S8. Dispersive Equations

Organizers:

- Luis Vega (BCAM and Universidad del País Vasco, Spain)
- Nicola Visciglia (Università di Pisa, Italy)

Speakers:

1. Riccardo Adami (Politecnico di Torino, Italy)
   Minimizing NLS energy on graphs
2. Valeria Banica (Université d'Évry, France)
   Large time behavior for the focusing NLS on hyperbolic space
3. Piero d'Ancona (Università di Roma La Sapienza, Italy)
   Global existence of equivariant wave maps with small data on rotationally symmetric manifolds
4. Patxi de La Hoz (Universidad del País Vasco, Spain)
   On the Generation of Random Numbers by means of the Vortex Filament Equation
5. Luca Fanelli (Università di Roma La Sapienza, Italy)
   Dispersive estimates for scaling critical Schrödinger operators
6. Damiano Foschi (Università di Ferrara, Italy)
   Sharp constants for Fourier restriction inequalities on the sphere
7. Susana Gutiérrez (University of Birmingham, United Kingdom)
   Self-similar solutions of the one-dimensional Landau-Lifshitz-Gilbert equation
8. Evelyne Miot (Université d'Orsay, France)
   Some examples of dynamics for nearly parallel vortex filaments
9. Gustavo Ponce (University of California Santa Barbara, USA)
   On unique continuation properties of solution to some dispersive equations
10. Javier Ramos (IMPA, Brazil)
    New Bounds on the Fourier Restriction Problem

11. Gigliola Staffilani (Massachusetts Institute of Technology, USA)
On the long time existence of some solutions to periodic supercritical NLS equations

12. Nikolay Tzvetkov (Université de Cergy-Pontoise, France)
Invariant measures and long time behavior for the Benjamin-Ono equation

13. Miren Zubeldia (University of Helsinki, Finland)
The inverse Robin boundary value problem in a half-space

Minimizing NLS energy on graphs

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Owing to the loss of translational symmetry, the Nonlinear Schroedinger Equation on star graphs consisting of at least two infinite edges shows the lack of ground states. In this talk it will be shown how to extend this result to a more general class of graphs with Kirchhoff's (i.e. free) conditions at vertices, and an example will be given in which, on the other hand, the ground state exists. It turns out that, in order for a ground state to exist, the key feature is the presence, in the structure of the graph of a so-called bottleneck.

Large time behavior for the focusing NLS on hyperbolic space

Valeria Banica

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In this talk I shall present some results on global existence, scattering and blow-up for the focusing nonlinear Schrödinger equation on hyperbolic space.

This is a joint work with Thomas Duyckaerts.

Global existence of equivariant wave maps with small data on rotationally symmetric manifolds

Piero D'Ancona

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In a joint work with Q. Zhang (Shanghai) we prove the existence of global wave maps of critical regularity with small data, defined on backgrowund metrics with rotational symmetry. The maps are assumed to be of equivariant type but we admit background metrics which are large at infinity. The class of admissible base manifolds is quite large and includes real hyperbolic spaces, spaces with polynomial and exponential growth at infinity, and small long range perturbations of the flat Minkowski space.

On the Generation of Random Numbers by means of the Vortex Filament Equation

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The evolution of the vortex filament equation (VFE),

$$X_t = X_s \wedge X_{ss},$$

taking a planar regular polygon of M sides as initial datum, provides a very good generator of pseudorandom numbers in a completely natural way. This essential randomness of (VFE) is in agreement with the randomness of the physical phenomena upon which it is based.

[1] De la Hoz, F., Vega, L., The vortex filament equation as a pseudorandom generator (2013); `http://arxiv.org/abs/1311.7274`.

Dispersive estimates for scaling critical Schrödinger operators

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We will show some recent results concerning with sharp time-decay estimates for Schödinger operators with scaling-critical electromagnetic operators. Via pseudoconformal transformation, an explicit representation formula for the propagators is obtained; this is the crucial step for the proof of sharp results in 2D and for some 3D-examples.

The results are obtained in collaboration with V. Felli (Milano Bicocca), M. Fontelos (ICMAT, Madrid) and A. Primo (UAM, Madrid).

[1] Fanelli, L., Felli, V., Fontelos, M., Primo, A., Time dispersion for scaling invariant electromagnetic Schrödinger flows, *Comm. Math. Phys.* **324** (2013), 1033–1067.

[2] Fanelli, L., Felli, V., Fontelos, M., Primo, A., Time decay of scaling invariant electromagnetic Schrödinger equations on the plane (2014).

Sharp constants for Fourier restriction inequalities on the sphere

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We consider adjoint Fourier restriction inequalities of the form

$$\|\widehat{f\sigma}\|_{L^p(\mathbb{R}^d)} \leq C(d,p,q)\|f\|_{L^q(\mathbb{S}^{d-1},\sigma)},$$

where σ is the standard surface measure on the unit sphere $\mathbb{S}^{d-1}$. We will discuss recent results [1, 2] about cases for which it is possible to compute the sharp constant $C(d,p,q)$ and to characterize the set of extremizers for the estimate.

[1] Foschi, D., Global Maximizers for the Sphere Adjoint Restriction Inequality (2013); http://arxiv.org/abs/1310.2510.
[2] Carneiro, E., Oliveira e Silva, D., Some Sharp Restriction Inequalities on the Sphere (2014); preprint.

Self-similar solutions of the one-dimensional Landau-Lifshitz-Gilbert equation

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We consider the one-dimensional Landau–Lifshitz–Gilbert equation (LLG), a model describing the dynamics for the spin in ferromagnetic materials.

Our main aim is the construction of a bi-parametric family of self-similar solutions for this model and the analytical study of these solutions with respect to the so-called Gilbert damping parameter. Our construction provides a family of global solutions of the LLG equation which are associated to a discontinuous initial data of infinite energy, and which are smooth and have finite energy for all positive times.

In the absence of damping, the LLG equation reduces to the Schrödinger map equation, and we recover some previous known results in this setting.

Some examples of dynamics for nearly parallel vortex filaments.

Evelyne Miot

CNRS - École Polytechnique, France

A system of equations combining the 1D Schrödinger equation and the point vortex system has been derived by Klein, Majda and Damodaran in order to describe the evolution of nearly parallel vortex filaments in 3D incompressible fluids. In this talk I will present some dynamics for this system such as travelling waves, collisions and finite-time blow-up. I will finally consider the case of pairs of filaments.

This is joint work with Valeria Banica and Erwan Faou.

[1] Banica, V., Faou, E., Miot, E., Collision of pairs of vortex filaments; preprint.
[2] Banica, V., Miot, E., Evolution, interaction and collisions of vortex filaments, *Differential Integral Equations* **26** (2013), 355–388.
[3] Banica, V., Miot, E., Global existence and collisions for certain configurations of nearly parallel vortex filaments, *Ann. Inst. H. Poincaré Anal. Non Linéaire* **29** (2012), 813–832.

On unique continuation properties of solution to some dispersive equations

Gustavo Ponce

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We shall discuss several results concerning unique continuation properties of solutions to some canonical dispersive equations and their relation with decay and persistent properties of the corresponding solutions flow. On these canonical dispersive models we shall include the generalized Korteweg-de Vries equation, the nonlinear Schrödinger equation, the Camassa-Holm equation and the Benjamin-Ono equation.

New Bounds on the Fourier Restriction Problem

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We use the trilinear approach, in a different way than Bourgain–Guth, and obtain new estimates for the linear restriction problem in dimension $d = 3$.

On the long time existence of some solutions to periodic supercritical NLS equations

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In this lecture I will consider the quintic, periodic, defocusing NLS in dimension 3. I will recall that in [1] it was proved that this initial value problem is almost surely locally well posed for certain data in $H^{1-\alpha}, 0 < \alpha \ll 1$, which is a supercritical Sobolev space for the equation. I will then move on showing that although the supercriticality of the problem makes it difficult to move from local to global, by approximating the problem with a finite dimension one, where a certain Gibbs measure is invariant, we can extend the well-posedness from a small interval of time to a finite arbitrary interval of time $[0.T]$, at least for some initial data.

This is joint work with Andrea Nahmod.

[1] Nahmod, A. and Staffilani G., Almost sure well-posedness for the periodic 3D quintic nonlinear Schrödinger equation below the energy space, *J. Eur. Math. Soc.*, to appear (2015).

Invariant measures and long time behavior for the Benjamin-Ono equation

Nikolay Tzvetkov

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The KdV and the Benjamin-Ono equations are basic models, derived from the water waves equations for the propagation of long, small amplitude one dimensional waves. The solutions of the KdV equations, posed on the torus are known to be almost periodic in time. The long time behavior of the Benjamin-Ono equation, posed on the torus is much less understood. In this talk, we will present some progress on this problem. Namely, we shall construct an infinite sequence of weighted gaussian measures which are invariant by the flow of the Benjamin-Ono equation. These measures are supported by Sobolev spaces of increasing regularities. The "probabilistic view point" is essential in our analysis. In particular our arguments are less dependent on the particular behavior of each trajectory, compared to previous works on the subject.

The talk is based on a series of works by Yu Deng (Princeton University), Nicola Visciglia (University of Pisa) and the speaker.

The inverse Robin boundary value problem in a half-space

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In this talk we show recent results related to the inverse Robin problem for the Schrödinger equation in a half-space. The potential is assumed to be compactly supported. We first solve the direct problem for dimensions two and three. We then show that the Robin-to-Robin map uniquely determines the potential q.