S4. Complex Analysis and Operator Theory

Organizers:

- Nicola Arcozzi (Università di Bologna, Italy)
- Filippo Bracci (Università di Roma "Tor Vergata", Italy)
- Manuel Contreras (Universidad de Sevilla, Spain)
- Daniel Girela (Universidad de Málaga, Spain)

Speakers:

1. Marco Abate (Università di Pisa, Italy)
   Angular derivatives for infinitesimal generators of one-parameter semigroups
2. Juan Bès (Bowling Green State University, USA)
   On the existence of special hypercyclic subspaces
3. José Bonet (Universidad Politécnica de Valencia, Spain)
   Abel's functional equation and eigenvalues of composition operators on spaces of real analytic functions
4. Filippo Bracci (Università di Roma "Tor Vergata", Italy)
   Open problems in Loewner theory in higher dimension
5. Olivia Constantin (University of Kent, United Kingdom)
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6. Chiara de Fabritiis (Università Politecnica delle Marche, Italy)
   Function Spaces of slice-Regular Functions
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   The Linear Fractional Model Theorem and Alexandrov-Clark measures
8. Graziano Gentili (Università di Firenze, Italy)
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   Parametric representation of univalent self-maps with given boundary fixed points

10. María J. Martín (University of Eastern Finland, Finland)
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11. Daniele Morbidelli (Università di Bologna, Italy)
    Carnot-Carathéodory distances and Palais' completeness theorem

12. Artur Nicolau (Universitat Autònoma de Barcelona, Spain)
    Inner Functions in Weak Besov Spaces

13. José Ángel Peláez (Universidad de Málaga, Spain)
    L^p*-behavior of reproducing kernels and applications*

14. Marco Peloso (Università degli Studi di Milano, Italy)
    Bergman kernel and projection on the unbounded Diederich–Fornæss worm domain

15. M. Carmen Reguera (University of Birmingham, United Kingdom)
    Sharp Hankel operators and de Saint-Venant's inequality

16. Luis Rodríguez Piazza (Universidad de Sevilla, Spain)
    p-summing composition operators on Hardy spaces

Angular derivatives for infinitesimal generators of one-parameter semigroups

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In this talk we shall present some recent results (see [1]) on the boundary behavior of infinitesimal generators of one-parameter semigroups of holomorphic self-maps of the unit ball $B^n \subset \mathbb{C}^n$. In particular we shall discuss a Julia-Wolff-Carathéodory theorem which completes the interesting results already obtained on this topic by F. Bracci and D. Shoikhet in [2].

[1] Abate, M., Raissy, J., A Julia-Wolff-Carathéodory theorem for infinitesimal generators in the unit ball (2014); http://arxiv.org/abs/1403.0739.
[2] Bracci, F., Shoikhet, D., Boundary behavior of infinitesimal generators in the unit ball, *Trans. Amer. Math. Soc.* **366** (2014), 1119–1140.

On the existence of special hypercyclic subspaces

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We explore and refine criteria for the existence of hypercyclic subspaces, to derive the following consequences. There exist frequently hypercyclic operators with $\mathcal{U}$-frequently hypercyclic subspaces and no frequently hypercyclic subspace. On the space of entire functions, each differentiation operator induced by a non-constant polynomial supports an $\mathcal{U}$-frequently hypercyclic subspace. A solution to a problem by Costakis and Sambarino [1] on the existence of common hypercyclic subspaces for a certain family of weighted shift operators is also provided.

[1] G. Costakis and M. Sambarino, Genericity of wild holomorphic functions and common hypercyclic vectors, *Adv. Math.* **182** (2004), 278–306.

Abel's functional equation and eigenvalues of composition operators on spaces of real analytic functions

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We present a full description of eigenvalues and eigenvectors of composition operators C_φ acting on the space $A(\mathbb{R})$ of real analytic function on the real line for a real analytic self map φ, as well as an isomorphic description of corresponding eigenspaces. We also completely characterize those self maps φ for which Abel's equation $f \circ \varphi = f + 1$ has a real analytic solution on the real line. Finally, we find cases when the operator C_φ has roots using a constructed embedding of φ into a so-called real analytic iteration semigroups.

We report on joint work with Pawel Domański (Univ. Poznań, Poland).

Open problems in Loewner theory in higher dimension

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One of the main tool in geometric function theory in one dimension is the Loewner theory. In higher dimensions, Loewner theory has been developped to some extent by several authors, but there are still basic open questions to be answered, due to new phenomena (like Fatou-Bieberbach domains and shears automorphisms) which do not appear in one dimension. I will talk about some of these problems, especially those related to a Bieberbach conjecture in several complex variable.

Fock spaces versus Bergman spaces with rapidly decreasing weights

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It is well-known that, once a weight decays fast enough, the corresponding Bergman space starts to differ considerably in some respects from standard weighted Bergman spaces. It turns out that these spaces behave in a similar way to Fock spaces and, therefore, techniques from one setting can be employed to gain insight into the other. We shall explore such problems focussing on issues like natural projections and dualities. Finally, we are also going to point out some differences between these two types of spaces, which are mainly due to the fact that the area of the domain is finite in one case, while infinite in the other.

This is joint work with J. Á. Peláez.

Function Spaces of slice-Regular Functions

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The notion of slice-regular (s-regular) function of a quaternionic variable was introduced some years ago by Gentili and Struppa in order to generalize holomorphic functions on the skew-field of quaternions $\mathbb{H}$. This theory had a striking development (an almost up-to-date references is [1], see also the reference therein), thanks also to its links with functional analysis, matrix analysis, geometry of orthogonal complex structures on domains of $\mathbb{R}^4$ and physics.

The interest on spaces of s-regular functions, defined either on the unit ball $\mathbb{B}$ or on more general domains contained in $\mathbb{H}$, like axially symmetric slice domains, is a natural consequence of the rich interplay and deep relation between the well-established theory of holomorphic functions and the new-born theory of s-regular functions. Indeed, in the last century a large amount of research on several spaces of holomorphic functions has been published (think about Hardy spaces, Bergman spaces, Fock spaces and their generalizations, just to name a few).

We introduce and discuss a definition of Hardy spaces in the unit ball $\mathbb{B} \subset \mathbb{H}$ and give results which relate them to the corresponding Hardy spaces of holomorphic functions on slices. In particular we are able to describe the s-regular functions belonging to the Hardy spaces in terms of their boundary values and to show the analogue of the factorization of the elements of these spaces in terms of inner, outer and singular functions.

We will also give some results on the behaviour of multiplication operators on these spaces. Indeed a natural development of the theory of spaces of s-regular functions is the study of suitable generalizations of several classes of operators (composition, multiplication, Toëplitz) which act on spaces of holomorphic functions.

Joint works with G. Sarfatti, Università di Bologna, and G. Gentili, Università di Firenze.

[1] Gentili, G., Stoppato, C., Struppa, D. C., *Regular functions of a quaternionic variable*, Springer Monographs in Mathematics, Springer, Berlin-Heidelberg, 2013.

The Linear Fractional Model Theorem and Alexandrov-Clark measures

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A remarkable result by Denjoy and Wolff states that every holomorphic self map φ of the open unit disc $\mathbb{D}$ of the complex plane, except the elliptic automorphisms, has an attractive fixed point to which the sequence of iterates $\{\varphi_n\}_{n\geq 1}$ converges uniformly on compacta: if there is not a fixed point in $\mathbb{D}$, then there is a unique boundary fixed point that does the job, called the *Denjoy-Wolff point*. The Denjoy-Wolff point provides a classification of the holomorphic self maps of $\mathbb{D}$ into four types: maps with interior fixed point, hyperbolic maps, parabolic automorphism maps and parabolic non automorphism maps. We completely determine the convergence of the Alexandrov-Clark measures associated to maps falling in each group of such classification.

This is joint work with Pekka Nieminen.

On quaternionic tori and their moduli space

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Quaternionic tori are defined as quotients of the skew field $\mathbb{H}$ of quaternions by rank-4 lattices. Using slice regular functions, these tori are endowed with natural structures of quaternionic manifolds (in fact quaternionic curves), and a moduli space in a 12-dimensional real space is then constructed to classify them up to biregular diffeomorphisms. The points of the moduli space correspond to suitable *special* bases of rank-4 lattices, which are studied with respect to the action of the group $GL(4, \mathbb{Z})$, and up to biregular diffeomeorphisms. All tori with a non trivial group of biregular automorphisms - and all possible groups of their biregular automorphisms - are then identified, and recognized to correspond to five different subsets of boundary points of the moduli space.

[1] G. Gentili, C. Stoppato, D. C. Struppa, *Regular functions of a quaternionic variable*, Springer Monographs in Mathematics, Springer, Berlin-Heidelberg, 2013.
[2] G. Gentili, C. Bisi, On Quaternionic Tori and their Moduli Space (2013), Università di Firenze, preprint.

Parametric representation of univalent self-maps with given boundary fixed points

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The classical Loewner Theory provides the so-called *parametric representation* for the much studied class $\mathcal{S}$ of all normalized univalent holomorphic functions in the unit disk $\mathbb{D} := \{z \in \mathbb{C}\colon |z| < 1\}$ via solutions of a controllable ODE, known as the (radial) *Loewner differential equation.* A closely related construction provides the parametric representation for the class of all univalent holomorphic self-maps $\varphi : \mathbb{D} \to \mathbb{D}$ with $\varphi(0) = 0$, $\varphi'(0) > 0$, as the reachable set of the Loewner ODE. These parametric representations and their modifications proved to be very useful in various applications such as *extremal problems*, including the famous Bieberbach Conjecture, *criteria for univalence and quasiconformal extendability*, Schramm's much celebrated *stochastic Loewner evolution (SLE)*, etc.

In the study of holomorphic self-maps $\varphi : \mathbb{D} \to \mathbb{D}$, an important role is played by boundary regular fixed points, *i.e.* points $\sigma \in \partial\mathbb{D}$ such that $\varphi(r\sigma) \to \sigma$ as $r \to 1^-$ and $\alpha_\varphi(\sigma) := \liminf_{z\to\sigma}(1 - |\varphi(z)|)/(1 - |z|) < +\infty$.

Probably the first attempt to construct a kind of parametric representation for univalent holomorphic self-maps with one boundary regular fixed point at $\sigma = 1$ and to apply it to extremal problems for such maps with given $\alpha_\varphi(1)$ was made by H. Unkelbach [*Math. Z.* **46** (1940), 329–336]. Such a parametric representation in a rigorous way was established in 2011 by V.V. Goryainov [a conference talk].

In this talk we consider the problem of parametric representation of univalent holomorphic self-maps of $\mathbb{D}$ for the case of *several given boundary regular fixed points.* A variant of this problem with the prescribed Denjoy–Wolff is also considered. We work in the framework based, on the one hand, on the semigroup approach going back to Ch. Loewner and developed a lot by V. V. Goryainov [see, e.g., *Math. USSR Sbornik* **57** (1987), 463–483], and on the other hand, on a general version of Loewner Theory recently suggested by F. Bracci, M. D. Contreras and S. Díaz-Madrigal [*J. Reine Angew. Math.* **672** (2012), 1–37; *Math. Ann.* **344** (2009), 947–962].

Some new results of the author's work in progress will be presented.

Order of affine and linear invariant families of harmonic mappings

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We study the order of affine and linear invariant families of planar harmonic mappings with bounded Schwarzian norm. By means of Marty's relations, we determine sharp estimates for its order that show consistency between the conjectured order and bound for the Schwarzian norm in the family S_H of sense-preserving univalent harmonic mappings in the unit disk.

This is a joint work with Professors Chuaqui and Hernández (Chile).

Carnot-Carathéodory distances and Palais' completeness theorem

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A classical theorem of Palais states that if a family of complete vector fields on a manifold generates a finite dimensional Lie algebra, then each vector field of such Lie algebra is complete. Using techniques from analysis in Carnot–Carathéodory spaces we show that such completeness result holds under a general finite–type assumption.

Inner Functions in Weak Besov Spaces

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Blaschke products in weak Besov spaces will be considered. A description in terms of the distribution of its zeros will be presented.

L^p-behavior of reproducing kernels and applications

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Let $\{B_\zeta^\omega\}_{\zeta\in D}$ be the reproducing kernels of a Bergman space A_ω^2, induced by a weight ω. We shall describe the asymptotic behavior of the L^p-means and the L_v^p-behavior of B_ζ^ω (or its derivatives), where v, ω belong to any of both classes of radial weights considered in [2, Section 1.2].

We shall apply this result to study the two weight inequality

$$\|P_\omega(f)\|_{L_v^p} \le C\|f\|_{L_v^p},$$

where P_ω is the Bergman projection

$$P_\omega(f)(z) = \int_D f(\zeta) B_\zeta^\omega(z)\, \omega(\zeta) dA(\zeta).$$

Joint work with J. Rättyä.

1. M. Dostanic, Unboundedness of the Bergman projections on L^p spaces with exponential weights, *Proc. Edinb. Math. Soc.* **47** (2004), 111–117.
2. J. A. Peláez and J. Rättyä, Weighted Bergman spaces induced by rapidly increasing weights, *Mem. Amer. Math. Soc.*, vol. 227, n. 1066, (2014).
3. J. A. Peláez and J. Rättyä, L^p-behavior of Bergman reproducing kernels, preprint (submitted).

Bergman kernel and projection on the unbounded Diederich–Fornæss worm domain

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We wish to study the Bergman kernel and projection on the unbounded worm

$$\mathcal{W}_\infty = \big\{(z_1, z_2) \in \mathbf{C} \times \mathbf{C}^* : \big|z_1 - e^{i \log |z_2|^2}\big| < 1\big\},$$

where $\mathbf{C}^* = \mathbf{C} \setminus \{0\}$.

We show that the Bergman space of $\mathcal{W}_\infty$ is not trivial. In this work we study its Bergman kernel K and projection P. We obtain an asymptotic expansion for K that allows us to describe its singularities at the boundary and to prove the following:

(1) For all $s > 0$, the Bergman projection P does not map the Sobolev space $W^s(\mathcal{W}_\infty)$ into itself.

(2) For $p \neq 2$, P does not map $L^p(\mathcal{W}_\infty)$ into itself.

This work is in collaboration with S. Krantz and C. Stoppato.

Sharp Hankel operators and de Saint-Venant's inequality

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De Saint-Venant inequality is an isoperimetric inequality that relates the torsional rigidity of a cylindrical object with the area of its cross-section. In this talk, we will present a new proof of this classical inequality using operator theory. In particular, we look for sharp estimates for Hankel operators with antianalytic symbols in the Bergman space. The estimate we obtain improves a classical inequality in operator theory for commutators of Toeplitz operators known as Putnam's inequality. This improvement answers a recent conjecture by Bell, Ferguson and Lundberg. The operator theory approach to isoperimetric inequalities was first used by Khavinson in 1985, who obtained the classical isoperimetric inequality that relates area and perimeter of the region using Putnam's inequality for the commutator of Toeplitz operators in the Hardy space.

This is joint work with J.-F. Olsen.

p-summing composition operators on Hardy spaces.

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Given an holomorphic map $\varphi\colon \mathbb{D} \to \mathbb{D}$ the composition operator C_φ with symbol φ is the linear operator

$$f \mapsto f \circ \varphi\,.$$

Thanks to Littlewood's Subordination Principle, it is known that C_φ is a bounded operator from the Hardy space $H^q(\mathbb{D})$ to itself. In this talk I will present some recent results obtained in collaboration with Pascal Lefèvre (Université d'Artois, France) about the p-summingness of the composition operator $C_\varphi\colon H^q(\mathbb{D}) \to H^q(\mathbb{D})$, for $1 < q < \infty$.

Let us recall that, for $1 \le p < +\infty$, a linear operator between two Banach spaces $T\colon X \to Y$ is called p-summing if T sends weakly p-summable sequences in X into strongly p-summable sequences in Y. Equivalently, T is p-summing if there exists a constant C such that, for every finite sequence $\{x_1, x_2, \dots, x_n\}$ in X, we have

$$\sum_{j=1}^{n} \|Tx_j\|^p \le C^p \sup\Big\{\sum_{j=1}^{n} |\langle x^*, x_j\rangle|^p : x^* \in X^*, \|x^*\| \le 1\Big\}\,.$$

We will see some necessary and sufficient conditions for $C_\varphi\colon H^q(\mathbb{D}) \to H^q(\mathbb{D})$ to be p-summing. This will include a complete characterization for $1 < q < 2$, and for $q > 2$, $p \ge 2$. Until now only the characterizations (due to Shapiro and Taylor [1]) for $q = 2$ and for $p = q > 2$ were known.

[1] J. H. Shapiro and P. D. Taylor, Compact, nuclear, and Hilbert-Schmidt composition operators on H^2, *Indiana Univ. Math. J.* **23**,471–496, 1973.

[2] C. Cowen and B. MacCluer, *Composition Operators on Spaces of Analytic Functions*, CRC Press, Boca Raton, 1995.