S25. Operator Algebras and Applications to Quantum Physics

Organizers:

- Daniele Guido (Università di Roma Tor Vergata, Italy)
- Fernando Lledó (Universidad Carlos III de Madrid and ICMAT, Spain)
- Gerardo Morsella (Università di Roma Tor Vergata, Italy)

Speakers:

1. Sara Azzali (Istituto Nazionale di Alta Matematica, Italy)
   Two-cocycle twists and Atiyah-Patodi-Singer index theory
2. Giovanni Landi (Università di Trieste, Italy)
   On the K-theory and K-homology of quantum lens spaces
3. Gandalf Lechner (Universität Leipzig, Germany)
   Operator Algebras and the Construction of Models in Quantum Field Theory
4. Fedele Lizzi (Università Federico II di Napoli, Italy)
   Noncommutative Geometry and the Standard Model of Particle Physics
5. Pierre Martinetti (Università Federico II di Napoli, Italy)
   Spectral geometry with a cut-off: topological and metric aspects
6. Carlos Palazuelos (Universidad Complutense de Madrid, Spain)
   Bell inequalities from a mathematical point of view
7. Enrique Pardo (Universidad de Cádiz, Spain)
   A unified treatment of Katsura and Nekrashevych C^*-algebras
8. Francesc Perera (Universidad Autónoma de Barcelona, Spain)
   The Cuntz semigroup and its impact into classification
9. Juan Manuel Pérez Pardo (Universidad Carlos III de Madrid, Spain)
   Quantum symmetries, self-adjoint extensions and reduction theory
10. Mathilde Perrin (Instituto de Ciencias Matemáticas, Madrid, Spain)
    Noncommutative de Leeuw theorems

11. Giuseppe Ruzzi (Università di Roma Tor Vergata, Italy)
    Nets of local C^*-algebras and QED representations

12. Ezio Vasselli (Università di Roma La Sapienza, Italy)
    Quantum fields in curved spacetimes and presheaves of superselection structures

Two-cocycle twists and Atiyah-Patodi-Singer index theory

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Projectively invariant elliptic operators appear in the mathematical description of magnetic fields, in particular in some models of the fractional quantum Hall effect [3, 4].

From the geometric point of view, these operators give very interesting invariants analogous to those studied in L^2-index theory for covering spaces, or more generally higher index theory.

In this talk, we will describe the construction of *eta* and *rho* invariants for Dirac operators, on the universal covering of a closed manifold, that are invariant under the projective action associated to a 2-cocycle of the fundamental group. We will see how to prove an Atiyah-Patodi-Singer index theorem in this setting, and discuss some of the geometric properties of the rho invariant [1].

[1] S. Azzali, C. Wahl, Two-cocycle twists and Atiyah–Patodi–Singer index theorem, 2013, http://arxiv.org/abs/1312.6373.

[2] M. Gromov, Kähler hyperbolicity and L^2-Hodge theory, *J. Diff. Geom.* **33** (1991), 263–292.

[3] M. Marcolli, V. Mathai, Twisted index theory on good orbifolds I. Noncommutative Bloch theory, *Commun. Contemp. Math.* **1** (1999), 553–587.

[4] M. Marcolli, V. Mathai, Twisted index theory on good orbifolds II. Fractional quantum numbers. *Comm. Math. Phys.* **217** (2001), 55–87.

[5] V. Mathai, The Novikov conjecture for low degree cohomology classes. *Geom. Dedicata* **99** (2003), 1–15.

On the K-theory and K-homology of quantum lens spaces

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We define quantum lens spaces as 'direct sums of line bundles' and exhibit them as 'total spaces' of certain principal bundles over quantum projective spaces. For each of these quantum lens spaces we construct an analogue of the classical Gysin sequence. We use the sequence to compute the K-theory and the K-homology of the quantum lens spaces, in particular to give explicit geometric representatives of their K-theory classes. These representatives are interpreted as 'line bundles' over quantum lens spaces and generically define 'torsion classes'. We work out explicit examples of these classes.

Operator Algebras and the Construction of Models in Quantum Field Theory

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This talk reviews recent progress in rigorously constructing quantum field theoretic models by operator-algebraic techniques, including in particular Tomita-Takesaki modular theory and deformations of C^*-algebras. The similarities and differences of this topic in comparison to (deformation) quantization and non-commutative geometry will be discussed. An overview of known results as well as an outlook to open questions will be given.

Noncommutative Geometry and the Standard Model of Particle Physics

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I will briefly describe the state of the art of the spectral approach to the standard model of particle interaction based on noncommutative geometry, in view of the discovery of the Higgs particle. I will introduce the spectral action and its consequences, and concentrate field theory and symmetries. I will then confront them with experimental data and indicate possible new directions. The talk is geared to mathematicians with little knowledge of the particle physics context. A short bibliography follows.

1. A. H. Chamseddine and A. Connes, The spectral action principle, *Comm. Math. Phys.* **186** (1997), no. 3, 731–750.
2. A. H. Chamseddine, A. Connes and M. Marcolli, Gravity and the standard model with neutrino mixing, *Adv. Theor. Math. Phys.* **11** (2007), 991–1089.
3. A. H. Chamseddine, A. Connes and W. D. van Suijlekom, Beyond the Spectral Standard Model: Emergence of Pati-Salam Unification, *JHEP* **1311** (2013), 132.
4. A. H. Chamseddine, A. Connes and W. D. van Suijlekom, Inner Fluctuations in Noncommutative Geometry without the first order condition, *J. Geom. Phys.* **73** (2013), 222–234.
5. A. Connes and J. Lott, Particle models and noncommutative geometry. Recent advances in field theory (Annecy-le-Vieux, 1990). *Nuclear Phys. B Proc. Suppl.* **18B** (1991), 29–47.
6. A. Connes, M. Marcolli, *Noncommutative Geometry, Quantum Fields and Motives*, American Mathematical Society, 2007.
7. A. Devastato, F. Lizzi and P. Martinetti, Grand Symmetry, Spectral Action, and the Higgs mass, *JHEP* **1401** (2014), 042.
8. M. Dubois-Violette, J. Madore and R. Kerner, Classical Bosons In A Noncommutative Geometry, *Classical Quantum Gravity* **6** (1989), 1709–1724.
9. M. Dubois-Violette, R. Kerner and J. Madore, Noncommutative Differential Geometry of Matrix Algebras, *J. Math. Phys.* **31** (1990), 316–322.
10. C. A. Stephan, New Scalar Fields in Noncommutative Geometry, *Phys. Rev. D* **79** (2009), 065013.

Spectral geometry with a cut-off: topological and metric aspects

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Inspired by regularization in quantum field theory, we study topological and metric properties of spaces in which a cut-off is introduced. We work in the framework of noncommutative geometry, and focus on Connes distance associated to a spectral triple $(\mathcal{A}, \mathcal{H}, D)$. A high momentum (short distance) cut-off is implemented by the action of a projection P on the Dirac operator D and/or on the algebra $\mathcal{A}$. This action induces two new distances. We individuate conditions making them equivalent to the original distance. We also study the Gromov-Hausdorff limit of the set of truncated states, first for compact quantum metric spaces in the sense of Rieffel, then for arbitrary spectral triples. To this aim, we introduce a notion of *state with finite moment of order* 1 for noncommutative algebras. We then focus on the commutative case, and show that the cut-off induces a minimal length between points, which is infinite if P has finite rank. When P is a spectral projection of D, we work out an approximation of points by non-pure states that are at finite distance from each other. On the circle, such approximations are given by Fejér probability distributions. Finally we apply the results to Moyal plane and the fuzzy sphere, obtained as Berezin quantization of the plane and the sphere respectively.

1. F. D'Andrea, F. Lizzi, and P. Martinetti, Spectral geometry with a cut-off: topological and metric aspects, preprint, 2013; http://arxiv.org/abs/1305.2605.

Bell inequalities from a mathematical point of view

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It is well known that two separated observers, each holding half of an entangled quantum state and performing appropriate measurements, can produce some correlations which cannot be explained classically. In this case we say that these correlations violate a Bell inequality. In this talk we will explain this phenomenon from a mathematical point of view and we will show how it is related with fundamental results in Banach space theory and also with its noncommutative version, the operator space theory.

A unified treatment of Katsura and Nekrashevych C^*-algebras

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We will present a family of C*-algebras, generalizing certain algebras introduced by Katsura in terms of C^*-algebras of topological graphs, and also certain algebras introduced by Nekrashevych in relation with self-similar group actions on n-adic rooted trees. The goal is to realize these new algebras as groupoid C^*-algebras, and also as Cuntz-Pimsner algebras. Then, we will explain how these pictures allow us to characterize some properties of the algebras, as to be simple, purely infinite or nuclear. These results generalize the corresponding results obtained by Katsura and Nekrashevych in their particular classes.

This is a joint work with Ruy Exel (Departamento de Matemática da Universidade Federal de Santa Catarina, Florianópolis, Brazil).

[1] Exel, R., Pardo, E., Representing Kirchberg algebras as inverse semigroups crossed products, preprint, 2013; http://arxiv.org/abs/1303.6268.

[2] Exel, R., Pardo, E., Graphs, groups and self-similarity, preprint, 2013; http://arxiv.org/abs/1307.1120.

The Cuntz semigroup and its impact into classification

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The Elliott Conjecture predicts the classification of amenable, simple, unital C*-algebras by means of an invariant of a K-theoretical nature. After the discovery of counterexamples to the conjecture in its boldest form (by Rørdam and Toms), various courses of action have been taken over the last five years that have led to breakthroughs in the Classification Programme.

The Cuntz semigroup $W(A)$ of a C*-algebra A is an important ingredient, both in the structure theory of C*-algebras, and also in the current format of the Classification Programme. It is defined analogously to the Murray-von Neumann semigroup $V(A)$ by using equivalence classes of positive elements instead of projections. The lack of continuity of $W(A)$, considered as a functor from the category of C*-algebras to the category of abelian semigroups, led to the introduction (by Coward, Elliott and Ivanescu) of a new category Cu of (completed) Cuntz semigroups. They showed that the Cuntz semigroup of the stabilized C*-algebra is an object in Cu and that this assignment extends to a continuous functor. We introduce a category W of (pre-completed) Cuntz semigroups such that the original definition of Cuntz semigroups defines a continuous functor from C*-algebras to W. There is a completion functor from W to Cu such that the functor Cu is naturally isomorphic to the completion of the functor W. We also indicate how the category Cu should be recasted, by adding additional axioms.

This is joint work with Ramon Antoine (UAB) and Hannes Thiel (Münster).

Quantum symmetries, self-adjoint extension and reduction theory.

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In this talk we will show how does the process of selecting a self-adjoint extension of a symmetric operator intertwine with the notion of Quantum symmetry. In particular, given a unitary representation of a Lie group G on a Hilbert space $\mathcal{H}$, we develop the theory of G-invariant self-adjoint extensions of symmetric operators. Moreover, we will apply von Neumann algebra technics to analyse the relation between the reduction theory of the unitary representation of the group G and the reduction of the G-invariant unbounded operator.

These results are joint work with A. Ibort and F. Lledó, [1].

[1] A. Ibort, F. Lledó and J.M. Pérez-Pardo, On self-adjoint extensions and Quantum symmetries, preprint, 2014; `http://arxiv.org/abs/1402.5537`.

Noncommutative de Leeuw theorems

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In 1965, Karel de Leeuw proved three fundamental transfer results for Fourier multipliers on $\mathbb{R}^n$, namely the restriction, periodization and compactification theorems. Shortly after, Saeki extended these results to any locally compact abelian group. We will present some noncommutative analogs of de Leeuw theorems, replacing the frequency group by a locally compact group G. In that case we study the boundedness of the Fourier multipliers defined on the noncommutative L_p-spaces of the group von Neumann algebra $\mathcal{L}$G. Although some completely bounded versions of de Leeuw theorems can be obtained rather easily by transference between Schur and Fourier multipliers (under amenability assumptions), the bounded results need new arguments. In particular, the noncommutative restriction theorem relies on an interesting result on almost multiplicative maps on L_p, and its general statement follows from a noncommutative form of Igari's lattice approximation.

Nets of local C^*-algebras and QED representations

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We discuss the "net of causal loop", a model independent net of local C^*-algebra over a 4-d globally hyperbolic spacetime encoding basic features of local gauge theories. We focus on a class of representations which admit an interpretation in terms of "combinatorial" connections with values on local algebras, and see how mappings between equivalent connections can be interpreted as gauge transformations. The contact between this combinatorial setting and the usual physical theories is explained. We show that to the quantum electromagnetic field, not free in general, there corresponds such a connection, and that the usual gauge transformations of QED correspond, as expected, to mappings between equivalent connections.

The talk is based on two joint works with F. Ciolli and E.Vasselli [1, 2].

[1] Ciolli, F., Ruzzi, G., Vasselli, E., Causal posets, loops and the construction of nets of local algebras for QFT, *Adv. Theor. Math. Phys.* **16** (2012), 645–691.

[2] Ciolli, F., Ruzzi, G., Vasselli, E., QED representation for the net of causal loops, preprint, 2013; http://arxiv.org/abs/1305.7059.

Quantum fields in curved spacetimes and presheaves of superselection structures

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We show that superselection structures on curved spacetimes are categories of sections of presheaves of symmetric tensor categories. As a consequence, when an embedding functor is given the superselection structure is a Tannaka-type dual of a flat group bundle, which becomes a natural candidate for the role of gauge group: to support this idea, we show that any flat group bundle acts on a field net on an arbitrary spacetime. We also give examples of gerbes of C^*-algebras, defined by Wightman fields and constructed using projective representations of the fundamental group of the spacetime.

[1] E. Vasselli, Presheaves of symmetric tensor categories and nets of C^*-algebras, *J. Noncommut. Geom.*, to appear; http://arxiv.org/abs/1210.4302.

[2] E. Vasselli, Quantum fields in curved spacetimes and presheaves of superselection structures, preprint; http://arxiv.org/abs/1211.1812.