# ONE-TAILED ASYMPTOTIC INFERENCES FOR THE RATIO OF PROPORTIONS

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When comparing two independent binomial proportions ($p_1$ and $p_2$), interest is normally focused on the parameters $d = p_2–p_1$ (difference) and $R = p_2/p_1$ (ratio). The parameter $d$ is more frequent in applied statistics, but the parameter $R$ is usually used in the field of medical research. In fact $R$ is better than $d$ in certain occasions (Agresti, 2002), so this work is focused on inferences about the relative risk.

Cai (2005) showed that the performance for discrete distributions of the same procedure can vary strongly depending on whether the confidence interval (CI in the following) has one or two tails, so each situation should be analyzed separately. The two-sided case has traditionally been the subject of a great deal of attention, but the case of one tail has aroused much less interest.

For the inferences to be coherent the one-sided CI should be obtained by inverting the one-sided test. This means two things: the first conclusion is that the definition for a method of inference can be made from the point of view of the test or from the point of view of the CI; the second conclusion is that evaluating a CI method is equivalent to evaluating its associated test method (if both are performed to the same nominal error $\alpha$).

Exact confidence intervals are computationally intensive and are not practicable for moderately large sample sizes because they require special computer programmes. This is why researchers have shown great interest in approximate methods, especially in asymptotic methods as they are generally simpler to apply.

The paper evaluates several asymptotic methods for obtaining a one-sided confidence interval for the ratio between two binomial proportions (most of them are new proposals) and comes to the conclusion that the optimal method is the classic Z-statistic with the criterion of Peskun, after adding 1 success and 1 failure to the original data. Specifically for extreme values of $\rho$, the best method is the log statistic of Woolf (1955) with estimators of proportions based on the Newcombe's procedure, applied to the increased data in $z^2_{\alpha/2}/4$.

**Keywords**: Ratio of proportions; One-sided asymptotic inferences; Adjusted Wald methods; Score method; Arcsine transformation.