A new adjusted confidence interval for a linear combination of independent binomial proportions

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The Wald method, used for constructing approximate confidence intervals for a linear combination of binomial proportions of $k \geq 2$ independent populations, depends on the unrestricted model assumed in the parameter estimation process. The parametric family of estimators given by $(X_i+h_i)/(n_i+2h_i)$, $h_i \geq 0$, is usually suggested for estimating the proportions p_i, $i = 1, \ldots, k$, where each family member leads to two global types of variants: the classic ($h_i = 0$) and the adjusted ($h_i > 0$). Recently, a parameter h_i that addresses the presence of extremal observations ($x_i = 0$ and $x_i = n_i$) was proposed for constructing an adjusted version, with improved performance, of the Wald large-sample interval. As we know, the choices for h_i suggested in the literature do not take into account the effect of the estimate found for p_i on the estimation of the linear combination, which is the primary analysis. We propose a new parameter h_i in order to define an estimate of p_i that also takes into account its weight in the linear combination.

A comparative simulation study was carried out to investigate the performance of the adjusted version based on our proposal for h_i. Comparing all variants for several settings, the adjusted Wald confidence interval obtained from the new variant exhibits similar performance in terms of the exact coverage probability and the expected interval length, and shows improved performance when imbalances among the k populations are observed between the weight of each proportion in the linear combination and the sample size drawn from the population.

Keywords: Approximate confidence intervals; linear combination of binomial proportions; Wald method.

This work was supported by Portuguese funds through the CIDMA − Center for Research and Development in Mathematics and Applications of the University of Aveiro, and the Portuguese Foundation for Science and Technology (Fundação para a Ciência e a Tecnologia), within project UID/MAT/04106/2013.