S22. Multiobjective Optimization and Variational Problems

Organizers:

- Monica Milasi (University of Messina, Italy)
- Gabriel Ruiz-Garzón (Universidad de Cádiz, Spain)

Speakers:

1. Monica Bianchi (Università Cattolica del Sacro Cuore, Italy)
   Linear openness of the composition of set-valued maps and applications to variational systems
2. Maria Bernadette Donato (University of Messina, Italy)
   A necessary optimality condition in vector optimization problems in an infinite dimensional space
3. María del Carmen Escribano Ródenas (University CEU San Pablo, Spain)
   Computers and Outranking Relations: an inseparable link reinforced over the time
4. Beatriz Hernández-Jiménez (Universidad Pablo de Olavide, Spain)
   Optimality conditions for efficiency in multiobjective problems with conic constraints. Characterization of solutions and Duality
5. Mariano Luque (Universidad de Málaga, Spain)
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6. Enrico Miglierina (Università Cattolica del Sacro Cuore, Italy)
   Geometric properties of ordering cones in infinite dimensional vector optimization
7. Monica Milasi (University of Messina, Italy)
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   Scalarization in set optimization
9. Gabriel Ruiz-Garzón (Universidad de Cádiz, Spain)
   Invexity and Variational-like Inequalities in Fuzzy Context

Linear openness of the composition of set-valued maps and applications to variational systems

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Given the generalized equation $0 \in H(x,p)$, where $H : X \times P \to 2^W$ and X, W, P are metric spaces, our aim is to investigate the regularity properties of the solution map $S = S(p)$. In particular, we are interested in the case $H(x,p) = G(F_1(x,p), F_2(x))$, and we infer Lipschitz continuity of the set-valued map S via suitable properties of the maps F_1, F_2 and G. The main tools are the Nadler Fixed Point Theorem and the Lim Lemma. We obtain, as a special case, the well-known result concerning the sum of two set-valued maps.

[1] Bianchi, M., Kassay, G., Pini, R., An inverse map result and some applications to sensitivity of generalized equations, *J. Math. Anal. Appl.* **339** (2013), 279–290.
[2] Durea, R., Strugariu, R., Chain rules for linear openness in metric space and applications, *Math. Program. Ser. A.* **143**, (2014), 147–176.
[3] Lim, T. C., On fixed point stability for set-valued contractive mappings with applications to generalized differential equations, *J. Math. Anal. Appl.* **110**, (1985), 436–441.

A necessary optimality condition in vector optimization problems in an infinite dimensional space

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In this talk a nonconvex vector optimization problem in infinite dimensional spaces is presented. In particular, a new necessary optimality condition is introduced for the infinite dimensional Lagrange multiplier rule. This result is achieved by using the tangent cone and by assuming the Hadamard differentiability of the maps.

Computers and Outranking Relations: an inseparable link reinforced over the time

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Outranking Relations are one of the most important plots of Multiple Criteria Decision Aid. Methods such as ELECTRE (Elimination and Choice translating algorithm) and PROMETHEE (Preference Ranking Organization Method for Enrichment Evaluations) are the great exponents and they are characterized as being applicable to one, increasingly, wide range of real problems and to be easily understandable by the decision-maker.

From the scientific point of view they are widely accepted as they have a solid mathematic back up and a rigorous axiomatic supports them.

In its origins, their application to multicriteria discrete problems of small size did not pose grater difficulties in calculation. However, as the dimension of the decision problems to solve was increased, it was born the need to have computers tools that facilitate the calculation and could process large amount of data. On the one hand, the major exponents of the French School developed the software for the ELECTRE and its different versions; on the other hand, the most outstanding representatives of the Belgian School, started with the software for the PROMETHEE and is subsequent versions.

In this paper we will study how the computer technology, through the development of more complete and sophisticated programs, has been introducing to Multicriteria Decision Aid Methods which belong to Outranking Relations family and has contributed to the substantial improvement of them. Mathematical structures that lie behind these methods allow achieving more robust and reliable solutions.

The presentations of real cases that we have been dealing with these methodologies will show the wide range of applications possibilities.

Optimality conditions for efficiency in multiobjective problems with conic constraints. Characterization of solutions and Duality

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Convexity and generalized convexity play a central role in mathematical programming in order to get optimality conditions and duality results. In this paper, for a multiobjective problem with conic constraints, we get necessary optimality conditions; and taking in mind Craven's notion of K-invexity function and Martin's notion of Karush-Kuhn-Tucker invexity, a new notion of generalized convexity is defined. With this new notion we get a sufficient optimality condition and prove that the generalized convexity notion defined is both necessary and sufficient to ensure every Karush-Kuhn-Tucker point is an efficient solution. A Mond-Weir and a Wolfe type dual problems are formulated and duality results are provided.

[1] Arana-Jiménez, M., Rufián-Lizana, A., Osuna-Gómez, R., and Ruiz-Garzón, G., Pseudoinvexity, optimality conditions and efficiency in multiobjective problems; duality, *Nonlinear Anal.* **68** (2008), 24–34.
[2] Craven, B.D., *Control and Optimization*, Chapman and Hall Mathematics, 1995.
[3] Hernández-Jiménez, B., Osuna-Gómez, R., Rojas-Medar, M.A., and Arana-Jiménez, M., Characterization of optimal solutions for nonlinear programming problems with conic constraints, *Optimization* **60** (5) (2011), 619–626.

Reference point approaches in Stochastic Multiobjective Programming

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Many real-world decisions involve dealing with optimization problems where several objective functions are maximized or minimized simultaneously and where, in many occasions, it is necessary to work with some kind of uncertainty. Several Multiobjective Programming (MOP) techniques have been designed in order to deal with these kinds of real-world problems. In the case when the unknown parameters of the problem are considered to be random variables with known probability distributions, the resulting multiobjective optimization problem is called a Stochastic Multiobjective Programming (SMOP) problem. While reference point based interactive methods are widely used in the field of deterministic MOP, just a few methods of this kind have been developed for SMOP problems. The main reason for this is the fact that a reference value for each objective is not enough in a SMOP problem, because the stochastic function will achieve different values with different probabilities. Therefore, the decision maker (DM) has to state for each objective function, the value he wishes to achieve and the desired probability to achieve this value. In order to take this information into account, the traditional scalarizing achievement functions have to be adapted to the SMOP philosophy. In this presentation, we show that three different scalarizing functions can be used for each iteration, depending on which information is regarded as reference point. Namely, we can fix the probability and consider the reference levels as reference point, we can fix the levels and consider the probability as reference point, or we can include all the information in the reference point. Theoretical results are given which prove the efficiency of the optimal solutions obtained in the three cases. On this basis, an algorithm is developed for SMOP problems, which determines three solutions in each iteration, making use of the information given by the DM.

Geometric properties of ordering cones in infinite dimensional vector optimization

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The aim of this talk is to present a study of the assumptions concerning the boundedness (or the compactness) of the base of the ordering cones widely use in the theory of vector optimization. Indeed, the assumption of the existence of a bounded base for the ordering cone is almost always satisfied in finite dimensional setting but it will be proved that in an infinite dimensional setting this type of assumptions are often unduly restrictive (see, e.g, [4]). This approach allow us to investigate some geometrical properties of the ordering cones which in turn affect geometrical aspect of the set of proper efficient solutions of a vector optimization problem. Indeed, it will be shown that the hierarchical structure of the relationships between various notions of proper optimality completely changes when the existence of a bounded base is not ensured. Finally, from a purely mathematical point of view, the study of these geometrical aspects of the cones allows to prove some interesting result about the structural property of the whole space. Indeed, a characterization of reflexivity of a given Banach space can be obtained. The talk is essentialy based on [1, 2].

[1] E. Casini, E. Miglierina, The geometry of strict maximality, *SIAM J. Optim.* **20** (2010), 3146–3160.
[2] E. Casini, E. Miglierina, Cones with bounded and unbounded bases and reflexivity, *Nonlinear Anal.* **72** (2010), 2356–2366.
[3] E. Casini, E. Miglierina, I.A. Polyrakis, F. Xanthos, Reflexive Cones, *Positivity* **17** (2013), 911–933.
[4] J. P. Dauer, R. J. Gallagher, Positive proper efficient points and related cone results in vector optimization theory, *SIAM J. Control Optim.* **28** (1990) 158–172.

Approximation of quasiconvex functions and application

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In this talk we discuss the problem of approximation of functions. In particular, for a given continuous and quasi convex function, we find an approximation in terms of continuous and strictly quasiconvex functions. Finally, we apply our results to find equilibria for an exchange market.

Scalarization in set optimization

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We consider a set-valued optimization problem, where the objective values are compared through quasi order relations between sets (set optimization). First we study a general scalarization scheme for a set optimization problem based on order representation and order preservation properties, which works in any abstract set ordered by a quasi order relation. Following this approach, we study a recent Gerstewitz scalarization mapping for set-valued optimization problems with K-proper sets and a solid ordering cone K. In particular we show a dual minimax reformulation of this scalarization. Moreover, in the setting of normed spaces ordered by not necessarily solid ordering cones, we introduce a new scalarization functional based on the so-called oriented distance. Finally, whenever the ordering cone is solid, by considering suitable generalized Chebyshev norms with appropriate parameters, we show that the three scalarizations studied in the present work are coincident. Using these scalarization mappings, we obtain necessary and sufficient optimality conditions in set optimization.

[1] C. Gutiérrez, B. Jiménez, E. Miglierina, E. Molho, Scalarization in Set Optimization with Solid and Non Solid Cones, *J. Global Optim.*, to appear.
[2] C. Gutiérrez, E. Miglierina, E. Molho, V. Novo, Pointwise well-posedness in set optimization with cone proper sets, *Nonlinear Anal.* **75** (2012), 1822–1833.

Invexity and Variational-like Inequalities in Fuzzy Context

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The aim of this work is generalize classical concepts of generalized convexity and relationships between solutions of variational-like inequalities and mathematical programming problems given in the finite dimensional case to a fuzzy context. We are going to show you some inconvenience of various definitions and how to solve them.

[1] Rufián-Lizana, A., Chalco-Cano, Y., Osuna-Gómez R. and Ruiz-Garzón, G., On invex fuzzy mappings and fuzzy variational-like inequalities, *Fuzzy Sets and Systems* **200** (2012) 84–98.

[2] Rufián-Lizana, A., Chalco-Cano, Y., Ruiz-Garzón, G. and Román-Flores, H., On some characterizations of preinvex fuzzy mappings, *TOP* (2013), published online.

[3] Ruiz-Garzón, G., Santos, L.B., Rufián-Lizana, A. and Arana-Jiménez, M., Some relations between Minty variational-like inequality problems and vectorial optimization problems in Banach spaces, *Comput. Math. Appl.* **60** (2010) 2679–2688.