S10. Functional Analysis and Partial Differential Equations

Organizers:

- Chiara Boiti (Università degli studi di Ferrara, Italy)
- J. Alberto Conejero Casares (Universitat Politècnica de València, Spain)
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Speakers:

1. Alessia Ascanelli (Università degli studi di Ferrara, Italy)
   Necessary and sufficient conditions for well-posedness of p-evolution equations with time and space depending coefficients
2. María José Beltrán Meneu (Universitat de València, Spain)
   Classical operators on the Hörmander algebras
3. José Manuel Calabuig Rodríguez (Universitat Politècnica de València, Spain)
   Smoothness on L^p of a vector measure
4. María del Carmen Gómez Collado (Universitat Politècnica de València, Spain)
   Fredholm weighted composition operators on weighted Banach spaces of analytic functions
5. Antonio J. Guirao Sánchez (Universitat Politècnica de València, Spain)
   Preserved extreme points and Morris norms
6. Enrique Jordá Mora (Universitat Politècnica de València, Spain)
   Smoothness of the norm in Banach spaces of vector-valued functions
7. Thomas Kalmes (Technische Universität Chemnitz, Germany)
   Surjectivity of augmented linear partial differential operators
8. Pedro J. Miana (Universidad de Zaragoza, Spain)
   Orthogonal expansions for families of operators
9. Michele Miranda (Università degli studi di Ferrara, Italy)
   Perimeters and short-time behaviour of the heat semigroup

10. Alessandro Morando (Università degli studi di Brescia, Italy)
Well-posedness of the linearized problem for MHD contact discontinuities in 2D

11. Alessandro Oliaro (Università di Torino, Italy)
Global regularity of twisted differential operators

12. Natalia Romero (Universidad de la Rioja, Spain)
Quadrature formulas for orthogonal polynomials

13. Pablo Sevilla Peris (Universitat Politècnica de València, Spain)
Convergence of Dirichlet and power series

14. André Suess (Universitat de Barcelona, Spain)
Random-field solutions to linear hyperbolic stochastic partial differential equations with variable coefficients

15. Pedro Tradacete Pérez (Universidad Carlos III de Madrid, Spain)
Optimal rearrangement invariant range for Hardy type operators

16. Sven-Ake Wegner (Bergische Universität Wuppertal, Germany)
Growth bound and spectral bound for semigroups on Fréchet spaces

Necessary and sufficient conditions for well-posedness of p-evolution equations with time and space depending coefficients.

Alessia Ascanelli

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In this talk we will consider anisotropic evolution equations of the form

$$\left(D_t + a_p(t)D_x^p + \sum_{j=0}^{p-1} a_j(t,x)D_x^j\right) u(t,x) = f(t,x)\,, \tag{1}$$

for an integer $p \geq 2$, where a_j are complex-valued coefficients depending on $(t,x) \in [0,T] \times R$. We assume that the principal symbol (in the sense of Petrowski) $\tau + a_p(t)\xi^p$ has the real characteristic $\tau = -a_p(t)\xi^p$: this is necessary to obtain a unique solution in H^∞ for the corresponding Cauchy problem. Equations of the form 1 are, for instance, Shrödinger's equation ($p = 2$), and (the linear part of) Korteweg- de Vries equation ($p = 3$). It's well-known [5] that to obtain a unique solution in H^∞ in the case $p = 2$ a suitable decay as $|x| \to \infty$ is needed for the imaginary part of the coefficient $a_1(t,x)$. We will give a set of decay conditions at infinity on the coefficients $a_j(t,x)$ sufficient to obtain a Cauchy problem well-posed in H^∞ for the equation (1). These conditions involve even derivatives of $\Im a_j(t,x)$ and odd derivatives of $\Re a_j(t,x)$. Then, we will discuss the necessity of that conditions. Finally, we will say something about semi-linear equations and higher order equations. The topics here outlined are developed in the papers [1, 2, 3, 4].

[1] Ascanelli, A., Boiti, C., Cauchy problem for higher order p-evolution equations, *J. Differential Equations* **255** (2013), 2672–2708.

[2] Ascanelli, A., Boiti, C., Zanghirati, L., Well-posedness of the Cauchy problem for p-evolution equations, *J. Differential Equations* **253** (2012), 2765–2795.

[3] Ascanelli, A., Boiti, C., Zanghirati, L., Well-posedness in Sobolev spaces for semi-linear 3-evolution equations, *Ann. Univ. Ferrara Sez. VII Sci. Mat.* (2014), to appear.

[4] Ascanelli, A., Boiti, C., Zanghirati, L., A necessary condition for H^∞ well-possedness of linear 3-evolution equations (2014), in preparation.

[5] Ichinose, W., Some remarks on the Cauchy problem for Schrödinger type equations, *Osaka J. Math.* **21** (1984), 565–581.

Classical operators on the Hörmander algebras

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We study the dynamics of the differentiation operator $Df(z) = f'(z)$, the integration operator

$$Jf(z) = \int_0^z f(\zeta)d\zeta,$$

and the Hardy operator

$$Hf(z) := \frac{1}{z}\int_0^z f(\xi)d\xi,$$

for $z \in \mathbb{C}$, on the radial (LB) or Fréchet Hörmander algebras of entire functions

$$A_p(\mathbb{C}) := \left\{ f \in H(\mathbb{C}) |\ \exists n \in \mathbb{N} : \sup_{z\in\mathbb{C}} |f(z)| \exp(-np(z)) < \infty \right\}$$

and

$$A_p^0(\mathbb{C}) := \left\{ f \in H(\mathbb{C}) |\ \forall n \in \mathbb{N} : \sup_{z\in\mathbb{C}} |f(z)| \exp\left(-\frac{p(z)}{n}\right) < \infty \right\},$$

respectively. We see that the Hardy operator is always continuous, power bounded, thus not hypercyclic and uniformly mean ergodic on them. We give sufficient conditions to ensure that D and J are continuous on these spaces and we study when the operators are hypercyclic, chaotic, power bounded and (uniformly) mean ergodic in terms of the order of growth of the growth condition p. For the more general weighted Fréchet algebras $HW(\mathbb{C})$, where W is an increasing sequence of weights, we also characterize when the differentiation operator is hypercyclic, topologically mixing or chaotic.

Smoothness on L^p of a vector measure

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It is well-known that for $1 < p < \infty$ the classical Lebesgue space $L^p(\mu)$ of scalar functions that are p-integrabe with respect to the scalar measure μ is smooth (and also uniformly Gâteaux or Fréchet smooth). In the case of the space $L^p(m)$ consisting of those scalar functions that are p-integrable with respect to a Banach space valued measure m, this result is not, in the general case, true. In this talk we show that with a topological condition if the Banach space X is smooth (resp. uniformly Gâteaux or Fréchet smooth) then the space $L^p(m)$ is also smooth (resp. uniformly Gâteaux or Fréchet smooth). Some examples and applications are also given.

This is a joint work with L. Agud, S. Lajara and E. A. Sánchez Pérez.

Fredholm weighted composition operators on weighted Banach spaces of analytic functions

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We study when weighted composition operators are Fredholm operators when they are considered between weighted Banach spaces of analytic functions $H_v^0(\mathbb{D})$ and $H_v^\infty(\mathbb{D})$ where $\mathbb{D}$ is the unit disk in $\mathbb{C}$.

This lecture is based on joint work with J. Bonet, D. Jornet and E. Wolf.

[1] Bonet, J., Domanski, P., Lindström, M., Pointwise multiplication operators on weighted Banach spaces of analytic functions, *Studia Math.* **137** (2) (1999), 177–194.

[2] Galindo, P., Lindström, M., Fredholm composition operators on analytic function spaces, *Collect. Math.* **63** (2) (2012), 139–145.

Preserved extreme points and Morris norms

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An extreme point of the unit ball of a Banach space X that remains an extreme point of the unit ball of its bidual X^{**} is called a *preserved extreme point*. Morris showed that every separable space containing a copy of c_0, admits an equivalent norm whose unit ball is rotund (every point of the unit sphere is an extreme point of the unit ball) but has no preserved extreme point. We present a way of extending this result to the non-separable setting, for instance to the weakly compactly generated Banach spaces containing a copy of c_0.

This is a joint work with V. Montesinos and V. Zizler.

[1] A. J. Guirao, V. Montesinos, and V. Zizler, A note on extreme points of C^∞–smooth balls in polyhedral spaces, to appear.

[2] A. J. Guirao, V. Montesinos, and V. Zizler, On preserved and unpreserved extreme points, to appear.

Differentiability of the norm in Banach spaces of vector-valued functions

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A well known criterion of Šmulyian states that the norm $\|\cdot\|$ of a real Banach space X is Gâteaux differentiable at $x \in X$ if and only if there is $f \in B_{X^*}$ weak star exposed by x. This f is called differential of the norm at x. If f is strongly weak star exposed in B_{X^*} then the norm is Fréchet differentiable at x.

We show that in this criterion B_{X^*} can be replaced by a convenient smaller set, requiring it only to be 1-norming and weak star compact. We apply this extended criterion to characterize the points of Gâteaux and Fréchet differentiability of the norm of some Banach spaces of harmonic and holomorphic functions which arise naturally in Analysis. Since all these concepts are valid for real Banach spaces, the spaces of holomorphic functions are considered as spaces of functions with values in $\mathbb{R}^2$.

Surjectivity of augmented linear partial differential operators

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In their deep paper [1] on the solvability of partial differential equations on the space of vector valued distributions, J. Bonet and P. Domański asked the following natural question: Let $P(D)$ be a surjective constant coefficient linear partial differential operator on the space of Schwartz distributions $\mathcal{D}'(X)$ over an open set $X \subseteq \mathbb{R}^n$. Is the augmented operator $P^+(D)$ always surjective on $\mathcal{D}'(X \times \mathbb{R})$ (where $P^+(x_1, \ldots, x_{n+1}) = P(x_1, \ldots, x_n)$)?

Due to a result of D. Vogt [4] it is known that for elliptic differential operators $P(D)$ the augmented operator $P^+(D)$ is always surjective on $\mathcal{D}'(X \times \mathbb{R})$ for every open set $X \subseteq \mathbb{R}^n$. For $n \geq 3$, we constructed in [2] a hypoelliptic differential operator $P(D)$ such that for a suitable open $X \subseteq \mathbb{R}^n$ $P(D)$ is surjective on $\mathcal{D}'(X)$ while $P^+(D)$ is not surjective on $\mathcal{D}'(X \times \mathbb{R})$, and in [3] it was shown that for open $X \subseteq \mathbb{R}^2$ surjectivity of $P(D)$ on $\mathcal{D}'(X)$ always implies surjectivity of the augmented operator.

The aim of the talk is to discuss the aforementioned results and to give some conditions on the polynomial P and the open set X ensuring an affirmative answer to the question on Bonet and Domański.

[1] Bonet, J., Domański, P., Parameter dependence of solutions of differential equations on spaces of distributions and the splitting of short exact sequences, *J. Funct. Anal.* **230** (2006), 329–381.

[2] Kalmes, T., The augmented operator of a surjective partial differential operator with constant coefficients need not be surjective, *Bull. London Math. Soc.* **44** (2012), 610–614.

[3] Kalmes, T., Some results on surjectivity of augmented differential operators, *J. Math. Anal. Appl.* **386** (2012), 125–134.

[4] Vogt, D., On the Solvability of $P(D)f = g$ for vector valued functions, *RIMS Kokyoroku* **508** (1983), 168–181.

Orthogonal expansions for families of operators

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In this talk we introduce Laguerre and Hermite expansions to approximate C_0-semigroups, C_0-groups and cosine functions. We give the rate of convergence of this orthogonal expansion to these families of operators and compare with other known approximations. To do that, we need to study special functions and the convergence of orthogonal series in Lebesgue spaces. To finish, we consider concrete examples of families of operators: shift, convolution and holomorphic semigroups where some of these results are improved.

The talk is based on joint work with Luciano Abadias.

[1] Abadias, L., Miana, P.J., C_0-semigroups and resolvent operators approximated by Laguerre expansions, (2013); http://arxiv.org/abs/1311.7542.

[2] Szegö, G., *Orthogonal polynomials.* American Mathematical Society Colloquium Publications XXIII, American Mathematical Society, Providence, RI, 1967.

Perimeters and short-time behaviour of the heat semigroup

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The connection between the theory of semigroup and that of sets with finite perimeter started from the seminal paper seminal paper of E.De Giorgi [4] concerning a new theory of surface measures; this link has been considered again quite recently by M.Ledoux [5], where a semigroup approach for the proof of the isoperimetric problem has been considered.

In this talk we shall start recalling these classic results and shall show how they have been generalized in several settings; on Riemannian manifolds [7], Carnot groups [3] and abstract Wiener spaces [1].

In the last part we shall show that the Ledoux approach contains a lot of geometrical informations, in the same spirit of the theory of sets with positive reach proposed by Federer; these are mainly the results contained in [6].

[1] Ambrosio, L., Maniglia, S., Miranda Jr., M., Pallara, D., *BV* functions in abstract Wiener spaces, *J. Funct. Anal.* **258** (3) (2010), 785–813.
[2] Angiuli, L., Miranda Jr., M., Pallara, D., Paronetto, F., *BV* functions and parabolic initial boundary value problems on domains, *Ann. Mat. Pura Appl. (4)* **188** (2) (2009), 297–331.
[3] Bramanti, M., Miranda Jr, M., Pallara, D., Two Characterization of *BV* functions on Carnot Groups via the Heat Semigroup, *Int. Math. Res. Not. IMRN* **17** (2012), 3845–3876.
[4] De Giorgi, E., Su una teoria generale della misura $(r-1)$-dimensionale in uno spazio ad r dimensioni, *Ann. Mat. Pura Appl. (4)* **36** (1954), 191–213.
[5] Ledoux, M., Semigroup proofs of the isoperimetric inequality in Euclidean and Gauss space, *Bull. Sci. Math.* **118** (6) (1994), 485–510.
[6] Miranda Jr., M., Pallara, D., Paronetto, F., Preunkert, M., Short-time heat flow and functions of bounded variation in R^N, *Ann. Fac. Sci. Toulouse Math. (6)* **16** (1) (2007), 125–145.
[7] Miranda Jr., M., Pallara, D., Paronetto, F., Preunkert, M., Heat semigroup and functions of bounded variation on Riemannian manifolds, *J. Reine Angew. Math.* **613** (2007), 99–119.

Well-posedness of the linearized problem for MHD contact discontinuities in 2D

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We study the free boundary problem for contact discontinuities in ideal compressible magnetohydrodynamics (MHD). Under the Rayleigh-Taylor sign condition on the jump of the normal derivative of the pressure satisfied at each point of the unperturbed contact discontinuity, we prove the well-posedness in Sobolev spaces of the linearized problem for 2D planar MHD flows.

The results presented are a joint work with Yuri Trakhinin and Paola Trebeschi.

[1] Morando, A., Trakhinin Y., Trebeschi P., Well-posedness of the linearized problem for MHD contact discontinuities (2013), preprint.

Global regularity of twisted differential operators

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A linear operator A on the Schwartz distribution space $\mathcal{S}'(\mathbb{R}^d)$ is said to be globally regular if the conditions $u \in \mathcal{S}'(\mathbb{R}^d)$, $Au \in \mathcal{S}(\mathbb{R}^d)$, imply that $u \in \mathcal{S}(\mathbb{R}^d)$. We focus in this talk on partial differential operators A with polynomial coefficients; it is known that a globally hypoelliptic operator (in the sense of Shubin) is globally regular. We prove global regularity for operators that are not globally hypoelliptic, by using techniques related to transformations of Wigner type. We analyze in particular second order operators, and we recover as a particular case the global regularity of the twisted Laplacian.

The talk is based on a joint work with Ernesto Buzano (Department of Mathematics, University of Torino, Italy)

Quadrature formulas for orthogonal polynomials

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In this talk we obtain Sobolev norms of orthogonal functions (Hermite, Laguerre and Jacobi functions) in terms of the zeros of these orthogonal polynomials, known as quadrature formulae. In particular we give new expressions of the 1-Lebesgue norm of these orthogonal functions.

The talk is based on joint work with Luciano Abadias and Pedro J. Miana.

[1] Abadias, L., Miana, P.J., Romero, N., Quadrature formulas for orthogonal polynomials (2014), preprint.
[2] Szegö, G., *Orthogonal polynomials.* American Mathematical Society Colloquium Publications XXIII, American Mathematical Society, Providence, RI, 1967.

Convergence of Dirichlet and power series

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A Dirichlet series is a formal series of the form $\sum_{n=1}^{\infty} a_n n^{-s}$, with $a_n \in \mathbb{C}$ and $s \in \mathbb{C}$. It is a well know fact that the maximal domains of convergence of Dirichlet series are half-plains. Then we can consider σ_c, σ_u, σ_a the abscissas that define the maximal half-plains on which a given Dirichlet series converges, converges uniformly and converges absolutely. Harald Bohr started in the 1910's the study of these abscissas. He considered the number

$$S = \sup\{\sigma_a - \sigma_u : \text{ Dirichlet series}\}$$

and showed that $S \leq 1/2$. The problem (sometimes called *Bohr's absolute convergence problem*) was open for over 15 years, until Bohnenblust and Hille proved that actually $S = 1/2$.

Recently these results have been revisited using tools and techniques of functional analysis. Starting from ideas of Bohr, Hedenmalm, Lindvist and Seip showed that there is a bijection between the algebra of Dirichlet series and the algebra of formal power series in infinitely many variables. This links Dirichlet series with the theory of holomorphic functions on c_0 and with Hardy spaces on the infinite dimensional torus. In this way Hedenmalm, Lindvist and Seip one side and Bayart on the other side defined different Banach spaces of Dirichlet series.

We will explore this identification to show how recent results on the convergence of power series on infinitely many variables recover results on convergence in different spaces of Dirichlet series. Also, by using this identification we will extend some of these results to Dirichlet series taking values on a Banach space (i.e. the coefficients a_n belong to a Banach space).

We report on some joint work with Bayart, Defant, Frerick and Maestre and with Carando and Defant.

Random-field solutions to linear hyperbolic SPDEs with variable coefficients

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In this talk we investigate stochastic partial differential equations whose partial differential operator is hyperbolic, of second-order or higher-order. We work in the setting of random-field solutions, see [2, 1], where the solution $u(t,x)$ can be evaluated as a real-valued random variable at every point in time and space, $(t,x) \in [0,T] \times \mathbb{R}^d$. In order to achieve this, we use the theory of pseudo-differential and Fourier integral operators to obtain an explicit formula for the fundamental solution associated to the hyperbolic partial differential operator and check that it satisfies the integrability conditions necessary for the existence and uniqueness of a random-field solution. The examples that we treat are: second-order strictly hyperbolic operators, second-order weakly hyperbolic operators, higher-order strictly hyperbolic operators and, if time permits, we compute the classic example of the stochastic wave equation in any spatial dimension explicitly and show how it fits into this theory.

This is a joint work with Alessia Ascanelli of the University of Ferrara.

[1] Dalang, R. C., Extending Martingale Measure Stochastic Integral with Applications to Spatially Homogeneous SPDEs, *Electron. J. Probab.* **4** (1999), 1–29.

[2] Walsh, J. B., An Introduction to Stochastic Partial Differential Equations, *École d'été de Probabilités de Saint Flour XIV, 1984*, Lecture Notes in Math. **1180**, Springer, 1986.

Optimal rearrangement invariant range for Hardy type operators

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We characterize, in the context of rearrangement invariant spaces, the optimal range space for a class of monotone operators related to the Hardy operator. The connection between optimal range and optimal domain for these operators is carefully analyzed.

Growth bound and spectral bound for semigroups on Fréchet spaces

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Let X be a complete metrizable locally convex space. A family of operators $T = (T(t))_{t\geq 0} \subseteq L(X)$ which satisfies the evolution property is a C_0-semigroup if in addition all its orbits are continuous. The generator $(A, D(A))$ is defined exactly as in the case of a Banach space.

In the talk, we first generalize the concept of the growth bound $\omega_0(T)$ from Banach spaces to Fréchet spaces. Then we employ classical and recent approaches for non Banach spectral theories to define the spectral bound $s(A)$.

We show on the one hand that the Banach space inequality $s(A) \leq \omega_0(T)$ extends to the new setting. On the other hand we prove that the forward shift on the space of all complex sequences endowed with the topology of pointwise convergence generates a uniformly continuous semigroup for which the above bounds are distinct—a phenomenon which cannot occur on a Banach space.