S23. Nonlinear stationary and evolution partial differential equations and their applications

Organizers:

- Carlos Escudero (Universidad Autónoma de Madrid & ICMAT, Spain)
- Alberto Ferrero (Università del Piemonte Orientale "Amedeo Avogadro", Italy)

Speakers:

1. Veronica Felli (Università di Milano-Bicocca, Italy)
   Unique continuation properties and essential self-adjointness for relativistic Schrödinger operators with singular potentials
2. Pier Domenico Lamberti (Università degli Studi di Padova, Italy)
   A few spectral perturbation problems for elliptic operators on variable domains
3. Gunnar Pruessner (Imperial College London, United Kingdom)
   A field theory for the Wiener Sausage
4. Aníbal Rodríguez-Bernal (Universidad Complutense de Madrid, Spain)
   Bounded and unbounded solutions of degenerate logistic equations
5. Diana Stan (Universidad Autónoma de Madrid, Spain)
   Porous Medium Equations with fractional pressure
6. José Ignacio Tello (Universidad Politécnica de Madrid, Spain)
   Mathematical models of chemotaxis of two species
7. Peicheng Zhu (Ikerbasque and UPV/EHU, Spain)
   Solutions to a model for interface motion by interface diffusion

Unique continuation properties and essential self-adjointness for relativistic Schrödinger operators with singular potentials

Veronica Felli

Dipartimento di Matematica e Applicazioni, Università di Milano-Bicocca, Via Cozzi 55, 2015 Milano, Italy
veronica.felli@unimib.it

I will present some results obtained in collaboration with M. M. Fall [1, 2] about unique continuation properties for a class of relativistic fractional elliptic equations with singular potentials. I will also discuss the problem of essential self-adjointness of the corresponding relativistic Schrdinger operator providing an explicit sufficient and necessary condition on the coefficient of the singular potential for essential self-adjointness, see [3].

[1] Fall, M. M., Felli, V., Unique continuation property and local asymptotics of solutions to fractional elliptic equations, *Comm. Partial Differential Equations* **39** (2014), 354–397.

[2] Fall, M. M., Felli, V., Unique continuation properties for relativistic Schrödinger operators with a singular potential, 2013, preprint; http://arxiv.org/abs/1312.6516.

[3] Fall, M. M., Felli, V., Sharp essential self-adjointness of relativistic Schrödinger operators with a singular potential, 2014, preprint, http://arxiv.org/abs/1403.4438.

A few spectral perturbation problems for elliptic operators on variable domains

Pier Domenico Lamberti

Dipartimento di Matematica, Università degli Studi di Padova, Via Trieste 63, 35121 Padova, Italy
lamberti@math.unipd.it

We survey several recent results concerning spectral perturbation problems for elliptic partial differential operators subject to homogeneous boundary conditions on variable domains in the N-dimensional Euclidean space. Broadly speaking, we discuss two types of results: qualitative and quantitative. The first provides information such as shape differentiability and optimization. The second concerns stability estimates, possibly sharp. Our analysis embraces elliptic operators of second and higher order such as polyharmonic operators, and systems such as the Lamé and the Reissner-Mindlin systems. In the case of the bi-harmonic operator, Dirichlet, Neumann, Intermediate and Steklov-type boundary conditions will be discussed.

All these results were obtained in several joint papers with J.M Arrieta, G. Barbatis, V. Burenkov, D. Buoso, M. Lanza de Cristoforis.

[1] Arrieta J. M., Lamberti P. D., Spectral stability results for higher-order operators under perturbations of the domain, *C. R. Math. Acad. Sci. Paris* **351** (2013), no. 19-20, 725–730.

[2] Barbatis G., Lamberti P. D., Spectral stability estimates for elliptic operators subject to domain transformations with non-uniformly bounded gradients, *Mathematika* **58** (2012), no. 2, 324–348.

[3] Buoso D., Lamberti P. D., Eigenvalues of polyharmonic operators on variable domains, *ESAIM, Control Optim. Calc. Var.* **19** (2013), 1225–1235.

[4] Buoso D., Lamberti P. D., Shape deformation for vibrating hinged plates, *Math. Methods Appl. Sci.* **37** (2014), 237–244.

[5] Burenkov V., Lamberti P. D., Sharp spectral stability estimates via the Lebesgue measure of domains for higher order elliptic operators, *Rev. Mat. Complut.* **25** (2012), no. 2, 435–457.

[6] Lamberti P. D., Lanza de Cristoforis M., Critical points of the symmetric functions of the eigenvalues of the Laplace operator and overdetermined problems, *J. Math. Soc. Japan* **58** (2006), no. 1, 231–245.

A field theory for the Wiener Sausage

Gunnar Pruessner

Department of Mathematics, Imperial College London
g.pruessner@imperial.ac.uk

The Wiener Sausage is a classic problem in probability theory that has been famously studied by Kolmogorov and Leontovich in 1933, as well as by a number of other very well known scientists, such as Spitzer, Kac and Luttinger, over the past one hundred years or so. The aim is to determine the moments of the volume traced out by a sphere attached to a Brownian particle. In the present work, I study the Wiener Sausage as a reaction diffusion process whose relevant observable follows the same limiting distributing as the original Wiener Sausage. That process allows for a treatment using renormalised field theory which I am going to present. The field theoretic approach is appealingly elegant and flexible, allowing for boundary conditions and the properties of the underlying lattice to be changed very easily, as well as for very simple implementation of variations of the process, such as introducing branching. Contrasting and comparing the present results with those about the Wiener Sausage in the literature also elucidates some of the deeper structure of field theory, such as the meaning of diagrams, momentum conservation and the implementation of hard core repulsion and immobile particles.

Bounded and unbounded solutions of degenerate logistic equations

Aníbal Rodríguez-Bernal

Departamento de Matemática Aplicada, Universidad Complutense de Madrid, 28040 Madrid, Spain
arober@ucm.es

We consider positive solutions of degenerate logistic equations in the case the space dependent coefficient of the absorption term vanishes in an arbitrary compact set.

We show how topological and geometric–measure properties of this "vanishing" set determine whether solutions remain bounded or become unbounded with time.

We also mention some results indicating in which parts of the vanishing set, the solutions become unbounded or not.

This is a joint work with J. M. Arrieta and R. Pardo.

Porous Medium Equations with fractional pressure

Diana Stan

Department of Mathematics, Universidad Autonoma de Madrid, Campus de Cantoblanco, 28049 Madrid, Spain
diana.stan@uam.es

We consider the following diffusion equation of porous medium type $u_t = \nabla(u^{m-1}\nabla p)$, with fractional pressure $p = (-\Delta)^{-s}(u)$, for $m > 1$, $0 < s < 1$ and $u(x,t) \geq 0$. To be specific, the problem is posed for $x \in R^N$, $N \geq 1$ and $t > 0$. The initial data $u(x,0)$ is assumed to be a bounded function with compact support or fast decay at infinity. The problem has been recently studied by Caffarelli and Vazquez, also by Biler, Karch and Monneau in the particular case $m = 2$. In a recent collaboration with Teso and Vázquez we prove how the nonlinearity has a strong influence on the finite propagation property of the solution. More exactly, we prove two different behaviors depending on the exponent m: for $m \geq 2$ the problem has finite speed of propagation, while for $m < 2$ has infinite speed of propagation.

[1] D. Stan, F. del Teso and J. L. Vázquez, Finite and infinite speed of propagation for porous medium equations with fractional pressure, *C. R. Math. Acad. Sci. Paris* **352** (2014), 123–128.

[2] L. Caffarelli and J. L. Vázquez, Nonlinear porous medium flow with fractional potential pressure, *Arch. Ration. Mech. Anal.* **202** (2011), 537-565.

[3] P. Biler, G. Karch. and R. Monneau, Nonlinear diffusion of dislocation density and self-similar solution, *Comm. Math. Phys.* **294** (1) (2010), 145–168.

Mathematical models of chemotaxis of two species

J. Ignacio Tello

Departamento de Matemática Aplicada, ETSI Sistemas Informáticos.
Universidad Politécnica de Madrid. Campus Sur, 28031 Madrid. Spain
j.tello@upm.es

We consider a mathematical model for the spatio-temporal evolution of two biological species in a competitive situation. Besides diffusing, both species move toward higher concentrations of a chemical substance which is produced by them- selves. The resulting system consists of two parabolic equations with LotkaVolterra type kinetic terms with non-local expressions and chemotactic cross-diffusion, along with an elliptic equation describing the behavior of the chemical. We study the question of the asymptotic stability. We identify parameter regimes for which indeed one of the species dies out asymptotically, whereas the other reaches its carrying capacity in the large time limit.

[1] J. Ignacio Tello, Mihael Winkler, Stabilization in a two-species chemotaxis system with logisitc source, *Nonlinearity* **25** (2012), 1413–1425.
[2] Mihaela Negreanu, J. Ignacio Tello, On a competitive system under chemotactic effects with non-local terms. *Nonlinearity* **26** (2013), 1083–1103.
[3] Christian Stinner, J. Ignacio Tello, Michael Winkler, Competitive exclusion in a two-species chemotaxis model, *J. Math. Biology* **2013** (2013), 1–20.

Solutions to a model for interface motion by interface diffusion

Peicheng Zhu

Ikerbasque & Department of Mathematics, University of the Basque Country UPV/EHU, Barrio Sarriena s/n, 48940 Leioa, Spain
peicheng.zhu@ehu.es

Two types of phase-field models for phase transitions driven by material forces have been proposed recently by H.-D. Alber and myself. The order parameter in one of the two models is conserved, and the model in this case consists of a linear elliptic system coupled to a degenerate parabolic equation of fourth order. This model can be applied to describe Sintering, an interface motion by interface diffusion. We then prove the existence of weak solutions to an initial boundary value problem for this model, however for one dimensional case only.