed into forecast horizon components to give a more complete description of the data patterns. These results show that the well known lead and lag patterns between output and inflation arise mostly in the medium term forecasts horizons. Second, the data summary method is used to investigate a rich New Keynesian model with many modeling features to see which of these features can reproduce lead, lag and persistence patterns seen in the data. We show that a simple general equilibrium model with persistent IS curve shocks and persistent supply shocks can reproduce the lead, lag and persistence patterns seen in the data.

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[†]Department of Economics, 327 Waters Hall, Kansas State University, Manhattan, KS, 66506 (USA), (785) 532-6342, Fax:(785) 532-6919, email: scassou@ksu.edu.

[‡]Corresponding author - Fundamentos del Análisis Económico II, Facultad de Ciencias Económicas y Empresariales, Universidad del País Vasco, Av. Lehendakari Aguirre 83, 48015 Bilbao (SPAIN), Phone: +34 946013779, Fax: +34 946017123, email: jesus.vazquez@ehu.es.

1 Introduction

The relationship between output and inflation has long been of interest in the monetary economics literature. Today, some consensus has formed about some important issues. For instance, the general view is that this relationship is at most weak in the long-run, reflecting a sort of classical dichotomy between nominal and real variables. On the other hand, the short run seems to be well described by some variant of the New Keynesian Phillips Curve (NKPC).¹ Yet despite the emerging consensus for using the NKPC to model the short run, there remains considerable disagreement about what form it should take. Numerous studies have shown that the strictly forward looking NKPCs are unable to replicate many of the empirical patterns found in the data.² However, one limitation of many of these studies is that they have focused on models consisting of just a single NKPC equation (i.e. the aggregate supply) and have overlooked the aggregate demand side of the economy and its interaction with the NKPC. This focus on single equation NKPC models often results in misleading conclusions.³ Less attention has been placed on more fully specified general equilibrium

¹The fact that output and inflation should be connected in the short-run does not imply the existence of a stable relationship between the two variables since both the sources of economic shocks and the way central banks monitor policy may change over time and across countries. See Walsh (2003, Chapter 1) and references therein for a summary of the main empirical regularities found in the monetary economics literature.

²The original NKPCs were founded on contracting ideas from Taylor (1980) and Calvo (1983). However, performance issues, such as the lack of inflation persistence, led to the introduction of backward looking terms to these basic rational agent models. Furher and Moore (1995) and Furher (1997, 2006) advocate a contracting idea from Buiter and Jewitt (1981) to motivate the backward looking term, while Galí and Gertler (1999) use an empirical motivation for the backward term along with a marginal cost structure substituted for the output gap. Numerous papers, including Coenen and Wieland (2005), Rudd and Whelan (2006) have explored the merits of these formulations. More recently, Ireland (2007), Lansing (2007), Cogley and Sbordone (2008) and others have explored models which add learning, unit roots or near unit roots to the discussion.

³For instance, as noted in Mankiw (2003, pp. 66-67), it is hard to identify the parameters featured in the NKPC when inflation-push shocks are relatively more important than output gap shocks. Put differently, the chances of identifying the NKPC model parameters are higher when output gap shocks dominate. Indeed, many of the papers relying on single equation NKPC models assume an exact relationship between current inflation, expected inflation and output gap (i.e. there are no inflation-push shocks), and the source of variation comes from the output gap (i.e. the variable driving in the NKPC). In addition, from a policy perspective, the single equation NKPC models are often associated with the possibility for an immediate and costless disinflation policy because current inflation is entirely determined by the expected path of future output gaps and if the central bank could commit to setting the path of future output gaps equal to zero, adjustment would occur immediately (Galí and Gertler (1999, pp. 203)). However, this argument does not hold when output

models.⁴ This paper fills this gap by investigating the short run performance of the NKPC in a small-scale general equilibrium model with several sources of persistence. We use a general equilibrium structure that is rich enough so as to reproduce the key comovement features seen in the data that are not easily matched in single equation NKPC models, but also is simple enough, in contrast with the recent medium-scale models as in Smets and Wouters (2003, 2007), so that it is possible to understand exactly which model features are necessary to match the dynamic patterns of output and inflation data.

This paper contributes to our understanding of the output and inflation relationship in three important ways. First, it provides a new data description method, that builds on techniques developed in den Haan (2000). This extension is designed to shed light on lead and lag comovements of the data. It not only identifies the lead and lag empirical regularities, but it also shows whether they are part of the short term or long term forces driving the data. Second, the paper uses these techniques to describe the lead, lag and contemporaneous comovement between output and inflation as well as inflation persistence. This description is particularly useful for the output and inflation application here where so much of the debate has centered on whether the NKPC is able to replicate dynamic patterns seen in the data. Third, a small-scale New Keynesian model (NKM) with a rich set of modeling features is described and then studied to see which of these features are important for generating the actual patterns. These model features include, a consumer utility function with generalized habit persistence, a hybrid NKPC à la Galí and Gertler (1999), a monetary policy rule that incorporates inflation, output and output growth as suggested by Smets and Wouters (2007) and persistence in the IS curve and the NKPC shock processes.

gap and inflation-push shocks are modelled so that they are highly persistent as part of the economic structure. Under such a structure, it is not feasible for a central bank to make a commitment about the path of future output gaps.

⁴Some exceptions include Rotemberg and Woodford (1997), Christiano, Eichenbaum and Evans (2005), Keen (2009), and Smets and Wouters (2003, 2007) which propose medium-scale general equilibrium models with many equations and, in some cases, numerous sources for variation.

The data description method is used in two important ways. First, lead, lag and persistence patterns of the data are described. Early work by Fuhrer and Moore (1995) documented these patterns and thus set the mark which most studies of the NKPC have sought to achieve. Our method refines the typical data summary to decompose the lead, lag and persistence patterns into forecast horizons, thus allowing one to judge whether the data patterns are more short term or long term in nature. We find a hump shape in our lead and lag diagrams which show that these patterns are arising from medium term components rather than short- or long-term components. Second, we use this data description in a fitting exercise which fits our rich NKM to the data and thus shows which of the modeling features are important for achieving data matches.

Our fitting exercises find several important results. First, we find that there are several ways to reproduce the lead and lag patterns between output and inflation. The key features of the model that are needed to achieve this dimension of fit are: 1) the model needs to have both demand equations and supply equations with their own stochastic elements; 2) the model needs to get the relative proportions for the supply and demand shock variances just right; and 3) the model needs to get the relative persistence for the supply and demand shocks just right. It is shown that these requirements can be satisfied, at least qualitatively, with a variety of alternative shock and persistence specifications as well as with different specifications for the consumption habit formation structure and the backward looking component of the hybrid NKPC. The intuition for this structure is relatively easy to understand from the impulse response functions provided below. The demand shocks produce a positive lead of output over inflation when the effects of these shocks are more persistent in inflation than output, and the supply shocks produce a negative lead of inflation over output when the effects of supply shocks last longer in output than inflation. By balancing these two dynamic features with the right variances for the demand and supply shocks and the proper persistence levels of the model, the lead and lag patterns between output and inflation can be reproduced.

A second stylized fact of the data is that both inflation and output are highly persistent, but inflation is more so. Reproducing this fact is more difficult. To achieve this feature of the data jointly with the lead and lag patterns, we find it necessary to have the right balance (size and persistence) between IS and inflation-push shocks. Most importantly, we find that persistence of the IS shock is key to the fit and that the backward looking component in the hybrid NKPC, and to a lesser extent the habit formation structure of the model, help to reproduce the observed inflation persistence. These results perhaps explain the difficulty that single equation models, without a demand side to the economy, have had in achieving persistence in inflation.

The rest of the paper is organized as follows. Section 2 is designed as a data section. It first reviews some of the data features described by Fuhrer and Moore (1995) and others. It then describes the extension of den Haan's (2000) method to analyze lead and lag comovements in the data and applies the method to the U.S. post-war output and inflation time series. Section 3 introduces our small-scale NKM with three key building blocks, an IS curve, a hybrid NKPC and a monetary policy rule. This model is designed to have a rich set of demand and supply shock structures as well as several sources of persistence. At the same time, the model is simple enough to clearly understand what are the key features necessary to produce the statistical patterns seen in the data. In Section 4, we follow a fitting approach designed to uncover which of the model features are needed to reproduce the output and inflation dynamics. Section 5 concludes.

2 Leads, lags and persistence in output and inflation data

In this section we investigate the lead, lag and persistence patterns in the output and inflation data for the U.S. The section is broken into three subsections. In the first subsection, we begin by reviewing some of the findings from Fuhrer and Moore (1995) which have become well known stylized facts for the literature in this area.

Next we describe an extension to the forecast error correlation methods in den Haan (2000) which allows a more complete picture of the data movements. Finally, we apply this new method to the output and inflation data, and see how it provides a richer summary of the dynamic movements of these two data series than the Fuhrer and Moore (1995) approach.

2.1 Review of lead, lag and persistence measurements

In order to understand our new forecast error correlation approach, it is helpful to first review some of the more familiar lead and lag facts first described in Fuhrer and Moore (1995) and emphasized by Galí and Gertler (1999) and many others later on. Fuhrer and Moore (1995) used a trivariate VAR to summarize the data on output gap, inflation and short-term interest rates. For the output gap they used (the log of) deviations of per capita nonfarm business output from a linear-fitted trend, for inflation they used the annualized growth rate in the implicit deflator for the nonfarm business output and for the short-term interest rate they used the 3-month Treasury bill rate.

Our analysis has four small differences from theirs. First, our plots only include the output and inflation data series and leave out the short-term interest rate plots. We left out the short-term interest rate plots to keep things simple and focus on the output and inflation dynamics, which are the main focus of most of the literature in this area. Second, for our analysis, output gap is obtained by implementing the Hodrick and Prescott (1997) filter to the data.⁵ Third, we consider the Fed funds rate as the short-term interest rate and fourth, we use a larger sample period (1965:1 to 2008:4) which also includes data from the last fifteen years that was not available to Fuhrer and Moore (1995).

The results of our calculations are provided in Figure 1. This figure shows that these small differences in the calculations have little effect on the dynamic patterns in

⁵This filter is designed to extract so called business cycle frequencies, that is frequencies corresponding to 2 and 8 year cycles, from the data. The result of this filter is a detrended data series which could also be interpreted as a measure of output gap.

the data. The autocorrelation functions for output and inflation, as well as the lead and lag correlation patterns, are almost identical to what was found by Fuhrer and Moore (1995). In particular, the diagonal elements in Figure 1 show that inflation is quite persistent, whereas as the output gap has somewhat less persistence. On the other hand, the off-diagonal elements together show the familiar lead and lag pattern of output and inflation, where output leads inflation when there is a positive correlation and inflation leads output when there is a negative correlation. Thus, a high level of output anticipates a high level of inflation about five quarters later (upper-right graph), while a high level of inflation is followed by a lower level of output about ten quarters later (lower-left graph).⁶ These correlations are interpreted as follows. The negative correlation of lagged inflation with current output is sometimes interpreted as indicating that high (low) inflation rates generally lead to tight (loose) monetary policy which reduces (increases) future output, while the positive current and future correlations are interpreted as indicating that high (low) output levels put (reduce) inflationary pressures on the economy leading to higher (lower) future inflation.

2.2 A new method for measuring leads, lags and persistence

In den Haan (2000) a new methodology for assessing the comovement of economic variables was developed.⁷ The method makes use of forecast errors for assessing comovement and is attractive for several reasons. First, the method does not require any modeling assumptions, such as a VAR ordering or structural assumptions on the error terms, to be applied. Second, it does not require that the data be detrended

⁶An almost identical empirical lead-lag pattern can also be obtained by using a univariate approach instead of a VAR approach. For instance, Galí and Gertler (1999) and Smets and Wouters (2007), following a univariate approach, report similar lead-lag patterns. These authors also plot leads and lags in the same diagram. Their depiction of the lead-lag pattern exhibits the well known *S*-shaped pattern with lagged values of inflation exhibiting negative correlations with current output, and current and future values of inflation exhibiting positive correlations with current output.

⁷In addition to den Haan (2000), other applications of this approach include den Haan and Sumner (2004) and María-Dolores and Vázquez (2008).

or that the variables in the model have identical orders of integration.⁸

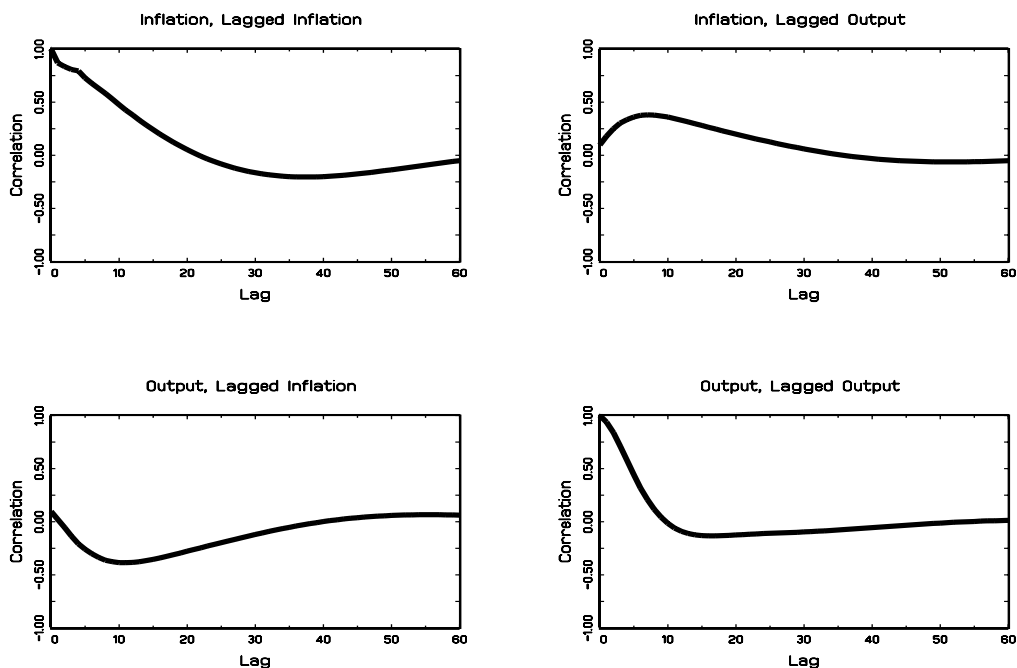


Figure 1: Autocorrelation and lead/lag pattern of output and inflation based on a VAR

Another salient feature of the den Haan (2000) approach is the interpretation for the sources of fluctuations. As in typical VAR methods, the fluctuations in both the data and thus in the forecast errors originate from some underlying structural shocks which could be associated with the various variables in the model. However, the method does not need to identify exactly which structural shocks play a role in any particular equation and can be left unspecified. One simply envisions that all of the structural shocks play some role in each of the model variables and the comovements

⁸Avoiding detrending of the data is useful because den Haan (2000, p. 5) argues that the negative correlation between output and prices often found in the data could be an artifact of common detrending procedures used to make the data stationary. Moreover, Fuhrer and Moore (1995) devoted several pages to discussing the order of integration of output, inflation and interest rates and nonconclusive evidence was found.

in the observed data are shaped by the importance of these structural shocks in the variables for which comovements are being investigated, but sorting out which of the structural shocks are important is not necessary.⁹

The focus in den Haan (2000) was on contemporaneous comovements of the economic variables, but for our investigation, we are interested in more than just that. Here we extend this methodology to look at not only the contemporaneous comovements, but also lead and lag comovements and autocorrelation functions in order to analyze inflation and output persistence. This provides a more complete description of the data dynamics. Such lead and lag and persistence analyses are familiar to readers of the modern dynamic macroeconomic literature. However, the technique here provides a broader format for describing the data dynamics than the approach used in the macroeconomic literature as well.

We begin by running a VAR of the form

$$X_t = \mu + Bt + Ct^2 + \sum_{l=1}^L A_l X_{t-l} + \varepsilon_t \quad (1)$$

where A_l is an $N \times N$ matrix of regression coefficients, μ , B , and C are N -vectors of constants, ε_t is an N -vector of innovations, and the total number of lags included is equal to L . The ε_t are assumed to be serially uncorrelated, but the components of the vector can be correlated with each other. For the application here, we run trivariate VARs, so $N = 3$. Also, following popular forecasting practice, we let $L = 4$, so there is one full year worth of lags in the VAR.

From this VAR, forecast errors can be computed for alternative forecast horizons. A particular N -vector of forecast errors can then be viewed as the cyclical component of X_t for a particular forecast horizon. Thus, the forecast errors associated with short-term horizons would tend to capture more of the high-frequency components of the data whereas long-term forecast errors would tend to emphasize relatively more low-frequency components. Each of these forecast errors, or cyclical components,

⁹One limitation of this approach is that it does not provide standard impulse response functions which show the responses of each endogenous variable to alternative structural shocks. However, den Haan (2000) views this as a positive feature as he notes that such standard impulse response analysis requires an identification structure which is often the subject of some dispute.

obtained from the different equations at various forecast horizons can then be used to compute contemporaneous correlations for the forecast errors from the different equations at various forecast horizons as in den Haan (2000).

In our analysis, we extend this approach by further using these forecast errors to compute cross correlations at various leads and lags as well as autocorrelation functions. In particular, the cross correlations are computed by matching forecast errors for the same forecast horizon where each forecast is based on an information set that differs with a specific number of leads or lags. These calculations provide a more complete dynamic perspective of comovement than the alternative approaches suggested by Fuhrer and Moore (1995), Galí and Gertler (1999) and den Haan (2000) by not only showing how the data comove at leads and lags, but also by showing how the data comove at leads and lags at alternative forecast horizons. These alternative forecast horizons thus tell us if the lead and lag patterns are arising due to more short term or more long term components of the data. In the next subsection, we show how this system of lead and lag correlations between forecast errors can be plotted against the forecast horizon to conveniently assess the lead and lag structure of the data.

2.3 New insights into the data comovements

This subsection is broken down into two smaller sections in order to keep the discussion clear. We begin by looking at the lead and lag results between output and inflation. Next, the persistence of inflation and output is discussed.

2.3.1 Lead and lag relationships between output and inflation

Figure 2 presents a set of six diagrams for the forecast error correlations between output and inflation. One common element in all the diagrams is the contemporaneous correlation which is plotted at various forecast horizons in each diagram by a dashed line. Each of the six diagrams then has a lead-lag pair in which a contemporaneous forecast error for output is matched with a lead (thick solid line) or a lag (thin solid line) forecast error for inflation. The upper left diagram has a lead-lag

pair in which the correlations are for inflation eight quarters, or two years, ahead or behind output, while the upper right diagram has a lead-lag pair corresponding to six quarters, the middle left diagram has a lead-lag pair corresponding to four quarters, the middle right has a lead-lag pair corresponding to three quarters, the lower left has a lead-lag pair corresponding to two quarters and the lower right has a lead-lag pair corresponding to one quarter. A useful comparison of these diagrams can be made with the off-diagonal graphs in Figure 1 by noting that if one focuses on the lead lines in Figure 2 and one moves upward through the diagrams (i.e. one moves through the diagrams with progressively longer leads), it is the same type of exercise as moving from the origin to the right in the upper-right diagram of Figure 1, while if one focuses on the lag lines in Figure 2 and one moves upward through the diagrams (i.e. moves through the diagrams with progressively longer lags), it is the same type of exercise as moving from the origin to the right in the lower-left graph in Figure 1.

Interpreting the diagrams borrows insights from both the Fuhrer and Moore (1995) and Galí and Gertler (1999) approach and the den Haan (2000) approach. As in Fuhrer and Moore (1995) and Galí and Gertler (1999), places where the lead correlation is higher than the contemporaneous correlation, one would interpret output as leading inflation. Furthermore, as in den Haan (2000), the horizontal axis represents the forecast horizon and provides information about whether the correlation occurs in the short run or long run. Situations in which the lead line exceeds the contemporaneous line toward the right edge of the diagram would indicate that output leads inflation at longer forecast horizons. Because alternative filters used in the literature (for instance, the Hodrick and Prescott filter used by Galí and Gertler (1999) or the linear-trend filter used by Fuhrer and Moore (1995)) are often set to isolate so called business cycle frequencies, our diagrams have as their highest forecast horizon 32 quarters (i.e. 8 years). We use forecast horizons as low as one quarter, so the left side of the diagrams consists of short term correlations. These correlations are typically low because of the high percentage of noise at short term forecast horizons.

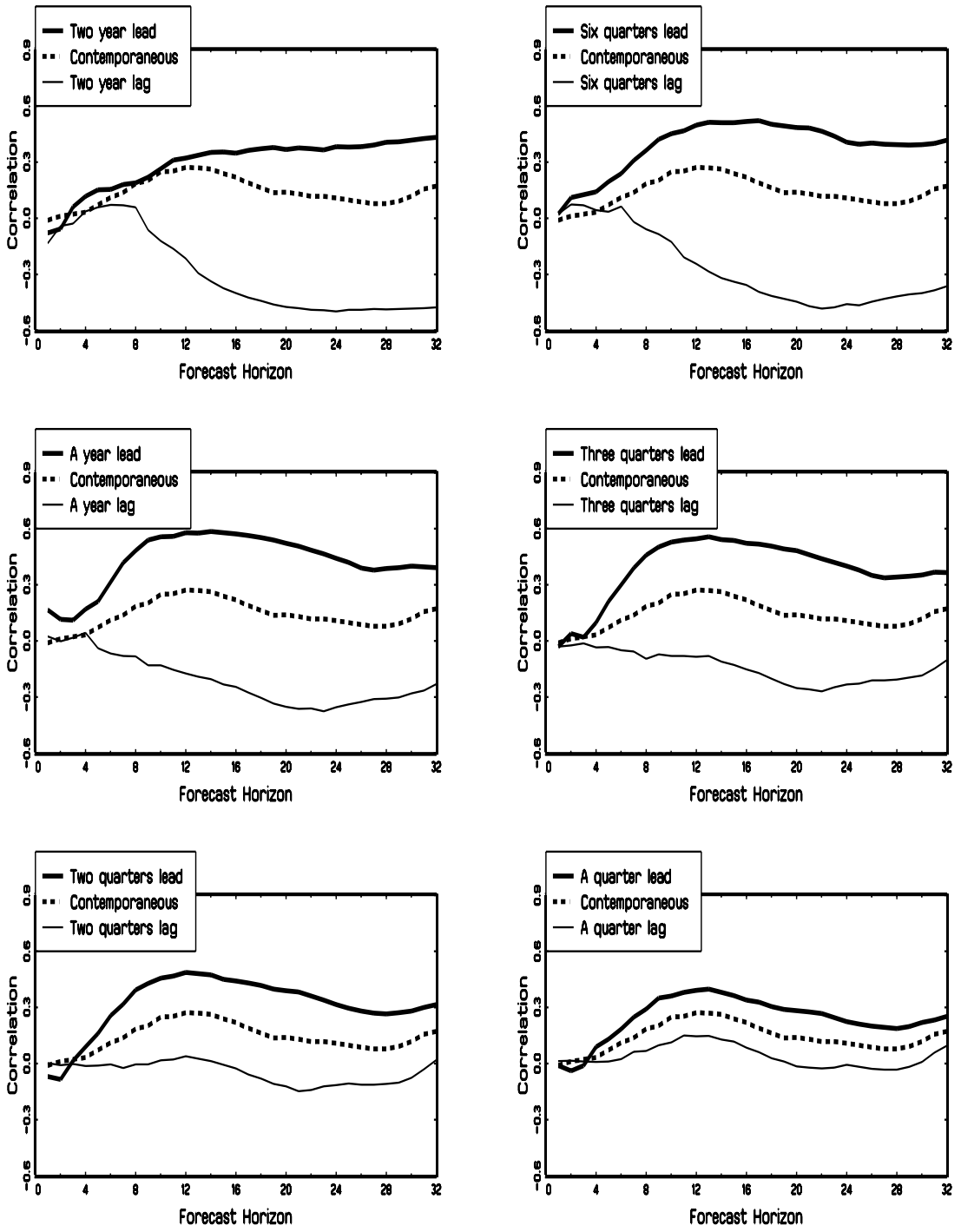


Figure 2: Actual comovement between output and inflation

To be more concrete about the actual results, let us start by walking through the middle right diagram in Figure 2.¹⁰ To conduct this analysis, it is important to recognize that the lead plot in essence decomposes the single three quarter correlation value in the upper right diagram of Figure 1, the lag plot in essence decomposes the single three quarter correlation value in the lower left diagram in Figure 1, and the contemporaneous correlation plot in essence decomposes the contemporaneous correlation value which is the left edge value of both the upper right and lower left diagrams in Figure 1.

First notice that the contemporaneous correlation plot in Figure 2 is relatively low and ranges between 0.1 and 0.25 over all the forecast horizons as emphasized by María-Dolores and Vázquez (2008). These values are in line with the contemporaneous correlation displayed on the left edge of the upper right and lower left diagrams in Figure 1. Next note that both the lead and lag lines are close to zero for the first 4 quarters. This is because the population moment (i.e. the population correlation) between the inflation forecast error four quarters ahead (or behind) and the current forecast error for output is zero. As one moves past the four quarter horizon, the lead line moves up positively and the lag line moves down negatively. These results indicate that at all forecast horizons, (i) high values of output lead to high values of inflation three quarters later, and (ii) high values for lagged inflation anticipate low values for output three quarters later. Both results are consistent with those displayed in Figure 1. What is new here is that the lead and lags have been broken down by forecast horizons. Since the forecast horizons are loosely related to frequencies, with short-term forecast horizon errors emphasizing high frequencies

¹⁰It is possible to use standard bootstrapping methods to find confidence bands around the correlation plots. Such confidence bands were generated using programs from den Haan's web site and showed sufficiently wide bands that the individual correlation plots were not significantly different from each other. However, as in Fuhrer and Moore (1995) and numerous others, we still interpret maximal correlations that are different from the contemporaneous correlation as indicating a lead or lag. Because the bands did not indicate significance, they are not provided here, but sample plots can be obtained from the authors upon request.

and the long term horizons emphasizing low frequencies, we see that the rising lead line and the falling lag line tells us that the positive lead and negative lag values in Figure 1 are due mostly to medium and longer term (low) movements (frequencies). Since the lead plot has a hump shape to it, we see that the medium term movements are somewhat more important than the long term movements for producing the lead of output over inflation. Similarly, since the lag plot has a cup shape to it, we see that the medium term movements are somewhat more important than the long term movements for producing the lag of output over inflation. Looking at the other diagrams in Figure 2 shows similar results with the curves spreading out for the medium term forecast horizons, but still maintaining a sizable lead or lag for the longer forecast horizons. These also indicate the values in Figure 1 are mostly due to the medium term movements, but the longer term movements also have a role.

It is also useful to note that, loosely speaking, the correlations in Figure 1 are recovered as the forecast horizon approaches infinity.¹¹ So looking at the middle left diagram in Figure 2 (with 1-year leads and lags) and focusing on the right edge of the lead and lag lines, we see that the right edge of the lead is somewhat higher than the right edge of the lead in the middle right diagram (with a 3-quarter lead) and the right edge of the lag is somewhat lower than the right edge of the lag -in the middle left diagram (with a 3-quarter lag). Recognizing this shows that Figure 2 also captures the *S*-shaped pattern described above in the discussion of Figure 1, where the *S*-shaped pattern terminology is used to describe the Galí and Gertler (1999) way of plotting the lead and lag results. What Figure 2 shows is that this *S*-shaped pattern not only exists at the aggregate, but it is also true across all forecast horizons and that the *S*-shape is due mostly to the medium term movements.

Finally, it is useful to understand economically, what these lead and lag plots are saying. The low correlation values at the short term horizons are indicating that

¹¹The “loosely speaking” qualification is important when considering situations when some variable in the VAR is non-stationary. In this case, the correlation coefficient of the forecast errors might not converge to the unconditional correlation coefficient of the two time series as the forecast horizon goes to infinity, but it can be estimated consistently, as shown in den Haan (2000), since forecast errors are stationary for a fixed horizon.

there is a relatively high amount of noise in the short term, thus driving down the correlations. On the other hand, the hump shape in which medium term forecast horizons are higher than long term forecast horizons point to a long run classical dichotomy where output and inflation movements are relatively independent. What the diagrams indicate is that most of the lead and lag correlations that people are familiar with in the Fuhrer and Moore (1995) analysis are arising from comovement between the data in the medium term. Furthermore, without the decomposition presented here, the aggregate numbers of the Fuhrer and Moore (1995) approach would not provide this insight.

2.3.2 Persistence of inflation and output

Figure 3 shows the autocorrelation functions for inflation and output. These functions have been computed using the forecast error decomposition and thus are useful for understanding whether the autocorrelations are due to short term or long term components of the data as well.

To understand how these plots are calculated, first focus on the solid line in each of the diagrams of Figure 3. These plots correspond to the first order autocorrelation value of inflation and output from the standard Box-Jenkins calculations, only here the first order autocorrelations correspond to the cyclical component of the variable associated with the forecast horizon enumerated on the horizontal axis. Each point in these plots is computed by computing a vector of n -step ahead forecast errors and then using this vector to compute the first order autocorrelation for the n -step ahead forecast horizon. The other plots are computed in a similar way. Each point in the two lag plot is computed by computing a vector of n -step ahead forecast errors and then using this vector to compute the second order autocorrelation for the n -step ahead forecast horizon.

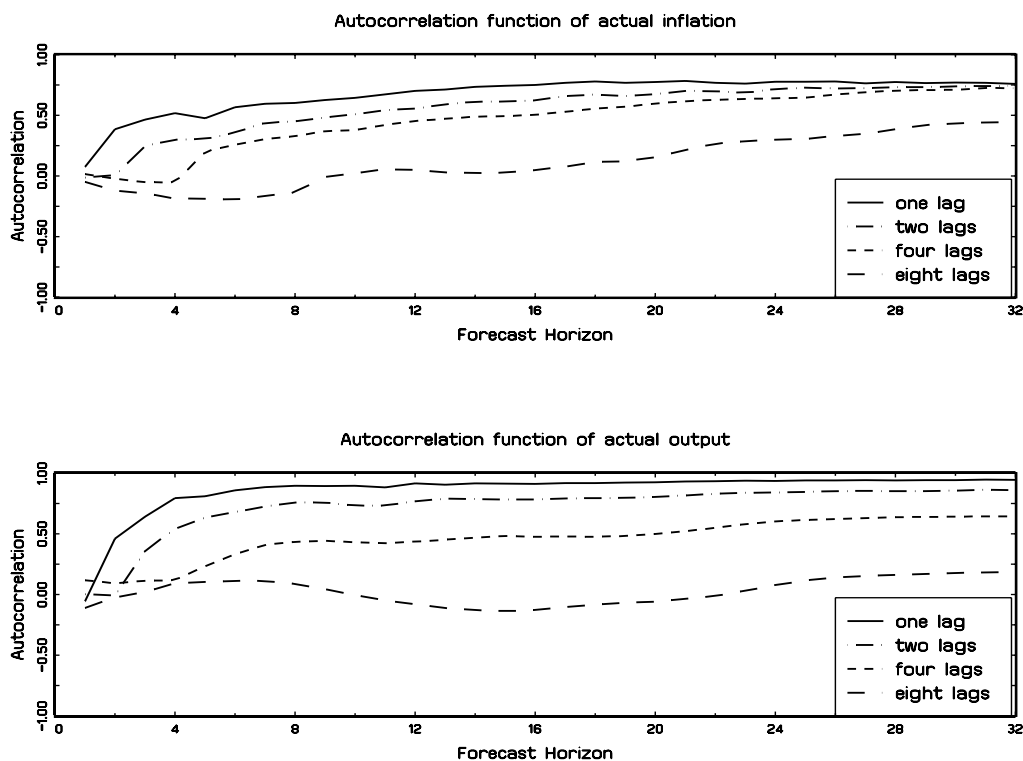


Figure 3: Inflation and Output Persistence

One useful point of reference to the Fuhrer and Moore (1995) calculations (i.e. the diagonal diagrams in Figure 1) is to note that the Box-Jenkins autocorrelation value is, loosely speaking, recovered as the forecast horizon approaches infinity.¹² This means that the right edge values of these diagrams are approximately equal to the values that Fuhrer and Moore (1995) compute in their plots. Focusing on this right edge, we see that the autocorrelation function for inflation falls off much more slowly than the correlation function for output. This illustrates the well known inflation persistence observation which so many NKM seek to match. By comparing the first order autocorrelation of inflation in Figure 1 (0.87) and Figure 3 (0.76) we observe

¹²Again, the “loosely speaking” qualification is important when considering situations when some variable in the VAR is non-stationary. In this case, the autocorrelation of the forecast horizon does not converge to the Box-Jenkins autocorrelation as the forecast horizon goes to infinity, but it can be estimated consistently for a fixed horizon.

that our cyclical component based on long-term forecast errors is less persistent than inflation itself. The intuition for this is simple. Our measures of cyclical inflation remove any linear and quadratic trend from actual inflation data thus reducing the long term correlation. These lower correlation results are in line with those described in Cogley and Sbordone (2008, p. 2111, Table 1) who also removed trends using a different technique.

Also of interest is to note that, as in the lead and lag analysis above, the forecast errors at different horizons can be interpreted as capturing more or less of the short term or long term components of the data, with the short term forecast horizons capturing more of the high frequency components of the data and the long term forecast horizons capturing more of the low frequency components of the data. Using this insight, we see that neither the inflation or output autocorrelation plots exhibit the hump shapes seen in the lead and lag analysis. This means that both the inflation and output persistence observed in the data is due to medium and long term components of the data.

3 A New Keynesian Model with built in persistence

This section describes a NKM with a number of different structures designed to induce persistence. Many of these structures are quite standard in the literature, so we will briefly review them. The purpose for describing this model is so that later we can explore which of these persistence structures are most effective at generating the type of comovement and persistence patterns seen in the output and inflation data.

The model is a general equilibrium model with three key equations that jointly influence the way that the economy behaves. In the IS curve, persistence is induced through a generalized habit persistence structure as well as through a shock process with persistence. In the NKPC, persistence is induced through a so called, “hybrid” structure suggested by Galí and Gertler (1999) as well as through a persistent shock process. And in the monetary policy rule, persistence is introduced through

a rule that incorporates several data features for guiding interest rates, including policy inertia and the standard connection to output and inflation, but also adding a connection to the growth rate of output as suggested by Smets and Wouters (2007).

3.1 An IS curve based on generalized habit persistence

The demand for goods, or IS curve, is based on an optimizing agent structure. Here we add to the standard IS derivation a generalized habit persistence formulation which induces considerable persistence in demand.

The IS curve is derived from a representative consumer optimization problem in which the consumer maximizes

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left(\frac{1}{1 - \frac{1}{\tau}} \right) \left(c_t - \frac{\gamma(\alpha c_{t-1} + \alpha^2 c_{t-2} + \alpha^3 c_{t-3} + \alpha^4 c_{t-4})}{\sum_{i=1}^4 \alpha^i} \right)^{\left(1 - \frac{1}{\tau}\right)} \right\}$$

subject to

$$c_t + s_t = y_t + R_t s_{t-1},$$

where c_t , y_t , s_t and R_t denote consumption, income, savings and gross real return at period t , respectively. The parameters β and τ denote the discount factor and the intertemporal elasticity of substitution, respectively. The parameters γ and α control the habit persistence structure. When $\gamma = 0$, habit persistence disappears and the associated IS curve collapses into the standard IS curve described in the basic NKM. When $\gamma > 0$, habit persistence is present and the parameter α comes into play. This parameter controls the way in which habit persistence enters the model. When $\alpha = 1$, the habit consists of an equally weighted index of consumption over the last four quarters. When $\alpha < 1$, the habit overweights the most recent consumption level, while when $\alpha > 1$, the habit overweights consumption four quarters earlier.¹³ As $\alpha \rightarrow 0$, the model approaches the standard one-period lag habit model.

¹³The habit formation structure introduced here could be a potential candidate for capturing seasonal patterns in the data. Since we are dealing with seasonal adjusted data in this paper, as in the related literature, there is no need to say much about this feature here. But this structure may be useful in other contexts where unadjusted data is important. Some authors have warned about the bias introduced in empirical analysis when considering seasonal adjusted data instead of raw (unadjusted) data.

Using standard optimization techniques followed by standard linearization methods, the IS curve can be shown to be given by

$$\begin{aligned}
& -\Delta\tilde{y}_{t-3} + \left(\frac{\gamma\beta}{\Omega} - \frac{1}{\alpha}\right)\Delta\tilde{y}_{t-2} + \left(\frac{\gamma\beta^2}{\Omega} + \frac{\gamma\beta}{\Omega\alpha} - \frac{1}{\alpha^2}\right)\Delta\tilde{y}_{t-1} \\
& + \left(\frac{\gamma\beta^3}{\Omega} + \frac{\gamma\beta^2}{\Omega\alpha} + \frac{\gamma\beta}{\Omega\alpha^2} - \frac{1}{\alpha^3}\right)\Delta\tilde{y}_t + \left(\frac{\gamma\beta^4}{\Omega} + \frac{\gamma\beta^3}{\Omega\alpha} + \frac{\gamma\beta^2}{\Omega\alpha^2} + \frac{\gamma\beta}{\Omega\alpha^3} + \frac{\Omega}{\gamma\alpha^4}\right)E_t[\Delta\tilde{y}_{t+1}] \\
& + \left(\frac{\gamma\beta^4}{\Omega\alpha} + \frac{\gamma\beta^3}{\Omega\alpha^2} + \frac{\gamma\beta^2}{\Omega\alpha^3} - \frac{\beta}{\alpha^4}\right)E_t[\Delta\tilde{y}_{t+2}] + \left(\frac{\gamma\beta^4}{\Omega\alpha^2} + \frac{\gamma\beta^3}{\Omega\alpha^3} - \frac{\beta^2}{\alpha^4}\right)E_t[\Delta\tilde{y}_{t+3}] \\
& + \left(\frac{\gamma\beta^4}{\Omega\alpha^3} - \frac{\beta^3}{\alpha^4}\right)E_t[\Delta\tilde{y}_{t+4}] - \left(\frac{\beta^4}{\alpha^4}\right)E_t[\Delta\tilde{y}_{t+5}] \\
& + \frac{\Omega}{\gamma\alpha^4}(\tau(\gamma - 1))\left(1 - \frac{\gamma}{\Omega}(\beta + \beta^2 + \beta^3 + \beta^4)\right)[i_t - E_t[\pi_{t+1}]] + \frac{\Omega}{\gamma\alpha^4}g_t = 0. \tag{2}
\end{aligned}$$

where Δ is the first-difference operator, $\Omega = \sum_{i=1}^4 \alpha^i$, and \tilde{y}_t , π_t and i_t denote output, inflation and the nominal interest rate deviations from their respective steady state values.¹⁴ Finally, g_t denotes an IS shock which is assumed to be governed by

$$g_t = \rho_g g_{t-1} + \varepsilon_{gt}, \tag{3}$$

where ε_{gt} are innovations which are identical and independently distributed over time with variance σ_g^2 .

3.2 The hybrid Phillips curve

The supply of goods is captured by a NKPC. Lubik and Schorfheide (2004) use a forward looking NKPC given by

$$\pi_t = \beta E_t \pi_{t+1} + \kappa(\tilde{y}_t - z_t), \tag{4}$$

where z_t captures exogenous shifts in the marginal costs of production and is often referred to as an inflation-push shock. It is assumed to be governed by

$$z_t = \rho_z z_{t-1} + \varepsilon_{zt}, \tag{5}$$

where ε_{zt} are independent over time, as well as from the ε_{gt} terms, and they have variance σ_z^2 . There is a general preference in the New Keynesian literature for a

¹⁴These calculations can be obtained from the authors upon request. The use of the first-difference operator is just to simplify the IS curve expression a little bit, which is nevertheless quite cumbersome. Moreover, notice that this way of writing the IS curve only makes sense whenever $\gamma \neq 0$.

strictly forward looking NKPC like this one because it can be motivated by the standard Calvo (1983) and Taylor (1980) contracting story as described in Galí and Gertler (1999). Under this formulation, κ measures the slope of the NKPC and is related to other structural parameters by

$$\kappa = \left(\frac{1}{\tau}\right) \frac{(1-\theta)(1-\beta\theta)}{\theta},$$

where θ denotes Calvo's probability, i.e. the fraction of firms that do not adjust prices optimally in a particular period.

Some presentations, such as Galí and Gertler (1999), further augment the NKPC to include a backward looking component such as

$$\pi_t = \frac{\beta}{1+\beta\omega} E_t \pi_{t+1} + \frac{\omega}{1+\beta\omega} \pi_{t-1} + \frac{\kappa}{1+\beta\omega} (\tilde{y}_t - z_t). \quad (6)$$

This NKPC is typically referred to as a hybrid NKPC. In these formulations, the additional component $\frac{\omega}{1+\beta\omega} \pi_{t-1}$ is typically motivated by an empirical need rather than microfoundations, and because of this lack of a formal foundation, the hybrid version is considered less attractive.

Since the strictly forward looking NKPC is a special case of the hybrid curve which imposes $\omega = 0$, we will work with this more general possibility. We wish to investigate the degree to which it is possible to match the data dynamics best. As we show below, it is possible to match the lead, lag and persistence characteristics of the data without the backward looking term. However, this term is helpful in improving the fit.

3.3 A persistent policy function

To complete the model, we consider a policy rule that is similar to the one used by Smets and Wouters (2007). According to this rule, nominal interest rate policy responds to output, inflation and the growth rate of output according to

$$i_t = \rho i_{t-1} + (1-\rho)[\phi_1 \pi_t + \phi_2 (\tilde{y}_t - z_t)] + \phi_3 [(\tilde{y}_t - z_t) - (\tilde{y}_{t-1} - z_{t-1})] + v_t, \quad (7)$$

where ϕ_1 , ϕ_2 and ϕ_3 are the sensitivities of policy to the various economic variables, ρ captures policy inertia and the shock v_t is independent over time and from the ε_{gt}

and ε_{zt} , and has variance σ_v^2 . One attraction of this formulation for policy is that it has the popular Taylor rule as a special case. The Taylor rule arises when $\rho = 0$ and $\phi_3 = 0$. In addition, the policy rule used by Lubik and Schorfheide (2004) is a special case when $\phi_3 = 0$.

3.4 Model simulations

The model is simulated using the method suggested by Lubik and Schorfheide (2003) that builds on Sims (2002) approach. This approach is straightforward to apply and simply requires writing the system of equations (2), (6), (7), (3) and (5), along with several identities involving forecast errors, in a matrix form

$$\Gamma_0 X_t = \Gamma_1 X_{t-1} + \Phi \varepsilon_t + \Pi \eta_t, \quad (8)$$

where

$$\begin{aligned} X_t &= (\tilde{y}_t, \tilde{y}_{t-1}, \tilde{y}_{t-2}, \tilde{y}_{t-3}, \pi_t, i_t, E_t \tilde{y}_{t+1}, E_t \tilde{y}_{t+2}, E_t \tilde{y}_{t+3}, E_t \tilde{y}_{t+4}, E_t \tilde{y}_{t+5}, E_t \pi_{t+1}, g_t, z_t)', \\ \varepsilon_t &= (\varepsilon_{gt}, \varepsilon_{zt}, v_t)', \\ \eta_t &= (\tilde{y}_t - E_{t-1} \tilde{y}_t, E_t \tilde{y}_{t+1} - E_{t-1} \tilde{y}_{t+1}, E_t \tilde{y}_{t+2} - E_{t-1} \tilde{y}_{t+2}, E_t \tilde{y}_{t+3} - E_{t-1} \tilde{y}_{t+3}, \\ &\quad E_t \tilde{y}_{t+4} - E_{t-1} \tilde{y}_{t+4}, \pi_t - E_{t-1} \pi_t)'. \end{aligned}$$

These equations are then programmed into computer code and simulated using routines available on the web.¹⁵

4 Fitting the model to the data

In this section, we investigate whether our general NKM is able to capture the lead, lag and persistence characteristics described in Section 2. In order to keep our exercise clear, we have organized this section into two subsections. In the first subsection, we describe our fitting approach. We then apply this approach and fit our model using a number of different groups of fitting targets. Because these different groups

¹⁵The GAUSS code for computing the equilibria of LRE models was downloaded from Schorfheide's web-site.

have different targets, the implied parameters resulting from the different exercises imply different performance characteristics for the fit model.

The next subsection describes the performance results of the various fitting exercises. That subsection is broken into two smaller sections, each focusing on a different aspect of the data characteristics. The first focuses on the lead and lag patterns between output and inflation, while the next one focus on the persistence patterns of inflation and output.

4.1 The empirical approach

The paper uses an empirical approach that is similar to methods matching impulse response functions as in Rotemberg and Woodford (1997) and Christiano, Eichenbaum and Evans (2005), among others.¹⁶ As in these papers, we split the model parameters in two groups. The first group is formed by the pre-assigned parameters β , τ and κ . Accordingly, we set $\beta = 0.99$, $\tau = 0.5$ and $\kappa = 0.25$ corresponding to standard values assumed in the relevant literature for the discount factor, the consumption intertemporal elasticity and the Phillips curve slope, respectively.¹⁷

The second group of parameters is chosen to minimize the value of a quadratic distance function. This objective has two key differences from the methods matching impulse response functions. First, we match a different set of moments, including lead and lag correlations between output and inflation, variances of inflation and output, and the autocorrelation function of inflation. Second, because we are not interested in testing, we do not compute standard errors and other formal statistics, and instead simply report the parameter values which match the summary statistics the best.

Given the aim of the paper, the alternative fitting exercises consider a set of statistics that describe the lead, lag and persistence patterns of output and inflation

¹⁶We also investigated a full-information Bayesian maximum-likelihood approach (see, for instance, Fernández-Villaverde and Rubio-Ramírez (2004) and Lubik and Schorfheide (2004)) to estimate the general equilibrium model suggested in this paper, but the parameter estimates resulted in a bad fit of the lead, lag and persistence patterns seen in actual data.

¹⁷Notice that these parameters are consistent with a Calvo's probability value equal to 0.81, which implies that firms roughly revise their optimal prices every five quarters on average.

data. As noted in footnote 1, these dynamic patterns are not likely (structural) stable. Therefore, they are not as appropriate for estimating structural parameters such as the ones assigned to the first group above, but are appropriate for the policy and shock process parameters. This insight further motivates the split of the model parameters in two groups.

The fitting approach works as follows. First, K summary statistics are obtained from the observed data. Using notation from Christiano, Eichenbaum and Evans (2005), these summary statistics are arranged in a vector denoted by $\widehat{\Psi}$. Then the model is simulated M times for an equal number of periods as the number of periods in the observed data.¹⁸ For each simulation, the same summary statistics are computed. These summary statistics are then averaged over the M simulations. Let $\Psi(\xi)$ denote the mapping from the remaining model parameters, which are denoted as a group by ξ , and the model summary statistics. These model summary statistics are then compared to the summary statistics in the data to compute a quadratic distance function of the statistical comparisons, which is then minimized via standard optimization methods. This distance function can be written formally as

$$J = \min_{\xi} \left[\widehat{\Psi} - \Psi(\xi) \right]' \left[\widehat{\Psi} - \Psi(\xi) \right],$$

where we are implicitly using the identity weighting matrix as in Rotemberg and Woodford (1997).

For our exercise, the K summary statistics include both correlation and variance information as well as moments capturing inflation persistence. All fitting exercises were based on calculations where the number of simulations, M , was set to 50.¹⁹ We carried out many different fitting exercises using different groups of statistics. The purpose for looking at these different groups of statistics is to evaluate the model performance when different characteristics were emphasized. The results of five of

¹⁸To be more precise, there are 177 observations in the data. To match this length, the model is simulated for 354 periods and the first half of the periods is discarded to move the model away from its initial conditions.

¹⁹A robustness exercise was carried out by using $M = 100$, and the results were not found to be sensitive to the choice of M .

these fitting exercises are presented in Table 1.

Column 1 of Table 1 provides a list of the parameter values that were used in these estimation searches while Column 2 shows the support interval chosen for each parameter. These support intervals reflect our view of the set of reasonable parameter values based on the estimates provided by a vast empirical literature estimating alternative versions of the NKM model. The remaining columns provide estimated parameter values for various different fitting exercises.

The results in Columns 3, 4 and 5 provide parameter values for fitting exercises that emphasized the lead and lag patterns between output and inflation described in Section 2 as well as the variances for output and inflation, while the fitting exercises in Columns 6 and 7 added additional information on the inflation persistence to the set of summary statistics. Our motivation for these exercises is described in the next subsection, but for now we will describe the exercises in a more mechanical fashion.

For the Columns 3, 4 and 5 results, 162 summary statistics were used. The first 160 summary statistics included the lead, lag and contemporaneous correlations that are plotted in the middle-left and bottom-right diagrams of Figure 2. Since the contemporaneous correlations in both diagrams are the same, in effect the summary statistics include the correlations represented by only 5 lines plotted: the two lead plots, the two lag plots and the one contemporaneous correlation plot. Each of these lines has 32 correlations, so the 5 together gave us 160 summary statistics. Since this is a large number and the information conveyed by those moments is somewhat redundant with the other four diagrams in Figure 2, we decided not to use the lead and lag plots from these other diagrams in the parameterization exercise, but nevertheless we also pay attention to them in assessing the ability of the model in reproducing the lead, lag and persistence patterns. Next, since these 160 summary statistics are only correlations, they do not necessarily fit variances very well. So, in order to target the variances better, we added 2 standard deviation statistics. These include the standard deviation of inflation and the standard deviation of Hodrick-Prescott detrended output. Altogether, this gave us 162 summary statistics for these exercises.

The differences between the Column 3, 4 and 5 exercises reflect an interest in investigating the range of possible model structures (i.e. habit formation and the backward looking component of the hybrid NKPC) that can capture the lead and lag patterns between output and inflation. The first exercise in Column 3 is unrestricted and all parameters were free to adjust. The results show that both habit formation and the backward looking component of the NKPC play a role as well as IS and inflation-push shocks, while the policy shock was essentially zero with a value of $\sigma_v = 1.1e - 06$. This unimportant role for monetary policy shocks to reproduce the lead and lag pattern is in line with the results in Smets and Wouters (2007, p.601) based on a cross-covariance decomposition of shock contributions. Since $\alpha > 1$, the habit persistence structure weights recent consumption more weakly than consumption further in the past.

To investigate if it was possible to obtain a good fit without the habit formation structure, a restricted fitting exercise, summarized in Column 4, in which $\gamma = 0$, was undertaken. This exercise resulted in higher values for the parameters characterizing persistence of the IS and inflation-push shocks (i.e. ρ_g and ρ_z) as well as higher values for the policy inertia parameter, ρ , while the backward looking parameter of the NKPC is smaller ($\omega = 0.44$). Next Column 5 constrained the backward looking parameter of the NKPC to be zero, $\omega = 0$. This exercise resulted in (i) the habit persistence structure becoming unimportant, and (ii) large values for all parameters characterizing persistence (ρ , ρ_g and ρ_z). As discussed below, the three parameterizations displayed in columns 3-5 do a reasonable job in matching the lead-lag patterns obtained from actual data. However, all of them fail dramatically in reproducing inflation persistence.

The Columns 6 and 7 fitting exercises were undertaken in an effort to improve the inflation persistence fit beyond what was achieved in the other exercises. Again, we will explain the details more fully below, but for now a mechanical explanation will suffice. Here, we added to the 162 summary statistics in the earlier exercises, additional summary statistics which were obtained from the inflation autocorrelation

plots in Figure 3. In particular, we added the 32 first-order autocorrelations of the inflation forecast errors and the 32 eight-order autocorrelations of the inflation forecast errors, bringing the total number of summary statistics up to 226. As in the case of the lead and lag plots, we focused only on a subset of the autocorrelation plots since the information in these plots is related to the information in the other plots and two plots proved to be sufficient to ensure a good match for all the autocorrelations. The fit in Column 6 presents the unrestricted search of the parameter space, while the one in Column 7 constrained the backward looking parameter of the NKPC to be zero.

The bottom panel in Table 1 shows the standard deviations of actual inflation and (Hodrick-Prescott detrended) output data together with the standard deviations of simulated data obtained from the alternative parameter values. In general, we see the model underestimates output volatility whereas the opposite is true for inflation volatility.

Table 1 - Estimation Results

		$K = 162$			$K = 226$	
		Restrictions			Restrictions	
	Support	None	$\gamma = 0$	$\omega = 0$	None	$\omega = 0$
γ	(0, 1)	0.2153	0.0000	0.0000	0.0048	0.2233
α	(0, ∞)	5.0015	11.2314	10.0434	2.0159	3.0040
ω	(0, 1)	0.9465	0.4436	0.0000	0.4464	0.0000
ρ	(0, 1)	0.9152	0.9703	0.9764	0.8772	0.8465
ϕ_1	(1, 2.5)	2.4882	2.4882	2.4878	2.4882	2.4881
ϕ_2	(0, 1)	0.0000	0.0000	0.0000	0.0000	0.0000
ϕ_3	(0, 1)	0.0000	0.0000	0.00	0.0000	0.0000
ρ_g	(0, 0.99)	0.8641	0.9828	0.9836	0.9679	0.9574
ρ_z	(0, 0.99)	0.7149	0.7357	0.7150	0.5389	0.6853
σ_g	(0, ∞)	0.0016	$3.6e - 04$	$2.9e - 04$	0.0011	0.0010
σ_z	(0, ∞)	0.0096	0.0111	0.0133	0.0135	0.0101
σ_v	(0, ∞)	$1.1e - 06$	$6.3e - 06$	$1.2e - 05$	$5.7e - 06$	$3.6e - 06$
Volatility statistics						
		$K = 162$			$K = 226$	
		Restrictions			Restrictions	
	Actual	None	$\gamma = 0$	$\omega = 0$	None	$\omega = 0$
σ_y	2.1101	1.0004	0.9409	1.1230	0.8238	0.7493
σ_π	2.3604	2.7483	2.7590	2.7460	2.7339	2.7423

4.2 Simulated data performance

4.2.1 Lead and lag patterns in inflation and output

The alternative parameterization results displayed in Table 1 do a good job in reproducing the lead and lag pattern between output and inflation seen in the data. These results show that there are many ways to achieve the lead and lag pattern. In fact, there are more ways to achieve a lead and lag fit than just the ones displayed, but we have only included these five to illustrate what is needed to get the pattern right. The basic requirements are: 1) the model needs to have both demand equations and supply equations with their own stochastic elements; 2) the model needs to get the relative proportions for the supply and demand shock variances just right (namely, σ_z larger than σ_g); and 3) the model needs to get the relative persistence for the demand and supply shocks just right (namely, ρ_g larger than ρ_z). In addition, both the habit persistence and the backward looking component of the NKPC do provide a slightly better fit of the lead-lag patterns, but they do not seem to be crucial.

Before turning to the simulated model results and understanding the specifics of how the model works, let us first consider a few more general insights about these requirements. The need for a general equilibrium structure stressed in this paper, with both demand and supply equations that have stochastic elements, can be recognized by noting the difficulty with which earlier papers that focused on single equation NKPC models had in achieving the lead and lag pattern. However, here, we see that such a pattern can be obtained under many different structures. Next, the relative variances and the relative persistence for supply and demand shocks is determined to a certain extent by whether the habit formation structure and the backward looking component of the NKPC are present. Moreover, a comparison of all parameterizations illustrates the unimportance of output gap, output growth and monetary shocks (i.e. σ_v is always small) in the policy rule.

To better understand how the model works, now focus on results from simulations based on the parameterization in Column 6.²⁰ Figure 4 plots lead and lag patterns

²⁰The rest of parameterizations in Columns 3, 4, 5 and 7 produce qualitatively similar diagrams

based on simulations of the model of length 177, which is the same length as the observed data. To compute these graphs, 50 simulations were generated, then the lead and lag patterns for each simulation were computed and finally the leads and lags were averaged across the 50 simulations.

Figure 4 shows that the general equilibrium NKM is able to reproduce some of the lead and lag patterns observed in actual data as shown in Figure 2. Although the model does fall short, in that it does not fully reproduce the size nor the hump shape seen in Figure 2, it is still a success relative to other papers which have had difficulty replicating the pattern. Focusing only on the comparison between the contemporaneous plots and the lead plots, we see that when the lead correlation is positive, output leads inflation for leads up to one year and that this lead is mostly due to medium and long term components in the simulated data. Focusing only on the comparison between the contemporaneous plots and the lag plots, we see that when the lead correlation is negative, inflation leads output at lags up to one year, and that the lead is due to medium and longer term components in the simulated data.

In order to dissect the origins for the lead and lag correlations, it is useful to consider impulse response functions. Figure 5 plots impulse response functions for the three economic variables in the model based on the parameterization in Column 6. These plots show the impulse responses for the IS curve (demand) shock and the NKPC (supply) shock. Each of these shocks is important for understanding a different part of the lead and lag pattern. In particular, the IS shocks are important for generating the lead of output over inflation, which occurs when output and inflation move together (the upper right plot of Figure 1), while the supply shocks are important for generating the lead of inflation over output, which occurs when output and inflation move in opposite directions (the lower left plot of Figure 1). Striking the right balance between the IS and supply shocks is crucial for reproducing this lead and lag pattern.

for the lead and lag plots.

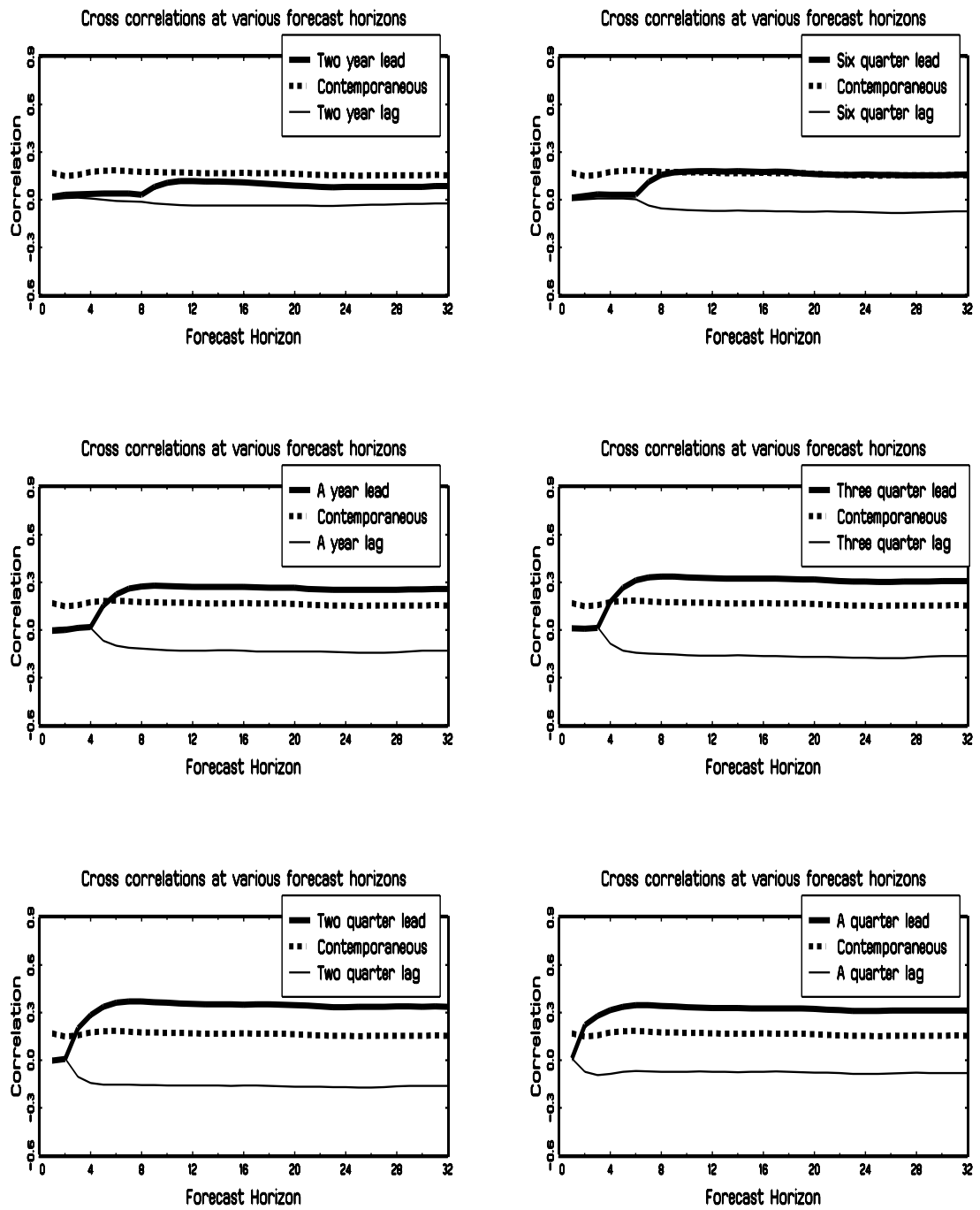


Figure 4: Simulated comovement between output and inflation

To be more clear about how the mechanics of the model work, first focus on the IS shock impulse. Note that a demand shock results in a spike up in both output and inflation, thus producing a positive relationship. To understand the lead of output over inflation, note that the impact on output dies out more quickly than the impact on inflation. This relatively persistent inflation value means that output portends future inflation.

On the other hand, to understand the lead of inflation over output, note that a supply shock results in a spike up in inflation and a spike down in output, thus producing a negative relationship. Next note that here, the negative output impact dies out more slowly than the positive inflation impact. In this case, the relatively persistent output value means that inflation portends future output. After some periods, the response of inflation also becomes negative reinforcing the positive correlation between output and inflation produced by IS shocks.

It is these two different response patterns which together produce the lead of output over inflation when the correlation between the two is positive, and the lead of inflation over output when the correlation between the two is negative, as seen in the off diagonal plots of Figure 1. However, to get these plots right, the response patterns need to be balanced just right. In other words, the demand and supply shocks need to be balanced just right. If either one overwhelms the other, then one of the lead relationships disappear. Furthermore, it is also important to emphasize that the degree to which there is persistence to the demand or supply shocks is also critical. This persistence impacts the persistence displayed in the impulse response functions and without that persistence, one of the lead relationships will also disappear.

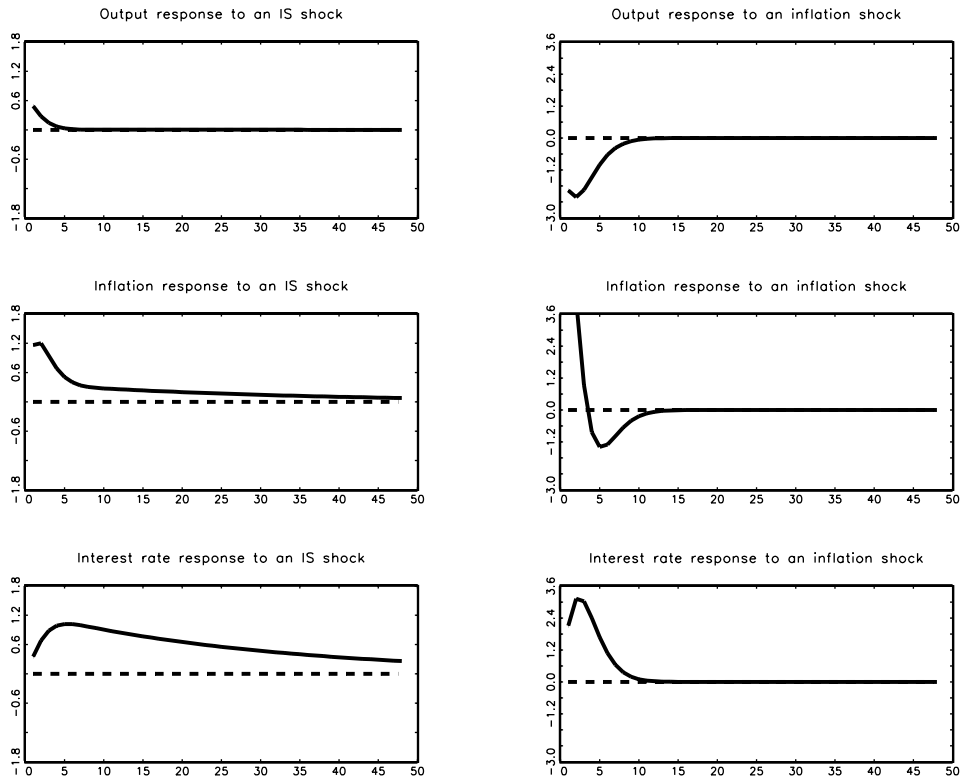


Figure 5: Impulse response functions

4.2.2 Inflation and output persistence

Two other key features of the inflation and output data are that, 1) both series are persistent and, 2) inflation is relatively more persistent than output. To investigate what modeling features were needed to match these two features, we began by investigating the model using the columns 3-5 parameterizations. These simulations (not shown here to save space) are not capable of reproducing the inflation and output persistence patterns observed in actual data.

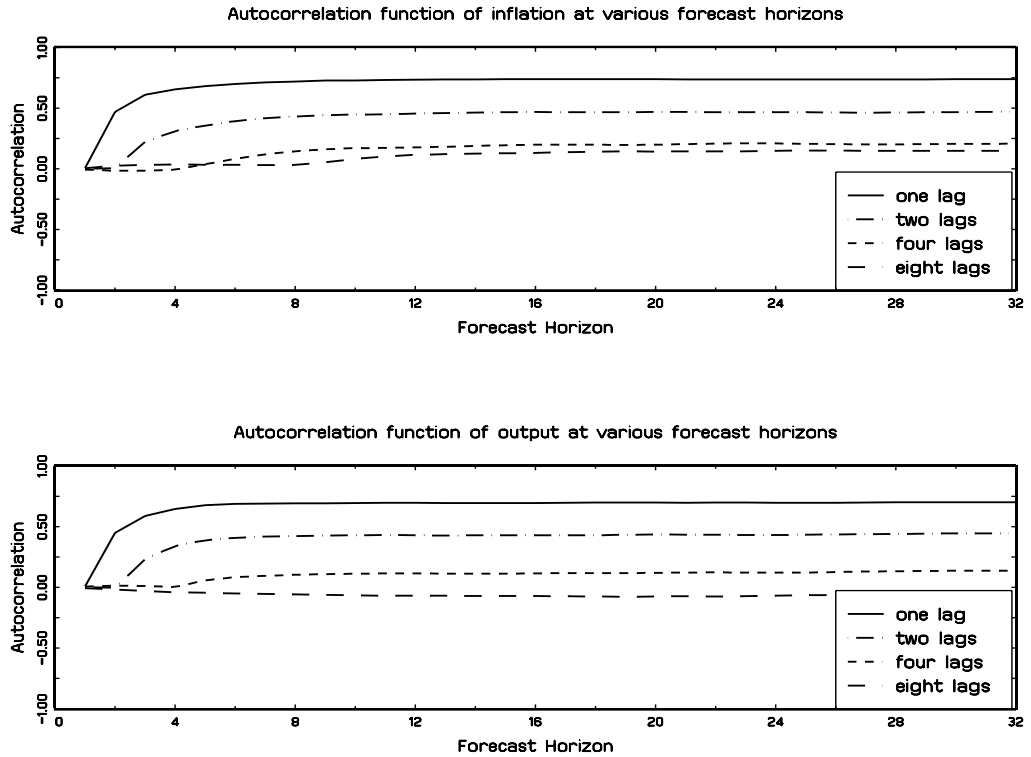


Figure 6: Simulated Inflation and Output Persistence (Column 6)

In an attempt to match the inflation persistence better, we undertook the parameterization exercises summarized by the results in Columns 6 and 7 of Table 1. These exercises added to the original 162 moments additional moments summarizing the autocorrelation patterns for inflation depicted in Figure 3. Figure 6 plots the inflation and output autocorrelations for the parameterization in Column 6. By comparing Figures 3 and 6, we see that this parameterization exhibits the two features seen in actual data: inflation and output are persistent and inflation is more persistent than output.

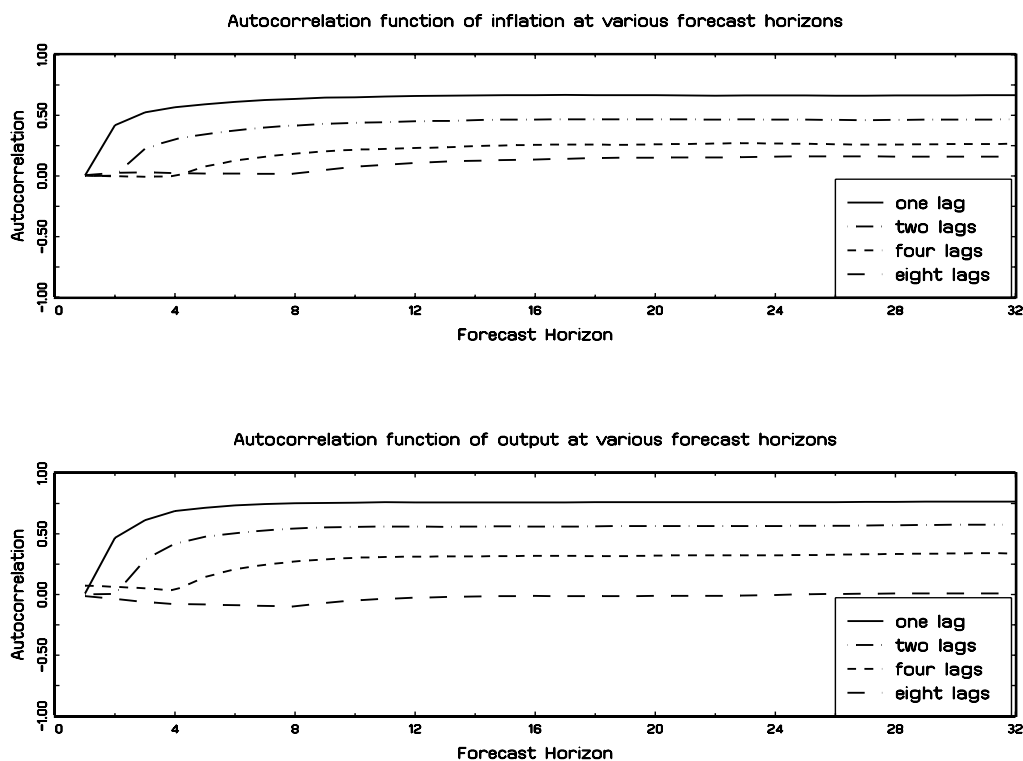


Figure 7: Simulated Inflation and Output Persistence (Column 7)

One last exercise, summarized in Column 7 of Table 1, is to investigate what happens if we restrict the backward looking component of the NKPC to be zero. Column 7 shows that this parameterization results in similar values of the alternative parameters, but the habit formation parameter, γ , increases to 0.22. Interestingly, this model qualitatively reproduces inflation and output persistence as shown in Figure 7, but the fit is worse than the one obtained in Figure 6 using the parameterization in Column 6. This exercise shows that in a general equilibrium model with both IS and supply shocks of just the right proportions (i.e. the Column 6 parameterization), we are able to achieve the lead, lag and persistence patterns seen in the data, but if we eliminate the backward looking term in the hybrid NKPC, then habit formation parameter increases. However, this feature is unable to provide quantitatively as good

a fit to the actual inflation persistence patterns as the one provided by considering the backward looking term in the NKPC.

5 Conclusion

This paper has contributed to our understanding of the relationship between output and inflation in three important ways. First, a new method that sheds light on lead and lag comovements of the data was described. This method not only identifies the lead and lag empirical regularities, but it also shows whether they are part of the short term or long term forces driving the data. Second, the paper uses these techniques to describe the lead, lag and contemporaneous comovement between output and inflation, as well as inflation persistence. Here we showed that the lead and lag patterns of the data arise mostly from data components that drive the medium term forecast horizons. Third, a New Keynesian model with a rich set of modeling features is described and then studied to see which of these features are important for generating the actual patterns. It was found that demand and supply shocks are important for replicating the lead and lag patterns in the data and that persistent IS shocks and the backward looking component of the NKPC were particularly important for achieving inflation persistence while monetary policy shocks did not play an important role.

These results provide insights relative to a number of previous studies. First, NKMs that only have a NKPC, and do not have demand equations, will have a limited ability to capture the data patterns well. Second, the model here is relatively simple compared to other general equilibrium models and shows that the right balance of persistence for demand and supply may be sufficient to explain the data patterns. In addition, the simple NKM presented here is attractive not only because of its ability to fit the data, but also because it is easy to understand the intuition behind the transmission mechanism of shocks.

Extensions of the analysis are worth considering. It is the nature of the business

cycle to be asymmetric. Bringing such asymmetry into the model structure may improve the performance. As was seen in the impulse response analysis, the impact of demand and supply shocks are quite different in terms of their impact and duration and this difference may be important in helping to understand the asymmetry of the business cycle. To pursue such analysis, further modifications to the time series methods introduced here that treat booms and busts differently may be helpful.

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Confidence Band Appendix - Not intended for publication

Figure A.1 shows the confidence bands associated with the contemporaneous and lead comovements between output and inflation which were displayed in Figure 2. This figure breaks apart some of the individual diagrams in Figure 2, so that only one plot is shown in each of the sub-figures. In particular, each of the sub-figures includes either a contemporaneous or a lead plot from Figure 2 along with 95% confidence intervals around the plot. The confidence bands were generated using a bootstrap method. As Figure A.1 shows, the confidence bands are quite wide and that the individual lead lines are not significantly different from the contemporaneous line. As we noted in the text, we still think that it is possible to interpret the leads and lags as we did in the paper in spite of the lack of statistical significance. Such an interpretation is consistent with conclusions in Fuhrer and Moore (1995), Galí and Gertler (1999) and numerous others.

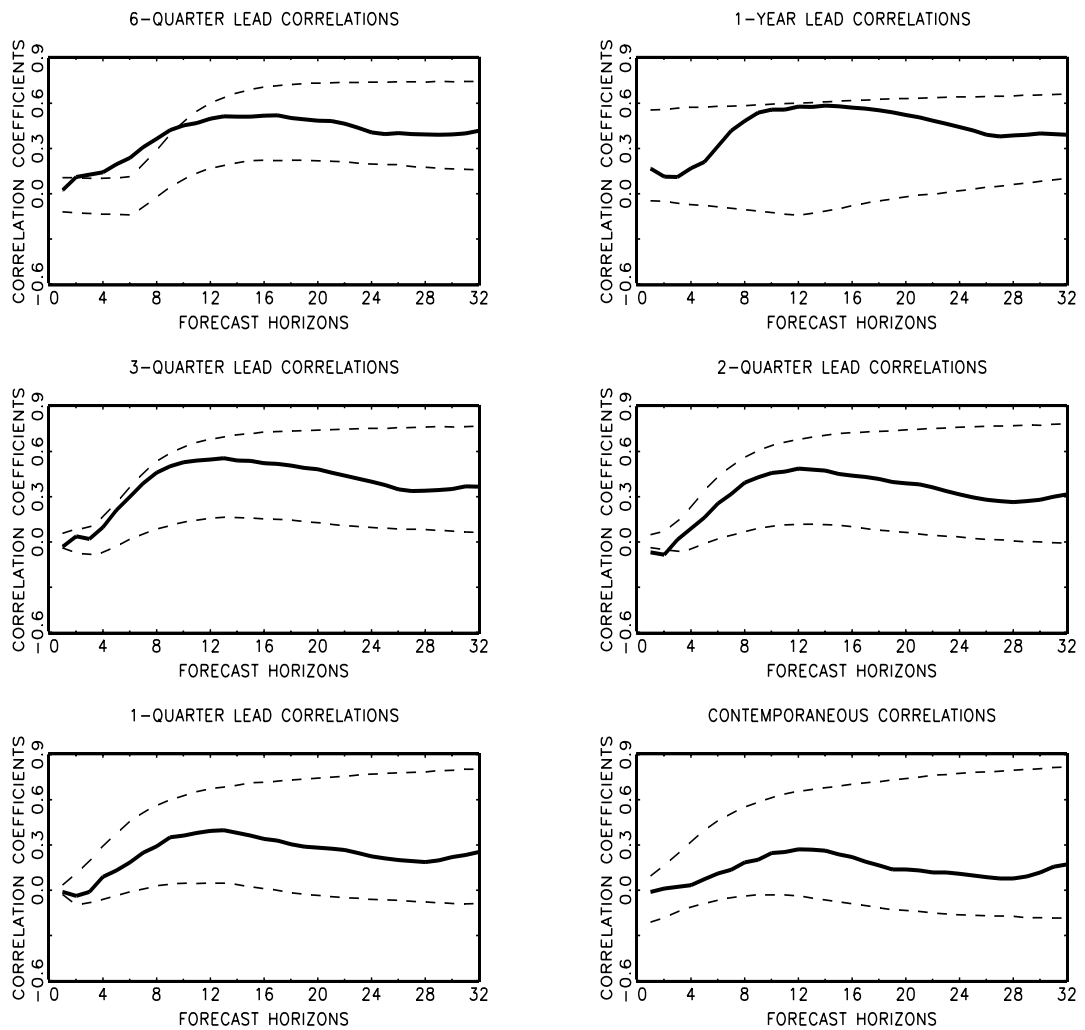


Figure A.1. Comovement between Output and Inflation (confidence bands)