Available online at www.sciencedirect.com
ScienceDirect
Energy Economics 30 (2008) 1850–1881
Energy Economics
www.elsevier.com/locate/eneco

Valuing flexibility: The case of an Integrated Gasification Combined Cycle power plant

Luis M. Abadie [a,1], José M. Chamorro [b,*]

[a] *Bilbao Bizkaia Kutxa, Gran Vía, 30, 48009 Bilbao, Spain*
[b] *University of the Basque Country, Dpt. Fundamentos del Análisis Económico I, Av. Lehendakari Aguirre, 83, 48015 Bilbao, Spain*

Received 4 October 2005; received in revised form 14 September 2006; accepted 10 October 2006
Available online 4 December 2006

Abstract

In this paper we analyze the choice between two technologies for producing electricity. In particular, the firm has to decide whether and when to invest either in a Natural Gas Combined Cycle (NGCC) power plant or in an Integrated Gasification Combined Cycle (IGCC) power plant, which may burn either coal or natural gas. Instead of assuming that fuel prices follow standard geometric Brownian motions, here they are assumed to show mean reversion, specifically to follow an inhomogeneous geometric Brownian motion.

First we consider the opportunity to invest in a NGCC power plant. We derive the optimal investment rule as a function of natural gas price and the remaining life of the right to invest. In addition, the analytical solution for a perpetual option to invest is obtained.

Then we turn to the IGCC power plant. We analyse the valuation of an operating plant when there are switching costs between modes of operation, and the choice of the best operation mode. This serves as an input to evaluate the option to invest in this plant.

Finally we derive the value of an opportunity to invest either in a NGCC or IGCC power plant, i.e. to choose between an inflexible and a flexible technology, respectively. Depending on the opportunity's time to maturity, we derive the pairs of coal and gas prices for which it is optimal to invest in NGCC, in IGCC, or simply not to invest.

Numerical computations involve the use of one- and two-dimensional binomial lattices that support a mean-reverting process for coal and gas prices. Basic parameter values are taken from an actual IGCC power plant currently in operation. Sensitivity of some results with respect to the underlying stochastic process for fuel price is also checked.

JEL classification: C6; E2; D8; G3

Keywords: Real options; Power plants; Technology choice; Mean reversion; Stochastic costs

* Corresponding author. Tel.: +34 946013769; fax: +34 946013891.
E-mail addresses: imabadie@euskalnet.net (L.M. Abadie), jm.chamorro@ehu.es (J.M. Chamorro).
[1] Tel.: +34 607408748; fax: +34 944017996.

0140-9883/$ - see front matter © 2006 Elsevier B.V. All rights reserved.
doi:10.1016/j.eneco.2006.10.004

1. Introduction

It is broadly accepted in the financial literature that traditional valuation techniques based on discounted cash flows are not the most appropriate tool for evaluating uncertain investments, especially in the presence of irreversibility considerations, or a chance to defer investment, or when there is scope for flexible management. In these cases, it is usually preferable to use the methods for pricing options, such as Contingent Claims Analysis or Dynamic Programming.

On the other hand, the energy sector is of paramount importance for the development of any society. Besides, its specific weight both in the real and financial sectors of the economy cannot be neglected. Therefore the use of inadequate instruments in decision making may be particularly onerous. In addition, the high sums involved in the energy industry, its operating flexibility and environmental impact, the progressive liberalization of the markets for its inputs and outputs along with many types of uncertainties, all of them render this kind of investments a suitable candidate to be valued as real options.

The aim of this paper is to use the real options methodology to assess decisions of investment in power plants. In particular, our firm is going to decide simultaneously the time to invest and the choice of technology. Specifically, we compare an inflexible technology (Natural Gas Combined Cycle, or NGCC henceforth) with a flexible one (Integrated Gasification Combined Cycle, or IGCC) which may burn either coal or natural gas. We derive the best mode of operation, the value of the investments and the optimal investment rule when there is an option to wait. We consider a mean-reverting stochastic process for fuel price, namely an Inhomogeneous Geometric Brownian Motion (IGBM), and also switching costs between modes of operation. The output (electricity) price, though, follows a deterministic path. Basic parameter values in our computations refer to an actual IGCC power plant currently in operation.

Restricting ourselves to the energy industry, we would mention Paddock, Siegel and Smith (1988) who apply real options theory to the valuation of undeveloped oil leases; they also provide empirical evidence of the superiority of option values over those derived from standard Net Present Value methods. Bjerksund and Ekern (1990) value an oil field when there is an option to defer operation and abandon the site. Lund and Oksendal (1991) comprise several theoretical and empirical studies on the valuation of oil investments and related issues. Kemma (1993) presents some practical case applications, with a focus on the use of real options theory in capital budgeting decisions by an actual oil firm. Ronn (2002) provides an array of applications of the real options approach to the valuation of different flexibilities embedded in the operation of energy assets by their owners.[2]

On the other hand, Bhattacharya (1978) investigates the accuracy of traditional methods of valuation when cash flows follow a mean-reverting process, as opposed to the standard Geometric Brownian Motion (GBM), and the ensuing biases in value. Laughton and Jacoby (1993, 1995) and Hasset and Metcalf (1995), among others, discuss several, possibly conflicting, influences of mean reversion on investment decisions; as they show, neglecting this pattern may bias investment choices either way. First there is a “risk-discounting” effect: mean reversion reduces the long-run uncertainty, and this should result in a lower risk premium or discount rate for cash flows far into the future. Thus reversion tends to increase the value of any claim to cash flows that increase with long-term prices. At the same time, though, lower uncertainty also tends to reduce directly the value of long-term options of any type because of the so-called “variance” effect; this in turn impinges on the “trigger” level to invest. But there is also a “realized-price” effect whereby

[2] None of the chapters analyses the choice between competing technologies for power generation.

investment would be less likely due to the fact that a lower variance means a lower probability for the cash flows to reach that "trigger" level. Robel (2001) develops some features of the IGBM process and analyses its implications on valuation. Insley (2002) assumes that timber prices follow an IGBM process and values forestry investments. Sarkar (2003) adopts this same stochastic process for analysing diverse investment decisions. Weir (2005) deals with the valuation of petroleum lease contracts assuming also an IGBM process for crude oil prices.

Finally, several papers have analyzed different flexibility options concerning the types of inputs or outputs involved, the mode of operation, or the kind of technology to adopt. First we single out Herbelot (1992), who studies the fulfillment of restrictions on SO_2 emissions either through the purchase of emission permits, or changing the fuel, or deleting polluting agents in the factory itself.[3] He and Pindyck (1992) focus on the flexibility to produce either one of two different products; the model can also be applied to the choice between alternative inputs. Kandel and Pearson (2002) extend Pindyck's (1988) model by giving the firm access to a second, fully reversible technology that requires no capital investment. This second technology produces the same output, but at a higher marginal cost. The fully reversible technology may be interpreted either as labour or as capital that may at any time be resold for its purchase price.[4] On the other hand, Kulatilaka (1988) develops a dynamic model that allows to value the flexibility in a flexible manufacturing system with several modes of operation, using a matrix of transition probabilities between states; in this paper, the importance of flexibility in the design of systems from the viewpoint of both engineers and competitors is emphasized. Later on, Kulatilaka (1995) analyses the choice between a flexible technology (fired by oil or gas) and two inflexible technologies to generate electricity. Some numerical results appear in Kulatilaka (1993), where he normalizes the gas price in terms of the oil price. A similar problem with two stochastic processes is dealt with in Brekke and Schieldrop (2000). This is perhaps the closest work to ours. Specifically, they focus on the choice between flexible and inflexible technologies when fuel prices are assumed to follow standard GBMs; the firm has a perpetual option to invest, and the flexible technology involves no switching costs. In contrast, the firm in our paper faces the choice of technology assuming that fuel prices display mean reversion, changes in fuel inputs entail switching costs, and the investment opportunity is available only for a finite time span (this requires a numerical solution as opposed to an analytical one).

The paper is organized as follows. In Section 2 we briefly introduce the NGCC and IGCC technologies. Section 3 presents the stochastic model for fuel prices (IGBM) and its main features. Next the differential equation for the value of a power plant and that for an option to invest in it are derived. In particular, first we consider a perpetual option to invest in an asset the value of which depends on fuel price; we obtain a solution in terms of Kummer's confluent hypergeometric function. Then we consider the case of a finite-lived option; the binomial lattices for one and two variables that will be used below are also shown.[5] The numerical results for the perpetual option serve later on as a benchmark to verify how the binomial lattices behave. In Section 4 we report some underlying parameter values which are taken from an actual power plant. Then we value an operating NGCC plant and the option to invest in it. We also value an operating IGCC plant (with switching costs between modes of

[3] A brief summary of this work appears in Dixit and Pindyck (1994).

[4] The latter interpretation does not seem to be the case in power generation, and the former one (labour) falls beyond the scope of this paper. In fact, Kandel and Pearson (2001) and Chen and Funke (2004) apply Kandel and Pearson's (2002) framework to the labour market.

[5] The binomial lattice approach is followed because of its suitability and acceptance by the industry. See Mun (2002).

operation) and the right to invest in this flexible technology. Finally we derive the optimal investment rule when it is possible to choose between the NGCC and IGCC alternatives. Since our assumption of mean reversion may be deemed inappropriate for the problem at hand, Section 5 shows some computations assuming instead that fuel prices follow a standard GBM. A final section with our main findings concludes.

2. The NGCC and IGCC technologies

2.1. Some features of electric power plants

The production of electricity can be viewed as the exercise of a series of nested real options to transform a type of energy (gas, oil, coal, or other) into electric energy. There are two sets of outstanding information. The first one has to do with the characteristics of the energy inputs used in the production process. The second one comprises the operation features of the electric power plants, among them: the net output, the rate of efficiency ("operating heat rate"), the costs to start and stop, the fixed costs, the starting and stopping periods, and the physical restrictions that prevent instantaneous changes between states. These factors determine the gap between the prices of the energy consumed and produced.

2.2. Natural Gas Combined Cycle (NGCC) technology

It is based on the employment of two turbines, one of natural gas and another one of steam.[6] The exhaust gases from the first one are used to generate the steam that is used in the second turbine. This system allows to reach a relatively high net efficiency.[7]

An NGCC power plant can be designed as a base load plant or as a peak plant; in the latter case, it only operates when electricity prices are high enough (usually during periods of strong growth in demand). We assume below that the firm runs a base load plant on a continuous basis.

The advantages of a NGCC power plant are (ELCOGAS, 2003):

a) Lower emissions of CO_2, estimated about 350 g/kW h, which allow an easier fulfillment of the Kyoto protocol;
b) Higher net efficiency, between 50% and 60%;
c) Low cost of the investment, about 500 €/kW installed;
d) Less consumption of water and space requirements, which allow to build in a shorter period of time and closer to consumer sites;
e) Lower operation costs, with typical values of 0.35 cents €/kW h.

On the other hand, it has some disadvantages:

a) The higher cost of the natural gas fired in relation to coal's;
b) The insecurity concerning gas supplies, since reserves are more unevenly distributed over the world;
c) The strong rise in the demand for natural gas, which can cause a consolidation of prices above historical levels.

[6] It is also known as Combined Cycle Gas Turbines or CCGT.

[7] The net efficiency refers to the percentage of the heating value of the fuel that is transformed into electric energy.

2.3. Integrated Gasification Combined Cycle (IGCC) technology

It is based on the transformation of coal into synthesis gas.[8] After the stage of gasification, it is time to clean the gas by means of washing processes with water and absorption with dissolvers. Thus an IGCC plant has significantly lower emissions of SO_2, NO_x and particles than standard coal-fired power plants. The emissions of CO_2 per kW h are also lower, but in this case the improvement derives mainly from the higher net efficiency of the cycle (typical current values approach 42%, though this technology is still in its first stages). Right now, the emissions of CO_2 from an IGCC power plant (about 725 g/kW h) are 20% lower than those from a coal plant. However, they are clearly higher than those emanating from an NGCC plant (around 350 g/kW h). Needless to say, the units for gasification, purifying, and other auxiliary systems raise the initial outlay and imply higher operation costs.

From the viewpoint of real options, it is necessary to stress that:

a) The investment in an IGCC plant can be considered as a strategic investment in a new technology; the ultimate results will depend on its final success or failure;
b) The IGCC power plant is a flexible technology concerning the possible fuels to use; apart from the synthesis gas, it may fire oil coke, heavy refinery liquid fuels, natural gas, biomass, and urban solid waste, among others. In this way, at any time it is possible to choose the best input combination according to relative prices.

2.4. The spark spread

Whatever the technology adopted, the firm earns a profit which is a function of the difference between power price and fuel costs; in the case of natural gas, this is known as the spark spread. Even though we model fuel costs as following a mean-reverting stochastic process, electricity price will be assumed to be deterministic. This may seem odd, so perhaps an explanation is in order.

First, electricity prices do not only reflect input prices but market forces and the institutional setting as well. Foreseeing what changes in technology, regulation, supply and demand will take place during the plant's useful life seems to be no easy task.

We consider a firm that decides simultaneously the time to invest and the choice of technology. Thus the firm can use either a pure technology that fires only natural gas or a flexible technology that can switch between coal and gas. Since both NGCC and IGCC power plants are assumed to be equal-sized and to produce only electricity, it is not obvious to us why a given (uncertain) behaviour of electricity price should favour one technology to the detriment of another.

Our firm is assessing the opportunity to build a base load power plant, i.e. one that operates in almost any plausible scenario. The plant has a deterministic useful life of 25 years. As a long-lived investment, it is an estimate of the average electricity price during the asset's life what actually matters, instead of short-term swings.

Empirical evidence supporting this view can be found, for instance, in Elliott et al. (2002): "The final major element of our analysis (for the electricity market in Alberta, Canada) is the

[8] This is mainly composed of carbon monoxide (CO) and hydrogen (H_2). It has several applications: (a) generation of electric energy in IGCC power plants; (b) production of hydrogen for diverse uses, like fuel cells; (c) as an input to chemical products, like ammonia, for manufacturing fertilizers; (d) as an input to produce sulphur and sulphuric acid.

mean reversion of the deseasonalised price. We find a very strong mean reversion, with a half-life measured in hours rather than days or months... This reduces the price risk considerably".

We have undertaken some preliminary work on the monthly average price (spikes included) in the Spanish wholesale electricity market (OMEL). Our results show that this price follows an AR (1) model with strong mean reversion (these results are available from the authors upon request). Therefore we have used the estimated 'equilibrium' price as a realistic estimate of the deterministic mean electricity price. Occasionally, we also consider a (deterministic) rate of growth of electricity price.

3. The stochastic model for the fuel price

Research on the behaviour of commodity prices has been intense for decades. Yet there is hardly a universal consensus on the stochastic process that best fits it. We provide below a few references that support our choice.

The starting point might be stated following Dixit and Pindyck (1994): "Are the prices of raw commodities and other goods best modeled as geometric Brownian motions or as mean-reverting processes? One way to answer this is to examine the data for the price variable in question". Unfortunately stationarity tests have fairly low power and unless a large number of observations are available over a long time period it is difficult to reject the hypothesis of a random walk, even if the series is in fact mean reverting. "As a result, one must often rely on theoretical considerations (for example, intuition concerning the operation of equilibrating mechanisms) more than statistical tests when deciding whether or not to model a price or another variable as a mean-reverting process".

At the same time, as Baker et al. (1998) point out: "Many commodities have traded futures or forward contracts, and the price series for these contracts are additional sources of information about the dynamics of the underlying spot price of the commodity. Even when data on spot prices does not provide clear evidence of reversion, data on futures prices often strongly supports the hypothesis that there is reversion in commodity prices". More recently, Cortazar and Schwartz (2003) share this view: "[The] random walk specification for commodity prices was used until a decade ago, when mean reversion in spot prices began to be included as a response to the evidence that volatility of futures returns declines with maturity". Note also that futures contracts typically trade with maturities far shorter than a power plant's life span.

As a consequence, Ronn (2002) concludes: "Empirically, spot prices for natural gas, electricity, industrial metals, and other commodities display mean reversion". And Pilipovic (1998) claims: "Energy markets require mean-reverting models. In fact, the price mean-reverting model turns out to do the best job of capturing the distribution of energy prices".

Anyway, we will have more to say on this issue in Section 5. In the end, one may always ask whether the results of the analysis are likely to change very much depending on the stochastic process for fuel price.

3.1. The Inhomogeneous Geometric Brownian Motion (IGBM)

In a model for long-term valuation of energy assets, it is convenient to keep in mind that prices tend to revert towards levels of equilibrium after an incidental change. Among the models which display mean reversion, we have chosen the Inhomogeneous Geometric Brownian Motion (or IGBM) process:[9]

$$dS_t = k(S_m - S_t)dt + \sigma S_t dZ_t, \tag{1}$$

where:

S_t	the price of fuel at time t.
S_m	the level to which fuel price tends in the long run.
k	the speed of reversion towards the "normal" level. It can be computed as $k = \log 2/t_{1/2}$, where $t_{1/2}$ is the expected half-life, i.e. the time required for the gap between S_t and S_m to halve.
σ	the instantaneous volatility of fuel price, which determines the variance of S_t at t.
dZ_t	the increment to a standard Wiener process. It is normally distributed with mean zero and variance dt.

Some of the reasons for our choice are:

a) This model satisfies the following condition (which seems reasonable): if the price of one unit of fuel reverts to some mean value, then the price of two units reverts to twice that same mean value.
b) The term $\sigma S_t dZ_t$ precludes, almost surely, the possibility of negative values.
c) This model admits as a particular solution $dS_t = \alpha S_t dt + \sigma S_t dZ_t$ when $S_m = 0$ and $\alpha = -k$; therefore it includes GBM as a particular case. In our opinion, this greater generality is by itself an advance over previous works.
d) The expected value in the long run is: $E(S_\infty) = S_m$; this is not so in Schwartz (1997) model, where $E(S_\infty) = S_m \left(e^{-\frac{\sigma^2}{4k}}\right)$.

3.2. The fundamental pricing equation

For our valuation purposes below we will adopt the risk-neutral valuation principle. The change from an actual process to a risk-neutral one is accomplished by replacing the drift in the price process (in the GBM case, α) with the growth rate in a risk-neutral world, $r - \delta$ where r is the riskless interest rate and δ denotes the net convenience yield.

In order to obtain the risk-neutral version of the IGBM process, we simply discount a risk premium (which, according to the CAPM, is)

$$\rho\sigma\phi S \tag{2}$$

to its actual rate of growth. In this expression, ρ is the correlation coefficient between the returns on the market portfolio and the fuel asset, and ϕ denotes the market price of risk, which is defined as:

$$\phi \equiv \frac{r_M - r}{\sigma_M}, \tag{3}$$

[9] This seems to be a well suited model for natural gas futures contracts traded at NYMEX (Pilipovic, 1998 chapter 4). It is also known as Integrated GBM or geometric Ornstein–Uhlenbeck process. See Bhattacharya (1978), Robel (2001), Insley (2002), Sarkar (2003), Weir (2005).

where r_M is the expected return on the market portfolio and σ_M denotes its volatility. Now let $\hat{S}_t$ denote the risk-neutral version of S_t; then:

$$d\hat{S}_t = \left[k\left(S_m - \hat{S}_t\right) - \rho\sigma\phi\hat{S}_t\right]dt + \sigma\hat{S}_t dZ_t. \tag{4}$$

If certain "complete market" assumptions hold, it can be shown that the value of an investment $V(S,t)$, which is a function of fuel price and calendar time, follows the differential equation:

$$\frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + [k(S_m - S) - \rho\sigma\phi S]\frac{\partial V}{\partial S} + \frac{\partial V}{\partial t} - rV + C(S,t) = 0, \tag{5}$$

where $C(S,t)$ is the instantaneous cash flow generated by the investment.[10]

3.3. Valuation of the option to invest

Next we want to derive the value F of an opportunity to invest in a project the value of which V depends on the price of an asset S that follows an IGBM process.

Starting from Eq. (5) above, in general F will depend on S and t. Since the option confers no cash flow to its owner, its value will satisfy:

$$\frac{1}{2}\sigma^2 S^2 \frac{\partial^2 F}{\partial S^2} + [k(S_m - S) - \rho\sigma\phi S]\frac{\partial F}{\partial S} + \frac{\partial F}{\partial t} - rF = 0. \tag{6}$$

3.3.1. Analytical solution to the perpetual investment option

In this case the term F_τ disappears in Eq. (6), which now can be expressed as:

$$\frac{1}{2}\sigma^2 S^2 F'' + [k(S_m - S) - \rho\sigma\phi S]F' - rF = 0. \tag{7}$$

This equation may be rewritten as:

$$S^2 F'' + (\alpha S + \beta)F' - \gamma F = 0, \tag{8}$$

where the following notation has been adopted:

$$\alpha \equiv -\frac{2(k + \rho\sigma\phi)}{\sigma^2}, \quad \beta \equiv \frac{2kS_m}{\sigma^2}, \quad \gamma \equiv \frac{2r}{\sigma^2}.$$

[10] This expression may be expressed in perhaps more familiar, yet equivalent, terms. For simplicity, define $\alpha \equiv (1/S)(E(S)/dt)$ and assume there is a rate of return shortfall (or the so-called convenience yield) δ. Thus $\mu = \alpha + \delta$ denotes the total expected rate of return. Now "this expected return must be enough to compensate the holders for risk. Of course it is not risk as such that matters, but only nondiversifiable risk. The whole market portfolio provides the maximum available diversification, so it is the covariance of the rate of return on the asset with that on the whole market portfolio that determines the risk premium. The fundamental condition of equilibrium from the CAPM says that $\mu = r + \phi\sigma\rho$" (Dixit and Pindyck, 1994, pp. 115). Hence, $\alpha + \delta = \mu = r + \phi\sigma\rho$. This implies that $\alpha - \phi\sigma\rho = r - \delta$. Therefore the differential equation for the value of the investment V can be equivalently stated in terms of the actual growth rate minus the risk premium, or the riskless interest rate minus the rate of return shortfall (or convenience yield). Eq. (5) shows the first choice (see also Trigeorgis, 1996 Section 3.8; or Hull, 1993 Appendix 12B). Note, though, that in our case δ is not constant; if we equate $(r - \delta)\ \hat{S}_t$ to the coefficient of dt in Eq. (4) the resulting expression for δ is a function of $\hat{S}_t$.

To find a solution to this equation, we define a function $h(\beta S^{-1})$ by

$$F(S) = A_0\left(\beta S^{-1}\right)^{\theta} h\left(\beta S^{-1}\right), \tag{9}$$

where A_0 and θ are constants that will be chosen so as to make $h(\bullet)$ satisfy a differential equation with a known solution. This function turns out to be (details of the proof are available from the authors):

$$h\left(\beta S^{-1}\right) = A_1 U(a,b,z) + A_2 M(a,b,z), \tag{10}$$

where $a=\theta$, $b=2\theta+2-\alpha$, and $z=(\beta S^{-1})$.[11] Now $U(a, b, z)$ is Tricomi's or second-order hypergeometric function, and $M(a, b, z)$ is Kummer's or first-order hypergeometric function. Therefore, the general solution to Eq. (9) will be

$$F(S) = A_0\left(\beta S^{-1}\right)^{\theta}(A_1 U(a,b,z) + A_2 M(a,b,z)). \tag{11}$$

The boundary conditions will determine whether A_1 or A_2 in Eq. (11) are zero. In our case, S refers to a fuel input so the firm faces stochastic costs. An increase in S entails a reduction in profits, so $F(\infty)=0$ and $z=0$, then $A_1=0$ and the term in Kummer's function remains.[12] The solution is:

$$F(S) = A_m\left(\beta S^{-1}\right)^{\theta} M(a,b,z), \tag{12}$$

with $A_m \equiv A_0 A_2$. The constant A_m and the critical fuel price S^* below which it is optimal to invest must be jointly determined by the remaining two boundary conditions:

a) Value-matching: $F(S^*)=V(S^*)-I(S^*)$,
b) Smooth-pasting: $F'(S^*)=V'(S^*)-I'(S^*)$.

3.3.2. Numerical solution to the non-perpetual option to invest

In this case F satisfies the partial differential Eq. (6) which now must be solved by numerical procedures. Given the American type of the options involved and the low number of sources of uncertainty the binomial lattice approach is used.

The time horizon T is subdivided into n steps, each of length $\Delta T=T/n$. Starting from an initial value S_0, at time i, after j positive increments (each of size u), the value of the fuel input is given by $S_0 u^j d^{i-j}$, where $d=1/u$.

Consider an asset whose risk-neutral price follows the stochastic differential Eq. (4):

$$d\hat{S} = \left[k\left(S_m - \hat{S}\right) - \rho\sigma\phi\hat{S}\right]dt + \sigma\hat{S}dZ.$$

Adopting the transformation $X=\ln\hat{S}$ and following standard procedures it can be shown that upward movements must be size $\Delta X = \sigma\sqrt{\Delta t}$; therefore $u = e^{\sigma\sqrt{\Delta t}}$ and $d = e^{-\sigma\sqrt{\Delta t}}$. The probability of an upward movement at node (i, j) is

$$p_u(i,j) = \frac{1}{2} + \frac{\hat{\mu}(i,j)\sqrt{\Delta t}}{2\sigma}, \tag{13}$$

[11] Taking the first and second derivatives of $F(S)$ and substituting them into Eq. (8) yields: $\theta^2+\theta(1-\alpha)-\gamma=0$. This quadratic equation allows to determine the positive value of θ, since the remaining parameters are known constants.

[12] $U(a, b, 0)=\infty$ if $b > 1$; this will be shown to hold with our parameter values below.

where

$$\hat{\mu}(i,j) \equiv \frac{k\left(S_m - \hat{S}(i,j)\right)}{\hat{S}(i,j)} - \rho\sigma\phi - \frac{1}{2}\sigma^2. \tag{14}$$

Now consider two assets the prices of which are governed by the following risk-neutral processes:

$$d\hat{S}_1 = \left[k_1\left(S_{m_1} - \hat{S}_1\right) - \rho_1\sigma_1\phi\hat{S}_1\right]dt + \sigma_1\hat{S}dZ_1, \tag{15}$$

$$d\hat{S}_2 = \left[k_2\left(S_{m_2} - \hat{S}_2\right) - \rho_2\sigma_2\phi\hat{S}_2\right]dt + \sigma_2\hat{S}dZ_2, \tag{16}$$

$$dZ_1 dZ_2 = \rho_{12}dt. \tag{17}$$

Defining $X_1 = \ln\hat{S}_1$ and $X_2 = \ln\hat{S}_2$, it may be shown that

$$\Delta X_1 = \sigma_1\sqrt{\Delta t}, \tag{18}$$

$$\Delta X_2 = \sigma_2\sqrt{\Delta t}, \tag{19}$$

$$p_{uu} = \frac{\Delta X_1\Delta X_2 + \Delta X_2\hat{\mu}_1\Delta t + \Delta X_1\hat{\mu}_2\Delta t + \rho\sigma_1\sigma_2\Delta t}{4\Delta X_1\Delta X_2}, \tag{20}$$

$$p_{ud} = \frac{\Delta X_1\Delta X_2 + \Delta X_2\hat{\mu}_1\Delta t - \Delta X_1\hat{\mu}_2\Delta t - \rho\sigma_1\sigma_2\Delta t}{4\Delta X_1\Delta X_2}, \tag{21}$$

$$p_{du} = \frac{\Delta X_1\Delta X_2 - \Delta X_2\hat{\mu}_1\Delta t + \Delta X_1\hat{\mu}_2\Delta t - \rho\sigma_1\sigma_2\Delta t}{4\Delta X_1\Delta X_2}, \tag{22}$$

$$p_{dd} = \frac{\Delta X_1\Delta X_2 - \Delta X_2\hat{\mu}_1\Delta t - \Delta X_1\hat{\mu}_2\Delta t + \rho\sigma_1\sigma_2\Delta t}{4\Delta X_1\Delta X_2}. \tag{23}$$

In the above expressions, p_{ud} stands for the risk-neutral probability of an upward movement in gas price and a simultaneous downward movement in coal price at a certain node; similarly for the probabilities p_{uu}, p_{du} and p_{dd}.

The branches of the lattice have been forced to recombine by taking constant increments ΔX_1 and ΔX_2 once the step length Δt has been chosen. Thus it is easier to implement the model in a computer. Nevertheless the probabilities change from one node to another by depending on $\hat{\mu}_1$ and $\hat{\mu}_2$ (see Eq. (14)). Besides, at any time the four probabilities must take on values between zero and one.

4. Valuation of alternative technologies

4.1. Representative underlying parameters adopted

By assumption, the firm is assessing the opportunity to build a base load power plant. We have used the standard values that appear in Table 1 (ELCOGAS, 2003).

Table 1
Basic underlying parameters

Concepts	Coal plant	IGCC	NGCC
Plant Size size Mw (P)	500	500	500
Production factor (%) (FP)	80%	80%	80%
Net efficiency (%) (RDTO)	35%	41%	50.5%
Unit investment cost €/Kw (i)	1186	1300	496
O&M cts. €/kW h (CVAR)	0.68	0.71	0.32

The Production Factor is the percentage of the total capacity used on average over the year. Using these data, the heat rate, the plant's consumption of energy, and the total production of electricity can be computed:

Heat rate: HR = 3600 / RDTO / 1,000,000, in GJ/kW h.[13]
Investment cost I = 1000 ∗ i ∗ P, in Euros.
Total annual production: A = 1000 ∗ P ∗ 365 ∗ 24 ∗ FP in kW h.
Fuel energy needs: B = 1000 ∗ P ∗ 365 ∗ 24 ∗ FP ∗ HR, in GJ/year.
Now with these formulae we estimate the parameters in Table 2.

4.2. Value of an operating NGCC plant

Our first purpose is to determine the value of a NGCC operating facility. The revenues are the receipts from the electricity generated, whereas the costs refer to the initial investment, the average operation costs and those of the fuel consumed, in this case natural gas. We consider a deterministic evolution of electricity prices and variable costs, both of which would grow at a constant rate r_a.

The time-0 value of revenues over a finite number of periods is computed according to the following formula:

$$\mathrm{PVR} = A.E.\frac{\left(1-e^{-\tau(r-r_a)}\right)}{r-r_a}, \qquad (24)$$

and variable expenditures amount to:

$$\mathrm{PVC}_{\mathrm{var}} = A.C_{\mathrm{var}}.\frac{\left(1-e^{-\tau(r-r_a)}\right)}{r-r_a}, \qquad (25)$$

where:

A	annual production : 3504 million kW h.
E	current electricity price : 0.035 €/kW h.
r_a	growth rate of electricity price and variable costs: 0%, for now.
r	riskless interest rate: 5%.
C_{var}	unit variable costs: 0.0032 €/kW h.
τ	estimated useful life of the power plant upon investment: 25 years.

[13] One kW h amounts to 3600 kilo joules (kJ), and one Giga joule (GJ) is a million kJ.

Table 2
Resulting parameter values

Concepts	Coal plant	IGCC	NGCC
Heat rate GJ/kW h	0.0103	0.0088	0.0071
Total Investment (mill€) (I)	593	650	248
Annual production (mill kW h)(A)	3504	3504	3504
Fuel Energy (GJ/year) (B)	36,041,143	30,766,829	24,979,010

The cost of the initial investment I is 248 million €. Thus:

$$PVR - PVC_{var} - I = 1,342,055,454\ €. \tag{26}$$

With regard to fuel costs, first we compute the present value of one fuel unit consumed per year over the whole life span of the plant (see Eq. (34) in the Appendix). Then multiplying by annual consumption we get the present value of fuel costs:

$$PVC = B\left[\frac{kS_m(1-e^{-r\tau})}{r(k+\rho\sigma\phi)} + \frac{S - \frac{kS_m}{k+\rho\sigma\phi}}{r+k+\rho\sigma\phi}\left(1-e^{-(r+k+\rho\sigma\phi)\tau}\right)\right], \tag{27}$$

where B is the annual fuel energy needed (GJ) and S stands for the natural gas price (€/GJ). Assuming the additional values: S_m=3.25 €/GJ, ρ=0, ϕ=0.40, k=0.25, σ=0.20, the fuel cost per unit consumed yearly during the facility's life is: 35.5498+3.3315S. Now multiplying this by annual consumption, B, we get: 887,999,971+83,217,315S. For the specific value S=5.45 €/GJ, we finally compute:[14]

$$NPV(NGCC) = PVR - PVC_{var} - I - PVC = 521,120\ €. \tag{28}$$

4.3. Valuation of the opportunity to invest in a NGCC plant

4.3.1. The perpetual option

Consider the case of the NGCC plant described in Table 1. As already seen, the present value of revenues minus the investment outlay and the present value of variable costs amounts to 1,342,055,454 €, whereas the present value of fuel costs is 887,999,971+83,217,315S. Thus the value of the investment if realized now is: $V(S)-I$=454,055,483−83,217,315S.

In this case, the boundary conditions are the following:

a) Value-matching: $F(S^*)=A_m(\beta(S^*)^{-1})^{\theta}M(a,\ b,\ \beta(S^*)^{-1})=454{,}055{,}483-83{,}217{,}315S^*=V(S^*)-I(S^*)$

b) Smooth-pasting: $F'(S^*) = A_m\left(\beta(S^*)^{-1}\right)^{\theta}\left[-\theta(S^*)^{-1}M\left(a,b,\beta(S^*)^{-1}\right)-\frac{a\beta}{b(S^*)^2}M(a+1, b+1,\beta(S^*)^{-1})\right] = -83,217,315 = V'(S^*)-I'(S^*)$

[14] The prices chosen derive from a cursory look at NYMEX Natural Gas futures contracts on October 18th 2004. In particular, S_m is approximated by the contract with the longest maturity, and S by that with the closest maturity. These contracts trade in US dollars per 10,000 million British thermal units.

From the specified parameter values we compute $\alpha=-12.5$, $\beta=40.625$, $\gamma=2.5$, and $\theta=0.1827$; then we derive $a=0.1827$ and $b=14.8654$.

Thus we have a system of two equations that will allow us to determine A_m and S^*. Substituting $A_m\ (\beta(S^*)^{-1})^{\theta}$ from the first condition into the second one we get an equation in S^* which has as its solution $S^*=2.7448$. Next it is easy to determine that $A_m=94{,}394{,}000$. Finally, the value of the option to invest for a gas price S is given by:

$$F(S) = 94,394,000\left(\frac{40.625}{S}\right)^{0.1827} M\left(0.1827, 14.8654, \frac{40.625}{S}\right).$$

In our case, for $S=5.45$, initially the option to invest is worth 153,870,000 €. Since NPV= 521,120 €, it is optimal to wait.

If there were no other option but to invest now or never, the hurdle price would result from $V(S^{**})=I(S^{**})$; hence we get $S^{**}=5.4563$. However, when the option is infinitely lived it is preferable to wait, since in the long run natural gas price is going to decrease from $S=5.45$ and swing around $S_m=3.25$, and then keep on waiting until it reaches $S^*=2.7448$ or below, so that the option value equals the net value of the investment. When the option's maturity is finite the threshold S^* will take on a value between 5.4563 and 2.7448.

4.3.2. The non-perpetual option

Now we want to determine the value of an option to invest and the optimal investment rule in a NGCC facility when the opportunity is only available from time 0 to T.

The cost of the initial investment is $Ie^{r_b t}$, with $0 \leq t \leq T$, where I is the initial disbursement at time $t=0$ and r_b is its rate of growth (both electricity price and variable costs are now supposed to be constant: $r_a=0$).

At the option's maturity, $t=T$, there is no other choice but to invest right then or not to invest. The decision to undertake the investment is adopted if the present value of revenues exceeds that of costs:

$$\text{NPV} = \text{PV(Revenues)}-\text{PV(Expenditures)} > 0.$$

We set up a binomial lattice with the following values in the final nodes:

$$W = \text{Max}(\text{NPV}, 0).$$

At previous moments, $0 \leq t \leq T$, depending on current fuel prices, we compute the present value of exercising the option to invest (NPV) and that of keeping the option alive. Then we choose the maximum value:

$$W = \text{Max}\left(\text{NPV}, e^{-r\Delta t}(p_u W^+ + p_d W^-)\right).$$

The lattice is solved backwards, and the solution provides the time-0 value of the option to invest. If we compare this with the value of an investment made at the outset, the difference will be the value of the option to wait. This option's value will always be nonnegative.

Concerning the investment rule, by changing the initial price of a fuel unit it is possible to determine the fuel price at which the option value switches from positive to zero. This will be the optimal exercise price at $t=0$. Similarly, arranging a binomial lattice for an investment opportunity with maturity $t < T$ and changing the fuel price, the optimal exercise price for intermediate moments is determined.

At time $t=T$, since there is no chance for further delay, investment is realized only if NPV > 0. The optimal rule to invest is found by computing the gas price for which NPV = 0; we have already seen

Table 3
Trigger price S^* with finite time to maturity

Term	$r_a=0$, $r_b=0$	$r_a=0$, $r_b=0.025$	$r_a=0$, $r_b=0.05$
0	5.4563	5.4563	5.4563
½	3.3268	3.5587	3.7823
1	3.2200	3.4417	3.6503
2	3.0864	3.3035	3.5107
3	3.0040	3.2250	3.4394
4	2.9480	3.1751	3.3982
5	2.9079	3.1413	3.3731
6	2.8782	3.1179	3.3575
7	2.8557	3.1012	3.3481
8	2.8386	3.0893	3.3423
9	2.8253	3.0808	3.3392
10	2.8151	3.0746	3.3378
∞	2.7448	–	–

that (for $r_a=r_b=0$) this price is $S_0^*=5.4563$. In the opposite case of an unlimited possibility to defer, with $r_a=r_b=0$, we know that $S_\infty^*=2.7448$, which comes from the analytical solution to the perpetual option. These results appear in Table 3. Critical fuel prices for intermediate maturities are derived from a binomial lattice with 1200 time steps in any case. They converge toward those of the perpetual option as the maturity of the investment opportunity increases.

The value of the option to invest increases with the option's maturity. In order to be exercised optimally, a longer-lived option requires a lower critical price for the fuel resource. It can also be observed that a higher investment growth rate, r_b, *ceteris paribus* quickens the time to invest: a higher S^* means a less stringent hurdle to overcome.

Next we analyse the optimal choice between investing or waiting as a function of the initial fuel price when the investment opportunity is available for 5 years. In principle, a lower fuel price *ceteris paribus* will render the power plant more valuable; this in turn will increase the value of the option to invest. Thus it is not obvious whether a cheaper natural gas will hasten the investment decision or not. As shown in Table 4, when $S_0=5.45$ the NPV is very low and the option to invest is worth more than 119 million €. As S_0 decreases, the investment option increases its value but less than the NPV, with the equality being reached when $S_0=2.9079$. For lower gas prices, it is preferable to invest immediately.

As is well known, the solution derived from binomial lattices may be very sensitive to the number (or length) of time steps considered. Now we analyse the convergence of S^* and also the robustness

Table 4
NPV and option value (thousand €) with $T=5$ years

S_0	NPV	Option value	Max (NPV, option)	Optimal decision
5.45	521	119,170	119,170	Wait
5.00	37,969	129,040	129,040	Wait
4.50	79,578	141,640	141,640	Wait
4.00	121,190	156,820	156,820	Wait
3.50	162,790	176,420	176,420	Wait
3.00	204,400	205,010	205,010	Wait
2.9079	212,070	212,070	212,070	Indifferent
2.50	246,010	245,900	246,010	Invest
2.00	287,620	287,450	287,620	Invest

Table 5
Convergence and sensitivity analysis of S^*

Steps	S^*	k	S^*	S_m	S^*	σ	S^*
120	2.9451	0.10	2.6668	2.75	2.8964	0.10	3.3005
240	2.9294	0.20	2.8555	3.00	2.9085	0.15	3.1042
1200	2.9079	0.25	2.9079	3.25	2.9079	0.20	2.9079
6,000	2.8981	0.30	2.9468	3.50	2.8732	0.25	2.7132
10,000	2.8963	0.40	3.0072	3.75	2.7576	0.30	2.5242

with respect to changes in k, S_m and σ. We restrict ourselves to the case of the option with 5 years to expiration.

According to the first two columns in Table 5, the value of the trigger price certainly changes as the number of steps varies. However the change is not large: increasing the number of steps from 120 to 10,000 (i.e. by a factor of more than 80) decreases S^* by less than 2%.

All the other columns have been derived with 1200 steps. Rises in reversion speed (k) somehow entail lower uncertainty; as a consequence the decision to invest does not call for such a low fuel price. Thus a four-fold increase in k increases S^* by 12% (fourth column). Therefore it looks unlikely that mild errors in the choice of k will lead to dramatic changes in the value of S^*. On the other hand, a higher equilibrium fuel price in the long run makes investment less attractive, so the trigger price to invest (sixth column) must be lower. A 36% increase in S_m induces a reduction in S^* of 5%. Finally, the greater the uncertainty in fuel prices the lower the critical price to trigger investment. Thus a three-fold increment in σ lowers S^* by 30% (last column).

4.4. Valuation of an operating IGCC plant

Now we must determine the value of an operating IGCC power station, both right upon the initial outlay and at any moment along its useful life to derive its remaining value. The firm faces two stochastic fuel prices, so there are two sources of risk and two-dimensional binomial lattices are needed. We set up two two-dimensional binomial lattices which refer to initial states consuming either coal or natural gas, respectively.

At the end of the plant's useful life its value is zero whatever the particular fuel that has been fired in the last period:

$$Wc = Wg = 0,$$

where W_c and W_g stand for the plant's (gross) value at the nodes of the coal and gas lattices, respectively.

At earlier times t we compute, for a time interval Δt, the profits by mode of operation. These are determined as the difference between electricity revenues and the sum of variable plus fuel costs:

$$\begin{aligned}\pi_c &= A.E.e^{r_a t}\frac{\left(1-e^{-\Delta t(r-r_a)}\right)}{r-r_a} - B_c \Delta t S_c - A.C_{\mathrm{var}_c}.e^{r_a t}\frac{\left(1-e^{-\Delta t(r-r_a)}\right)}{r-r_a},\\ \pi_g &= A.E.e^{r_a t}\frac{\left(1-e^{-\Delta t(r-r_a)}\right)}{r-r_a} - B_g \Delta t S_g - A.C_{\mathrm{var}_g}.e^{r_a t}\frac{\left(1-e^{-\Delta t(r-r_a)}\right)}{r-r_a},\end{aligned} \tag{29}$$

where:

π_c Net profits from operating with coal.
πg Net profits from operating with natural gas.

S_c current coal price (€/GJ).
S_g current natural gas price (€/GJ).
$A.E.e^{r_a t}\frac{\left(1-e^{-\Delta t(r-r_a)}\right)}{r-r_a}$ value at time t of revenues from electricity over the period Δt.
$A.C_{\text{var}}.e^{r_a t}\frac{\left(1-e^{-\Delta t(r-r_a)}\right)}{r-r_a}$ value at time t of variable costs incurred over Δt.
B_c the coal energy needed per year in GJ.
B_g the natural gas energy needed per year in GJ.
$B_c \Delta t S_c$ Costs of coal consumed during Δt.
$B_g \Delta t S_g$ Costs of natural gas consumed during Δt.
$I(c \rightarrow g)$ Switching cost from coal to gas.
$I(g \rightarrow c)$ Switching cost from gas to coal.

If initially the IGCC plant was consuming coal, the best of two options is chosen:[15]

a) continue: the present value of the coal lattice is obtained, plus the profits from operating in coal-mode at that instant.
b) switch: the present value of the gas lattice is obtained, plus the profits from operating in gas-mode at that instant, minus the costs to switching from coal to gas, $I(c \rightarrow g)$.

Thus the binomial lattices will take on the following values:[16]

$$\begin{aligned} Wc = & \text{Max}\left(\pi_c + e^{-r\Delta t}(p_{uu}Wc^{++} + p_{ud}Wc^{+-} + p_{du}Wc^{-+} + p_{dd}Wc^{--}), \pi_g - I(c \rightarrow g) + \right. \\ & \left. + e^{-r\Delta t}(p_{uu}Wg^{++} + p_{ud}Wg^{+-} + p_{du}Wg^{-+} + p_{dd}Wg^{--})\right). \end{aligned} \tag{30}$$

Similarly, when the initial state corresponds to operating with natural gas, we would compute:

$$\begin{aligned} Wg = & \text{Max}\left(\pi_c - I(g \rightarrow c) + e^{-r\Delta t}(p_{uu}Wc^{++} + p_{ud}Wc^{+-} + p_{du}Wc^{-+} + p_{dd}Wc^{--}), \right. \\ & \left. \pi_g + e^{-r\Delta t}(p_{uu}Wg^{++} + p_{ud}Wg^{+-} + p_{du}Wg^{-+} + p_{dd}Wg^{--})\right). \end{aligned} \tag{31}$$

Finally, at time zero the optimal initial mode of operation is chosen by:

$$\text{Max}(Wc, Wg).$$

In this way, we have derived the value of a flexible plant in operation.

Note that, in this computation, the cost of the initial investment plays no role but it could be included at the outset in order to compare this outlay with the present value of expected profits.

Table 6 shows the parameter values adopted in the base case.[17] Some of them are taken from Table 1 but are repeated here for convenience.

Table 7 shows the value of the plant as a function of switching costs. These figures have been derived from a lattice with 300 steps (one for each month of useful life) regardless of switching costs.

[15] We have not considered the option not to operate, though it could be taken into account easily. It could be included by a third lattice, corresponding to an idle initial state. At every time we should maximize over three possible values, taking into account the switching costs between states. If we denote the idle state by p, in this case there could be a stopping cost, $I(c \rightarrow p)$ or $I(g \rightarrow p)$, and a restarting cost, $I(p \rightarrow c)$ or $I(p \rightarrow g)$. If restarting costs were very high, stopping could amount to abandonment.
[16] We follow a similar procedure to Trigeorgis (1996), pp. 177–184.
[17] Volatility values for coal and gas prices, as well as their correlation coefficient, have been computed from yearly average prices gathered by the US Energy Information Administration. Note that these prices are expressed in US dollars per million Btu.

Table 6
An IGCC power plant: base case

Concepts	Coal mode	Gas mode
Plant Size size Mw (P)	500	500
Production Factor (FP)	80%	80%
Net efficiency(%) (RDTO)	41.0%	50.5%
Unit Investment cost €/Kw (i)	1300	1300
Operation cost (€cents/kW h) (CVAR)	0.71	0.32
Fuel price (€/GJ)	1.90	5.45
Reversion value S_m(€/GJ)	1.40	3.25
Reversion speed (k)	0.125	0.25
Plant's useful life (years)(τ)	25	25
Risk-free interest rate (r)	0.05	0.05
Market price of risk (ϕ)	0.40	0.40
Volatility (σ)	0.05	0.20
Correlation with the market (ρ_1, ρ_2)	0	0
Correlation between fuels (ρ_{12})	0.15	0.15
Investment cost rate of growth (r_b)	0	0
Electricity price rate of growth (r_a)	0	0
Switching costs (€)	20,000	20,000

When there are no switching costs, the value of the plant exceeds the initial disbursement (650,000,000 €) by 52,662,000 €. Thus it is worth an 8.10% more than the amount disbursed. As switching costs swell, the plant's value drops.

Now the value of flexibility may be computed as the difference between the net values with finite and infinite switching costs. For example, when these are nil, flexibility is worth 10,675,000 €, just 1.5% of the initial investment. Note that flexibility in the IGCC plant may be valuable because of reasons other than harnessing at any instant the best fuel option. For instance, it may be due to failures in elements that are necessary to operate in coal-mode but do not prevent the plant from operating in gas-mode (thus averting the total stopping of the facility), or if there are problems concerning supplies of a certain kind of fuel.

The plant's value as a function of coal and natural gas prices is shown in Fig. 1. It can be observed that value drops as the price of either fuel rises. Also, for low gas prices, the value of the plant decreases smoothly as coal price swells, since this is not the preferred fuel resource. However, for high gas prices, the plant burns mainly coal; if this gets dearer, an abrupt fall in value ensues.

Table 8 shows the gross value of the plant as a function of its remaining life and the growth rate of electricity price and variable cost (r_a) in the base case with switching costs of 20,000 €. For the sake of consistence, we have used one step per month again.

Whatever the level of r_a, the plant's value diminishes as its life span comes to an end. Note also that at least two factors are at work here. For concreteness, consider the plant's value with 1 and 2 years to closure. Clearly the reversion effect is stronger over 2 years than over 1 year; thus one would expect the plant's value with 2 years to be more than double that with 1 year to operate. Yet the influence of the discount rate $r=0.05$ pushes against this trend. For constant electricity prices ($r_a=0$), the effect of time discounting prevails since 77,806,000 euros is less than 2 × 39,044,000 euros. This is not the case, though, with rising electricity prices ($r_a=0.03$), so now the reversion effect more than offsets the discounting effect.

Concerning the convergence of our results, as already mentioned, figures in Table 7 correspond to a lattice with 300 steps (regardless of switching costs). Take, for instance, switching costs of 20,000 €; the plant's net value amounts to 52,534,000 €. The first two columns in Table 9 show that if we

Table 7
Gross and net value (thousand €) of an IGCC plant

Switching costs (€)	Plant's value	Plant's value — initial investment
0	702,662	52,662
10,000	702,598	52,598
20,000	702,534	52,534
50,000	702,345	52,345
100,000	702,129	52,129
1,000,000	700,049	50,049
∞	691,987	41,987

consider 75 time steps the net value falls to 47,662,000 €, whereas with 1200 steps it rises to 53,693,000 €. Thus a 16-fold increase in the number of steps increases the plant's NPV by 12.65%.

In the case of infinite switching costs there is no scope for flexibility and the cheapest fuel resource is used exclusively. Given current fuel prices, operation turns out to be only in coal-mode. Thus the above formulas for the NGCC plant apply, but now they refer to coal. Again, the value of the plant changes with the number of steps in the lattice. As shown in the central columns of Table 9, with 3000 steps we reach a net value of 43,353,000 €, a figure which is very close to 43,594,000 € derived from an analytical procedure akin to that for a NGCC plant.

Finally let us consider a plant with 10 years to closure. According to Table 8, with 10 × 12 = 120 monthly time steps, the gross value of the plant (r_a = 0:00) is 361,090,000 €. Again, the last two columns in Table 9 show that the plant's value increases with the number of steps, but the increase is not relatively large: with 1500 time steps it jumps to 362,050,000 €.

4.5. Valuation of the opportunity to invest in an IGCC plant

Let us assume that the investment opportunity is available from time 0 to time T (the non-perpetual option). In this case, the procedure is very similar to that for the optimal timing to invest in a

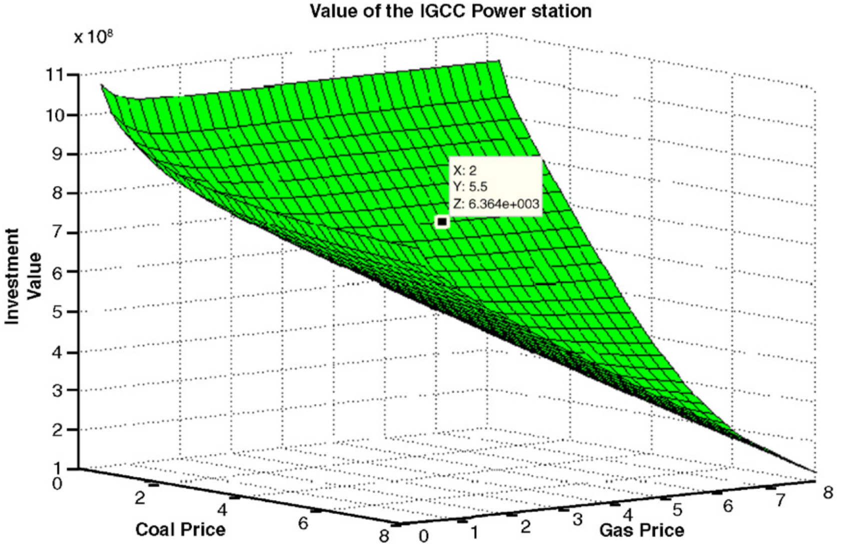

Fig. 1. Gross value (€) of an IGCC plant (a = 0.00, τ = 25).

Table 8
Gross value (thousand €) of an operating IGCC plant as a function of the growth rate of electricity price and variable cost (r_a)

Remaining life	r_a=0.00	r_a=0.03
25 years	702,534	1,245,700
20 years	613,835	999,120
15 years	501,490	742,186
10 years	361,090	479,990
5 years	191,020	223,980
4 years	153,950	175,460
3 years	116,150	128,500
2 years	77,806	83,409
1 year	39,044	40,477
0 year	0	0

NGCC plant. Yet it is much more time-consuming. The firm must choose the optimal time to invest. Thus at every moment between 0 and T it must compare the continuation value (i.e. that of the option to invest when kept unexercised) and the value of an immediate investment. Since the firm can decide to invest at any time given current fuel prices, the plant's useful life can start at any moment at the prevailing fuel prices. This implies that the above task of valuing an operating IGCC power plant now must be solved at every node of the two-dimensional binomial tree.

At the option's expiration, the firm must decide whether to invest then (if the plant's net value is positive) or not to invest:

$$W = \text{Max}\left(\text{NPV}_{\text{igcc}}, 0\right).$$

Assuming again switching costs of 20,000 €, Table 10 shows pairs of coal and natural gas prices that imply an option value equal to zero (that is, the plant's value matches initial outlay) at maturity.

From the first two columns, if the IGCC plant is to be worthy for a very high coal price it is necessary that natural gas prices remain below 0.9897 €/GJ. This is due to the fact that the reversion process pushes gas prices towards 3.25 €/GJ. However, if gas price were to grow arbitrarily high, the plant's net value would reach zero for a coal price of 2.1410 €/GJ, which is above its reversion value (S_m= 1.40 €/GJ). For lower coal prices, its value would even switch to positive. This seems reasonable since the IGCC plant is mainly designed to operate with coal. These values are shown in Fig. 2.

At previous moments, the best of the two options (to invest or to continue) has to be chosen:

$$W = \text{Max}\left(\text{NPV}_{\text{igcc}}, e^{-r\Delta t}\left(p_{\text{uu}}W^{++} + p_{\text{ud}}W^{+-} + p_{\text{du}}W^{-+} + p_{\text{dd}}W^{--}\right)\right). \quad (32)$$

At t=0, the value obtained from the lattice is compared with that of an investment realized right then. The difference is the value of the option to wait.

Table 9
Convergence analysis of gross and net values (thousand €)

Steps	Net value (switch: 20,000 €)	Steps	Net value (switch: ∞)	Steps	Gross value (switch: 20,000 €)
75	47,662	300	41,987	120	361,090
300	52,534	3000	43,353	1500	362,050
1200	53,693	∞	43,594	3,000	362,090

Table 10
Critical boundary to invest in IGCC

At maturity ($t=T$)		2 years to maturity ($t=T-2$)	
Gas (€/GJ)	Coal (€/GJ)	Gas (€/GJ)	Coal (€/GJ)
∞	2.14	∞	1.57
5.4563	2.23	5.4563	1.56
5.45	2.23	5.45	1.56
5.00	2.24	5.00	1.56
4.50	2.26	4.50	1.56
4.00	2.30	4.00	1.56
3.50	2.37	3.50	1.56
3.25	2.45	3.25	1.57
3.00	2.58	3.00	1.57
2.50	3.07	2.50	1.59
2.00	4.06	2.25	1.85
1.50	6.26	2.00	2.98
1.00	18.92	1.50	5.15
0.9897	∞	0.96	∞

Using a lattice with quarterly steps for the option to invest, and one with monthly steps to value the plant (300 as before), we get the valuations in Table 11 for the base case, as a function of the time to maturity of the investment option.

The second column refers to the plant's value under the assumption of a "now or never" investment in the flexible technology. It includes the value of the plant's flexibility. With 25 years ahead in operation we have: 702,534,000 − 650,000,000 = 52,534,000 € (see Table 8).

The value of the option increases with its maturity, given the starting point of fuel prices (both above their reversion levels). The highest yearly increase takes place in the first period (8,058,000 €); henceforth that increase is much lower since the reversion effect is stronger in the initial periods.

The optimal investment rule in an IGCC plant can be derived following a procedure akin to that used for the NGCC technology. Nonetheless, in this case we have to compute the combinations of coal price and gas price for which the option is worthless.

At any time t, a range of initial prices for natural gas is chosen; then, for each one of them, we compute the coal price for which the option switches from positive to zero. For an option to wait up to two years, we get the last two columns in Table 10. Again, the higher the price of one fuel, the cheaper the other one: since coal and gas are substitute resources, there must be a trade-off in their prices for the option's value to stabilize at a certain level (in this case, zero).

These results are shown in Fig. 2. It can be observed that the new boundary has shifted downwards and to the left, in relation to the case in which there is no option to wait (the first two columns in Table 10). In other words, it makes sense to give up ("kill") the option to wait if fuel prices are now relatively lower than before but not otherwise. The upper locus divides the price space into two regions; above, the best decision is not to invest, and below it is optimal to invest. Similarly, above the lower locus the firm should wait, but it should invest immediately if fuel prices happened to fall below this locus.

Finally, let us consider for a moment the surface in Fig. 1. Fixing natural gas price at $S_g = 5.00$ €/GJ, the resulting graphic would be a downward sloping curve in the plant's value / coal price plane. Subtracting from these gross values the investment disbursement, and restricting coal prices to the range from 1.4 €/GJ to 1.9 €/GJ, the lower locus in Fig. 3 arises. For convenience, in addition to the plant's NPV, the value of the option to invest has also been drawn. It can be observed that optimal

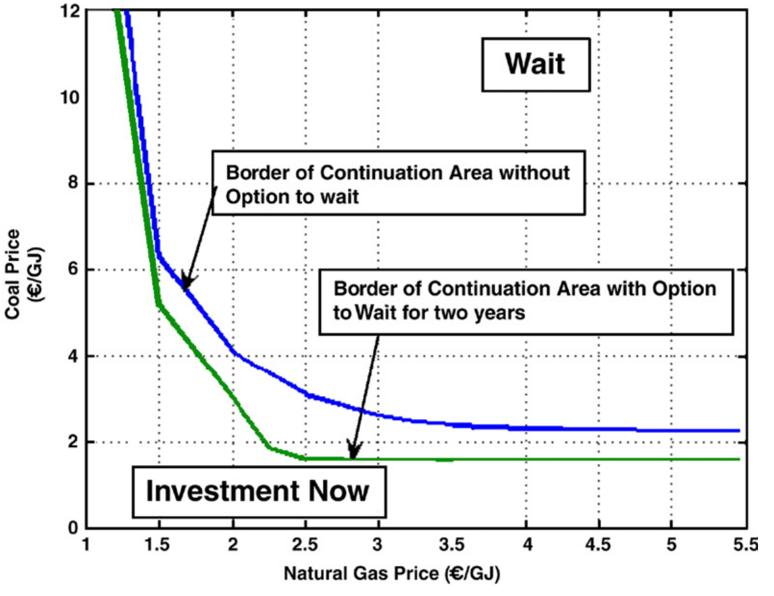

Fig. 2. Optimal boundary between invest/not invest (upper locus, $t=T$) and invest/wait (lower locus, $t=T-2$) in an IGCC power plant.

exercise of the option will take place as soon as coal price falls to 1.56 €/GJ. If this is higher, it will be better to wait.

4.6. Valuation of the opportunity to invest in NGCC or IGCC

Up to now we have considered both the flexible and the inflexible technologies in isolation. When there is an opportunity to invest in either one of the two technologies, at each moment we face the choice:

a) to invest in the inflexible technology (NGCC),
b) to invest in the flexible technology (IGCC),
c) to wait and at maturity give up the investment.

Table 11
NPV and option value (thousand €) of an IGCC plant

Term (years)	NPV	Option value	Option value — NPV
5	52,534	76,398	23,864
4	52,534	74,179	21,645
3	52,534	71,048	18,514
2	52,534	66,651	14,117
1	52,534	60,592	8,058
0.5	52,534	56,835	4,301
0	52,534	52,534	0

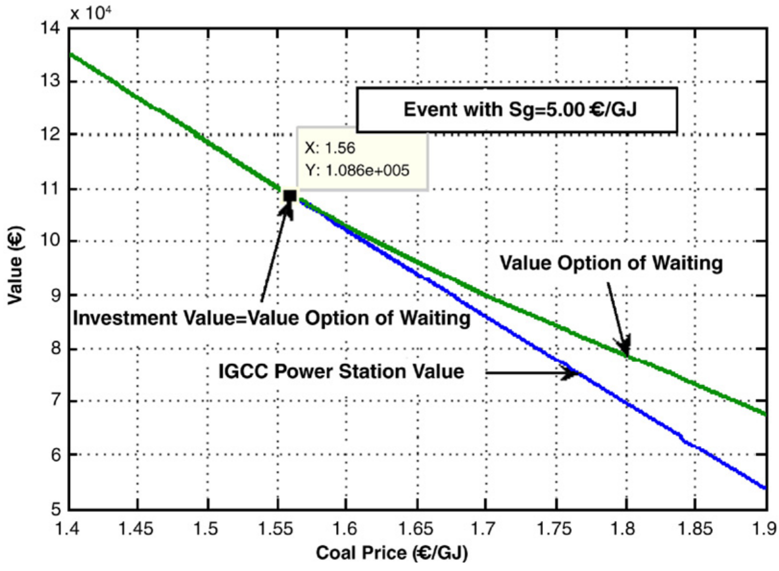

Fig. 3. NPV of the IGCC plant (for S_g=5.00 €/GJ) and value of the option to invest with 2 years to maturity, both as a function of coal price.

At expiration ($t=T$), since there is no remaining option, the best alternative among the three available ones is chosen:

$$W = \text{Max}\left(\text{NPV}_{\text{igcc}}, \text{NPV}_{\text{ngcc}}, 0\right),$$

where the value NPV_{igcc} at each node is derived from a two-dimensional binomial lattice with the fuel prices at that node.

At time T, if the only possibility is to invest in the NGCC technology, the investment is realized whenever natural gas price is lower than 5.4563 €/GJ; see Table 3. Similarly, if the only alternative is to invest in the IGCC technology, the plant is built when the pairs of gas price and coal price lay below the boundary resulting from Table 10 ($t=T$). Now Fig. 4 shows both decision rules, but it must be remembered that, in this case, it is not possible to choose the best possibility. Thus, for $S_g > 5.45$ €/GJ it does not pay to invest in a NGCC plant. Concerning the IGCC facility, it is optimal to invest for pairs of fuel prices below the decreasing locus, but not otherwise. Note from Table 10 ($t=T$) that this locus intersects the vertical border at a coal price S_c=2.23 €/GJ.

Now assume that it is possible to choose between the two alternatives (Fig. 5). Clearly, if both fuel prices are relatively high, none of the technologies will be adopted and there will be no new investment in these power plants. There are also many pairs (S_g, S_c) such that the value of the investment is positive for both plants; consequently the plant with the highest value will be chosen. Thus, given a high coal price, if S_g drops enough the NGCC will become the technology of choice. Alternatively, given a high gas price, if S_c falls enough the IGCC technology will be preferred. This area will be divided into two by a boundary starting at the point (S_g=5.4563, S_c=2.2325) and pointing towards the origin; along this line there is indifference between investing in IGCC and NGCC.

At previous moments ($t < T$), the choice is:

$$W = \text{Max}\left(\text{NPV}_{\text{igcc}}, \text{NPV}_{\text{ngcc}}, e^{-r\Delta t}\left(p_{\text{uu}}W^{++} + p_{\text{ud}}W^{+-} + p_{\text{du}}W^{-+} + p_{\text{dd}}W^{--}\right)\right).$$

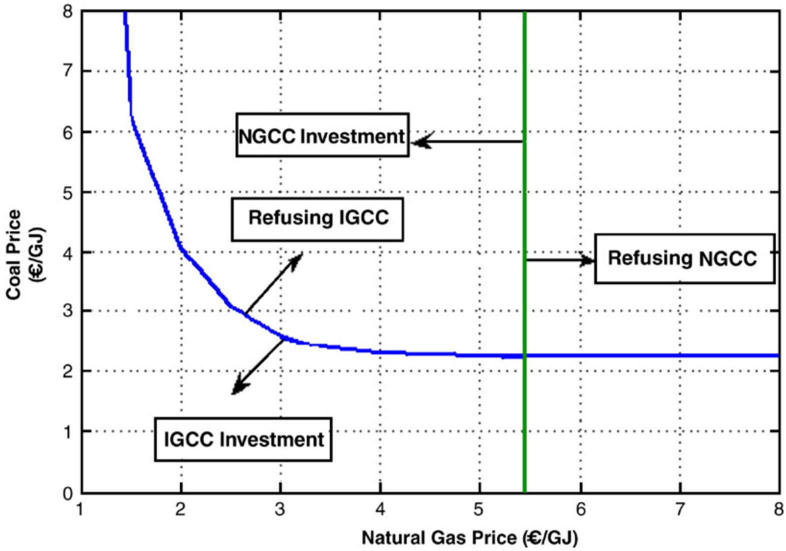

Fig. 4. Decision regions ($t=T$) when each technology is considered in isolation.

This computing procedure is iteratively followed until the initial value is obtained. At that instant, the NGCC technology will be adopted if $W=\text{NPV}_{ngcc}$; similarly, the IGCC technology is chosen if $W=\text{NPV}_{igcc}$. If there is no investment at $t=0$, this means that the best decision is to wait.

Along the optimal exercise boundary the value of the option drops to zero. In principle, there may be two curves, one for the IGCC plant and another one for the NGCC plant. See Fig. 6 for an

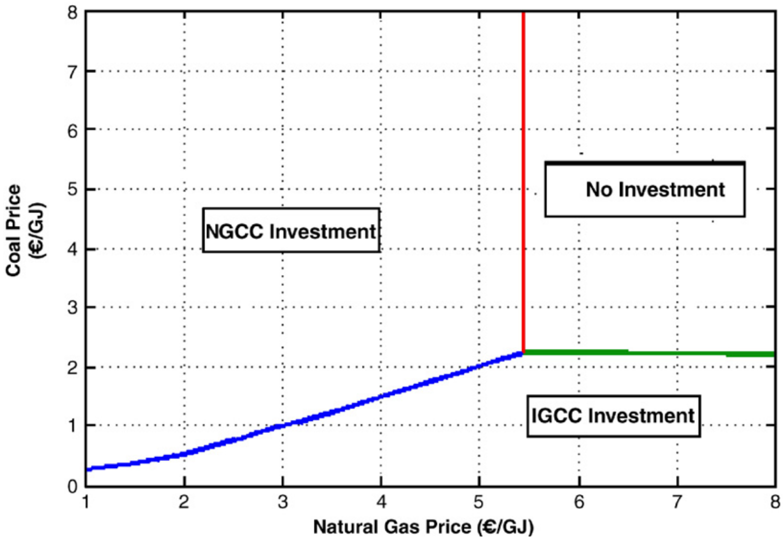

Fig. 5. Investment/Continuation regions ($t=T$) when choosing between the two technologies.

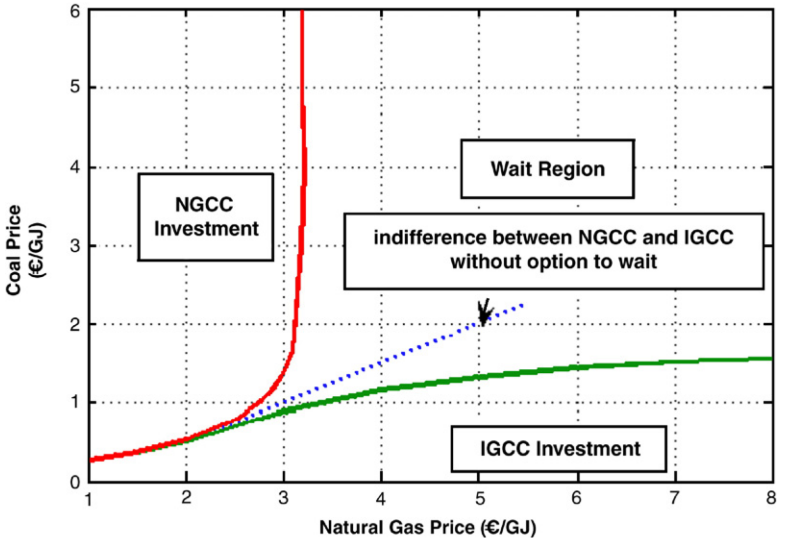

Fig. 6. Investment/Continuation regions when both technologies are on offer and there is an option to wait up to 2 years.

option to invest in either technology with 2 years to maturity; again, we set up a binomial lattice with quarterly steps for this option to invest. It can be observed that:

a) In order to invest, prices must be rather lower than those when there is no option to wait. Thus the new locus for the NGCC moves leftward from the vertical line at $S_g=5.45$ €/GJ before; and the new locus for the IGCC moves downwards from the flat line at $S_c=2.23$ €/GJ. The reversion effect, given initial fuel prices, promotes this behaviour.
b) For a very high natural gas price, there will be investment in an IGCC plant if coal price falls below 1.57 €/GJ; this is the same that we computed for the IGCC investment option when this was the only technology available (Table 10, $t=T-2$).
c) Similarly, for a very high coal price, there will be investment in a NGCC plant if gas price is lower than 3.17 €/GJ. This value is slightly different from the homologous in Table 3, namely 3.0864 €/GJ, derived from a lattice with 1200 steps. Now only 8 quarterly steps are being used, yet trigger prices differ by less than 3%.
d) For fuel prices close to the iso-value line between immediate investment in NGCC and IGCC, now the best choice is just to wait and see how uncertainty unfolds. The waiting zone expands into the regions of immediate investment driving a wedge along the indifference boundary (at T) between NGCC and IGCC.

The shape of the three optimal regions in Fig. 6 somehow resembles that in Brekke and Schieldrop (2000), which is derived as the analytic result of a perpetual option to invest. In our case the solution is numerical because the option to invest is finite-lived.

5. A geometric Brownian motion for fuel price

All the above numerical results have been derived under the assumption that mean reversion in fuel prices is relevant for the problem at hand. To what extent are they affected by it? How would they change if a standard GBM were instead adopted? These are legitimate concerns that we want to address somehow. Below we show some computations under this new assumption that lend themselves easily to comparison with former results. We do not seek to replicate all the analyses, but to allow the reader to have a glimpse at this issue and judge by herself. The stochastic processes under consideration are:

$$(\text{GBM})dS_t = \mu S_t dt + \sigma S_t dZ_t,$$

$$(\text{IGBM})dS_t = k(S_m - S_t)dt + \sigma S_t dZ_t.$$

Ideally the parameters of these processes should be estimated rigorously from actual fuel prices. Instead we have approximated some numerical values for all of them bar one, μ. Concerning its value, and in order for the comparisons to be meaningful, we adopt the following criterion: the total cost of consuming one unit of fuel per year over the plant's useful life must be the same regardless of the particular process that governs fuel price, be it GBM or IGBM. In the simple case in which $\phi=0$ or $\rho=0$ we have (see Appendix):

$$\frac{S\left[1-e^{-(r-\mu)T}\right]}{r-\mu} = \frac{S_m(1-e^{-rT})}{r} + \frac{S-S_m}{r+k}\left[1-e^{-(r+k)T}\right].$$

In the absence of a fully-fledged econometric estimation of μ from historical price series, we compute the value of μ that makes both sides equivalent.

5.1. Only the NGCC technology available

In the case of natural gas the parameter values are: $S_m=3.25$ €/GJ, $k=0.25$, $\sigma=0.20$, $T=25$, $S=5.45$ €/GJ and $r=0.05$. Hence: $\mu=-0.04106216$.[18]

Note that the above criterion impinges directly on the value of the operating NGCC plant. Specifically, given that fuel costs are forced to remain the same and these are the only item assumed to be stochastic, the net present value of the plant is not affected by the choice between the two processes.

The value of the option to invest, though, behaves differently. First we analyse the perpetual option to invest in a NGCC plant. In this case, the option value must satisfy the following differential equation:

$$\frac{1}{2}\sigma^2 S^2 F'' + \mu S F' - rF = 0.$$

The solution to this equation is:

$$F(S) = A_1 S^{\gamma_1} + A_2 S^{\gamma_2}.$$

It may be argued that if fuel price grows arbitrarily high the option will be worthless, so $A_1=0$ and hence $F(S)=A_2 S^{\gamma_2}$. Taking the first and second derivatives, and substituting F, F' and F'' into the differential equation we get:

$$\frac{1}{2}\sigma^2\gamma_2^2 + \gamma_2\left(\alpha - \frac{1}{2}\sigma^2\right) - r = 0.$$

This is a quadratic equation. For $\sigma=0.20$ the negative root turns out to be: $\gamma_2=-0.671255612$.

[18] In fact, this value implies similar expected gas prices by the plant's mid life ($t=12.5$) under both stochastic processes:

$$(\text{GBM})E(S_t) = S_0 e^{\mu t} = 3.26,$$
$$(\text{IGBM})E(S_t) = S_m + (S_0 - S_m)e^{-kt} = 3.35.$$

Table 12
Trigger price S^* with finite time to maturity

Term	r_a=0, r_b=0	r_a=0, r_b=0.025	r_a=0, r_b=0.05
0	5.4521	5.4521	5.4521
½	2.7579	3.0118	3.2660
1	2.6757	2.9171	3.1497
2	2.5659	2.7820	2.9846
3	2.4886	2.6901	2.8815
4	2.4322	2.6258	2.8130
5	2.3900	2.5792	2.7648
6	2.3577	2.5442	2.7299
7	2.3326	2.5175	2.7038
8	2.3128	2.4966	2.6840
9	2.2970	2.4802	2.6688
10	2.2842	2.4671	2.6571
∞	2.1898	–	–

The values of A_2 and S^* remain to be determined. We resort to the value-matching and smooth-pasting conditions:

$$F(S^*) = V(S^*) - I(S^*),$$

$$F'(S^*) = V'(S^*) - I'(S^*).$$

With regard to V, as before the present value of revenues minus that of variable costs and the investment outlay amounts to 1,342,055,454 € (see expression (26)). Now the present value of fuel costs must be deducted; in terms of fuel price, S, with our parameter values:

$$\frac{S\left[1-e^{-(r-\mu)T}\right]}{r-\mu}B = 246,153,090\ \ S.$$

Thus the boundary conditions state that:

$$A_2(S^*)^{\gamma_2} = 1,342,055,454 - 246,153,090\ \ S,$$

$$A_2\gamma_2(S^*)^{\gamma_2-1} = -246,153,090.$$

It is straightforward to solve for the trigger price; now S^*=2.1898 €/GJ (instead of 2.7448 when the IGBM was assumed). Intuitively, mean reversion imposes narrower barriers to the price path

Table 13
NPV and option value (thousand €) with T=5 years

S_0	NPV	Option value	Max(NPV, Option)	Optimal decision
5.45	521	285,850	285,850	Wait
5.00	111,290	332,080	322,080	Wait
4.50	234,370	390,690	390,690	Wait
4.00	357,440	457,700	457,700	Wait
3.50	480,520	534,340	534,340	Wait
3.00	603,600	622,710	622,710	Wait
2.50	726,670	727,580	727,580	Wait
2.3900	753,750	753,750	753,750	Indifferent
2.00	849,750	849,660	849,750	Invest

Table 14
Gross and net value (thousand €) of an IGCC plant

Switching costs (€)	Plant's value	Plant's value — Initial investment
0	799,890	149,890
10,000	799,850	149,850
20,000	799,810	149,810
50,000	799,680	149,680
100,000	799,530	149,530
1,000,000	797,980	147,980
∞	691,692	41,692

than the GBM. In this sense the adoption of a GBM increases volatility. This in turn translates into a lower fuel price for the option to be exercised.

As for the non-perpetual option, the starting point is the risk-neutral version of the GBM:

$$\frac{d\hat{S}}{\hat{S}} = (\mu - \rho\sigma\phi)dt + \sigma dZ.$$

Define $X = \ln\hat{S}$; then applying Ito's Lemma:

$$dX = \left(\mu - \rho\sigma\phi - \frac{1}{2}\sigma^2\right)dt + \sigma dZ = \hat{\mu}dt + \sigma dZ.$$

It can be shown that $\Delta X = \sigma\sqrt{\Delta t}$, $u = e^{\sigma\sqrt{\Delta t}}$ and $d = e^{-\sigma\sqrt{\Delta t}}$. The probability of an upward movement is given by $p_u = \frac{1}{2} + \frac{\hat{\mu}\sqrt{\Delta t}}{2\sigma}$, where $\hat{\mu}$ is now a constant.

Table 12 below is the homologous to Table 3 under the GBM. Again, as the option's maturity increases the trigger price decreases; also, this converges toward 2.1898 as maturity approaches infinity. Finally, the values in Table 12 are lower than those in Table 3; in other words, the conditions to exercise the option to invest are more stringent under the GBM.

Similarly, Table 13 is the homologous to Table 4 under the GBM. The option to invest expires in 5 years' time. As can be seen, greater uncertainty induces a higher option value. Under the IGBM it was optimal to invest as soon as fuel price dropped to 2.9079 €/GJ. Now the price must fall further, to 2.3900. In other words, there are less chances for the investment to be undertaken.

Table 15
Gross value (thousand €) of an operating IGCC plant as a function of the growth rate of electricity price and variable cost (r_a)

Remaining life	r_a=0.00	r_a=0.03
25 years	799,810	1,367,700
20 years	674,270	1,074,300
15 years	528,270	775,860
10 years	363,940	484,820
5 years	186,770	219,870
4 years	150,370	171,940
3 years	113,650	126,020
2 years	76,503	82,109
1 year	38,689	40,122
0 year	0	0

Table 16
Critical boundary to invest in IGCC ($t=T$)

Gas price (€/GJ)	Coal price (€/GJ)
∞	2.03
5.45	2.55
5.00	2.69
4.50	3.00
4.00	4.39
3.90	5.71
3.81	∞

5.2. Only the IGCC technology available

Concerning the operating IGCC plant, the relevant parameters for coal are: $S_m=1.40$ €/GJ, $k=0.125$, $\sigma=0.05$, $T=25$, $S=1.90$ €/GJ and $r=0.05$. With these values we compute: $\mu=-0.01818148$. Table 14 closely resembles Table 7, apart from the assumption of a GBM price process. Again, at current fuel prices the value of the flexible plant is not very sensitive to switching costs. Anyway the value is significantly higher than before; presumably flexibility is more valuable when fuel prices are more volatile.

On the other hand, the value of the plant depends on its remaining useful life. Table 15 is parallel to Table 8. As before, the value decreases as the plant approaches closure. This is so regardless of the electricity price's growth rate. Nonetheless, there is a quirk: the GBM enhances the plant's value when this has a long time ahead to operate; however, with 5 years or less before closure, the plant is less valuable than under the IGBM. It is as if greater volatility spells greater opportunities in the early years but becomes more of a threat in the final ones.

Last, consider the option to invest in an IGCC plant. At maturity, there is no further chance for new information to be gained from waiting. Also, the above results show that a new operating plant is more valuable under the GBM assumption. Thus it is not surprising that now there are more pairs of fuel prices for which the optimal decision is to invest. Just compare Table 16 with Table 10 ($t=T$). For a given gas price, the coal price that triggers investment is higher. In graphical terms, the boundary that divides the prices space into two regions turns steeper and the "invest" region becomes larger. From the numerical values, this shift appears to be significant.

6. Concluding remarks

Many investments in the energy sector can be conveniently valued as real options. Frequently appearing features are the operating flexibility and the possibility to delay the investment or even not to invest at the final moment. Special attention in the valuation must be paid to the nature of the stochastic processes that govern the underlying variables.

In this paper we address the choice between competing technologies for producing electricity under the assumption that input fuel prices follow a mean-reverting process, namely an IGBM. In particular, we have analyzed the valuation of a non-perpetual opportunity to invest in an IGCC power plant, as opposed to the alternative of an NGCC power plant. Actual parameters from an operating plant have been used.

First we have computed the value of an operating NGCC plant. This is relatively easy since there is no special flexibility in its usage. Then the value of a perpetual option to invest in it has been derived. It serves as a benchmark or limiting case for the finite-lived option. This has been valued by means of a binomial lattice for different maturities and growth rates of electricity price and investment outlay. The optimal investment rule for a given term as a function of natural gas price has also been analyzed.

Second we have valued an IGCC plant in operation using two two-dimensional lattices, depending on whether initially the plant burns coal or natural gas. The value of the plant has been computed as a function of switching costs between modes of operation and for different useful life spans with constant or growing electricity prices. The value of the operating flexibility in the IGCC power plant seems to be low (at current fuel prices well above their long-run levels). Next the non-perpetual option to invest in an IGCC plant has been considered, assuming again that this is the only technology available. In this case, an optimal locus of fuel prices arises above which it is optimal to wait. As could be expected, the longer the option's maturity the closer the locus is to the origin in the fuel prices space.

Third we have assumed that both technologies are on offer. Since the firm has always the opportunity to not invest, there are now three regions in the prices space. Obviously, there are pairs of fuel prices such that the optimal decision remains the same whatever the time to the option's expiration. In other cases, as maturity approaches waiting ceases to be optimal and the firm chooses to invest. Decision regions are clear-cut anyway.

At this point several qualifications can be made. The choice of the appropriate stochastic model for fuel prices has already been addressed to some extent. This is basically an empirical matter, yet we have not run any kind of test on actual fuel prices series. Instead we have adopted a relatively general stochastic process which encompasses more traditional specifications as particular cases; as for the parameter values, we have taken hopefully sensible values used elsewhere. In this sense the paper just shows how to apply a specific methodology to the valuation and management of investments in certain power plants. With regard to the flexible plant it must be stressed that we have restricted ourselves to flexibility on the input side; any consideration about optional working units which may give rise to an array of final products has thus been neglected (even though they may actually become additional sources of value). This helps to keep matters simple in that there are only two sources of risk. On the other hand, power utilities (at least in the European Union) now face a new carbon market which, regardless of whether it is seen as a threat or an opportunity, no doubt will influence decision making by utilities. Nevertheless carbon is not an issue in our choice between the aforementioned technologies.

This paper may be extended in several ways. Whenever actual market data are available they should be the natural starting point to check which stochastic process fits best and/or to get numerical estimates of the parameter values. There are well developed markets for a number of energy resources and also for electricity in some countries. Concerning the valuation model, inclusion of more sources of risk could be justified to account for output price uncertainty, or the uncertain price of carbon allowances, for instance. Obviously this would render the numerical results more palatable. In doing so, though, binomial lattices are no longer a workable approach and one must resort to other numerical techniques, probably involving Monte Carlo simulation.

Acknowledgements

We want to thank seminar participants at the 9th Conference on Real Options (Paris 2005), XIII Foro Europeo de Finanzas (Madrid 2005), the 1st Annual Congress of the Spanish Association for Energy Economics (Madrid 2006), and the 2nd Conference of the Spanish

Portuguese Association of Natural Resource and Environmental Economics (Lisbon 2006) for their helpful comments. We also thank the Editor Prof. Richard Tol for his guidance and two anonymous referees for their remarks; they have enriched the paper and improved the presentation. We remain responsible for any errors. Chamorro acknowledges financial support from research grant 9/UPV00I01.I01 14548/2002.

Appendix. Valuation of an annuity under mean reversion

Our aim is to value an asset V which pays Zdt continuously over a finite number of periods τ of remaining asset's life, with X following a mean-reverting process of the type:

$$dX = k(X_m - X)dt + \sigma X dZ_t.$$

It can be shown that the asset A satisfies the differential equation:

$$\frac{1}{2}V_{XX}\sigma^2X^2 + (k(X_m - X) - \rho\sigma\phi X)V_X - rV - V_\tau = -X, \tag{33}$$

where it is assumed that the existing traded assets dynamically span the price X. As in Section 3, let ρ denote the correlation with the market portfolio, and φ the market price of risk. The solution $V(X, \tau)$ to the differential equation must satisfy the following boundary conditions:

a) At $\tau=0$ the value must be zero: $V(X, 0)=0$.
b) Bounded derivative as $X \to \infty$: $V_X(\infty, \tau) < \infty$.
c) Bounded derivative as $X \to 0$: $V_X(0, \tau) < \infty$.

Using Laplace transforms we get:

$$\frac{1}{2}h_{XX}\sigma^2X^2 + (k(X_m - X) - \rho\sigma\phi X)h_X - h(r+s) = -\frac{X}{s}.$$

Rearranging:

$$\frac{1}{2}h_{XX}\sigma^2X^2 - (k+\rho\sigma\phi)Xh_X - h(r+s) = -kX_m h_X - \frac{X}{s}.$$

The general solution has the form:

$$h(X) = A_1X^{\beta_1} + A_2X^{\beta_2} + \frac{X - \frac{kX_m}{k+\rho\sigma\phi}}{s(s+r+k+\rho\sigma\phi)} + \frac{kX_m}{(k+\rho\sigma\phi)s(s+r)}.$$

The derivative is bounded; thus $A_1=0$. Besides, $h(0)=0$; therefore $A_2=0$.

The solution simplifies to:

$$h(X) = \frac{X - \frac{kX_m}{k+\rho\sigma\phi}}{s(s+r+k+\rho\sigma\phi)} + \frac{kX_m}{(k+\rho\sigma\phi)s(s+r)}.$$

With the first and second derivatives, $h_x = \frac{1}{s(s+r+k+\rho\sigma\phi)}$, $h_{XX}=0$, it is possible to show, by substitution, that the differential equation applies.

At this moment, the inverse Laplace transforms are taken. To do so we use formula 29.3.12 in Abramowitz and Stegun (1972), the final result being:

$$V = \frac{kX_m(1-e^{-r\tau})}{r(k+\rho\sigma\phi)} + \frac{X-\frac{kX_m}{k+\rho\sigma\phi}}{r+k+\rho\sigma\phi}\left(1-e^{-(r+k+\rho\sigma\phi)\tau}\right). \tag{34}$$

This formula may be useful to compute the present value of fuel costs over the whole life of a plant with inflexible technology, like an NGCC or coal plant.[19]

A series of particular, frequently used, cases may be derived from the above general solution:

a) If $\phi=0$ or $\rho=0$, then the formula reduces to

$$V = \frac{X_m(1-e^{-r\tau})}{r} + \frac{X-X_m}{r+k}\left(1-e^{-(r+k)\tau}\right).$$

b) If $\tau \to \infty$:

$$V = \frac{kX_m}{r(k+\rho\sigma\phi)} + \frac{X-\frac{kX_m}{k+\rho\sigma\phi}}{r+k+\rho\sigma\phi}. \tag{35}$$

In this case, it can be observed that the project value is the sum of two components: one related to the reversion value and another one which is a function of the initial difference between the observed value and the "normal" level of X.

c) When it is a perpetuity and also $X_m=0$ and $k+\rho\sigma\phi=-\alpha$, then:

$$V = \frac{X}{r-\alpha}. \tag{36}$$

d) When it is a perpetuity and also $X_m=0$ and $k+\rho\sigma\phi=-r+\delta$, then:

$$V = \frac{X}{\delta}. \tag{37}$$

References

Abramowitz, M., Stegun, I., 1972. Handbook of Mathematical Functions. Dover Publications.

Baker, M.P., Mayfield, E.S., Parsons, J.E., 1998. Alternative models of uncertain commodity prices for use with modern asset pricing methods. Energy Journal 19 (1), 115–148.

Bjerksund, P., Ekern, S., 1990. Managing investment opportunities under price uncertainty: from "last chance" to "wait and see" strategies. Financial Management 19 (3), 65–83.

Bhattacharya, S., 1978. Project valuation with mean-reverting cash flow streams. Journal of Finance XXXIII (5), 1317–1331.

Brekke, J.A., Schieldrop, B., 2000. Investment in flexible technologies under uncertainty. In: Brennan, M.J., Trigeorgis, L. (Eds.), Project flexibility, agency and competition. Oxford University Press, pp. 34–49.

Chen, Y.-F., Funke, M., 2004. Working time and employment under uncertainty. Studies in Nonlinear Dynamics and Econometrics 8 (3) (article 5).

Cortazar, G., Schwartz, E.S., 2003. Implementing a stochastic model for oil futures prices. Energy Economics 25, 215–238.

Dixit, A.K., Pindyck, R.S., 1994. Investment Under Uncertainty. Princeton University Press.

[19] See Bhattacharya (1978), Eq. (15), and Sarkar (2003), Eqs. (2)–(4).

ELCOGAS, 2003. Integrated Gasification Combined Cycle Technology: IGCC. Its Actual Application in Spain.
Elliott, R., Sick, G., Stein, M., 2002. Price interactions of baseload changes and electricity demand shocks. In: Ronn, E.I. (Ed.), Real Options and Energy Management. Risk Books.
Hasset, K.A., Metcalf, G.E., 1995. Investment under alternative return assumptions: comparing random walks and mean reversion. Journal of Economic Dynamics and Control 19, 1471–1488.
He, H., Pindyck, R.S., 1992. Investments in flexible production capacity. Journal of Economic Dynamics and Control 16, 575–599.
Herbelot, O., 1992. Option valuation of flexible investments: The case of environmental investments in the electric power industry. Ph.D. thesis, Massachusetts Institute of Technology.
Hull, J., 1993. Options, Futures and Other Derivatives. 2nd edition. Prentice Hall.
Insley, M., 2002. A real option approach to the valuation of a forestry investment. Journal of Environmental Economics and Management 44, 471–492.
Kandel, E., Pearson, N.D., 2001. Flexibility versus commitment in personnel management. Journal of Japanese and International Economics 15, 515–556.
Kandel, E., Pearson, N.D., 2002. Option value, uncertainty, and the investment decision. Journal of Financial and Quantitative Analysis 37 (3), 341–374.
Kemma, A., 1993. Case studies on real options. Financial Management 22 (3), 259–270.
Kulatilaka, N., 1988. Valuing the flexibility of flexible manufacturing systems. IEEE Transactions on Engineering Management 35 (4), 250–257.
Kulatilaka, N., 1993. The value of flexibility: the case of a dual-fuel steam boiler. Financial Management 22 (3), 271–279.
Kulatilaka, N., 1995. The value of flexibility: a general model of real options. In: Trigeorgis, L. (Ed.), Real options in capital investment. Praeger.
Laughton, D.G., Jacoby, H.D., 1993. Reversion, timing options and long-term decision making. Financial Management 22 (3), 225–240.
Laughton, D.G., Jacoby, H.D., 1995. The effects of reversion on commodity projects of different length. In: Trigeorgis, L. (Ed.), Real options in capital investment. Praeger.
Lund, D., Oksendal, B., 1991. Stochastic Models and Option Values. North-Holland.
Mun, J., 2002. Real Option Analysis. John Wiley and Sons, Inc.
Paddock, J., Siegel, D., Smith, J., 1988. Option valuation of claims on physical assets: the case of offshore petroleum leases. Quarterly Journal of Economics 103 (3), 479–508.
Pilipovic, D., 1998. Energy Risk. McGraw-Hill.
Pindyck, R.S., 1988. Irreversible investment, capacity choice and the value of the firm. American Economic Review 78, 969–985.
Robel, G.F., 2001. Real options and mean-reverting prices. The 5th Annual Real Options Conference.
Ronn, E.I., 2002. Real Options and Energy Management. Risk Books.
Sarkar, S., 2003. The effect of mean reversion on investment under uncertainty. Journal of Economic Dynamics and Control 28, 377–396.
Schwartz, E.S., 1997. The stochastic behavior of commodity prices: implications for valuation and hedging. Journal of Finance 52 (3), 923–973.
Trigeorgis, L., 1996. Real Options — Managerial Flexibility and Strategy in Resource Allocation. The MIT Press.
Weir, J.G., 2005. Valuation of petroleum leases as real options. The 9th Annual Real Options Conference.