S30. Symplectic geometry and special metrics

Organizers:

- Diego Conti (University of Milano Bicocca, Italy)
- Marisa Fernández (University of the Basque Country UPV/EHU, Spain)
- Anna Fino (University of Torino, Italy)
- Luis Ugarte (University of Zaragoza, Spain)

Program:

1. Daniele Angella (Università degli Studi di Parma, Italy)
   Cohomological properties of symplectic manifolds

2. Sergey Grigorian (University of Texas-Pan American, USA)
   Flows of G_2-structures

3. Stefan Ivanov (University of Sofia "St. Kliment Ohridski", Bulgaria)
   The Lichnerowicz-Obata sphere theorems on a quaternionic contact manifold

4. Spiro Kariagiannis (University of Waterloo, Canada)
   Centro-affine geometry and curvature of the moduli space of G_2 metrics

5. Victor Manero (Universidad del País Vasco/Euskal Herriko Unibertsitatea, Spain)
   Ricci soliton metrics induced by closed G_2 forms and the Laplacian flow

6. Vicente Muñoz (Universidad Complutense de Madrid, Spain)
   Formality of Kähler orbifolds and Sasakian manifolds

7. Johannes Nordström (University of Bath, UK)
   Disconnecting the G_2 moduli space

8. Álvaro Pelayo (Washington University in St. Louis, USA)
   Classical and Quantum Integrable Systems

9. Andrew Swann (Aarhus University, Denmark)
   Twists and special holonomy

10. Luigi Vezzoni (University of Torino, Italy)
    The Calabi-Yau equation on almost-Kähler torus fibrations

11. Raquel Villacampa (Centro Universitario de la Defensa de Zaragoza, Spain)
    Symplectic harmonicity and primitive cohomologies

Cohomological properties of symplectic manifolds

Daniele Angella

Istituto Nazionale di Alta Matematica, Dipartimento di Matematica e Informatica, Università degli Studi di Parma, Parco Area delle Scienze 53/A, 43124 Parma, Italy
angella@mail.dm.unipi.it

Both in complex and in symplectic geometry, Bott-Chern and Aeppli cohomologies provide further tools for the study of non-Kähler manifolds. We provide an inequality *à la* Frölicher that relates the Betti numbers and the dimensions of the complex, respectively symplectic Bott-Chern cohomology. Such an inequality provides also a characterization of cohomological decomposition properties.

We present results obtained in joint works with: A. Tomassini, H. Kasuya, F. A. Rossi, M. G. Franzini, S. Calamai, G. Dloussky.

Flows of G_2-structures

Sergey Grigorian

Department of Mathematics, University of Texas-Pan American,
1201 W University Dr., Edinburg, TX 78539-2999, USA
grigorians@utpa.edu

G_2-structures on 7-dimensional manifolds play a very important role in both geometry and physics. One of the ways of better understanding the relationships between different types of G_2-structures is to study their flows. In this talk, we will consider Laplacian flows of either closed or co-closed G_2-structures. Since the Laplacian is itself determined by the underlying G_2-structure, these flows give rise to non-linear partial differential equations. We will show that these flows share many similarities, such as the corresponding flow of the associated metric being equal to the Ricci flow to the leading order, but also some major differences. It turns out that unlike the flow of closed G_2-structures, the Laplacian flow of co-closed G_2-structures is not even weakly parabolic. We then show that this flow can be modified to make it weakly parabolic at least in certain directions and prove short-time existence and uniqueness of solutions for this new flow.

The Lichnerowicz-Obata sphere theorems on a quaternionic contact manifold

Stefan Ivanov

Faculty of Mathematics and Informatics, University of Sofia "St. Kl. Ohridski", blvd. James Bourchier 5, 1164, Sofia, Bulgaria and
Institute of Math., Bulgarian Academy of Sciences
ivanovsp@fmi.uni-sofia.bg

Lichnerowcz showed that on a compact Riemannian manifold of dimension n for which the Ricci curvature is greater than or equal to that of the unit n-dimensional sphere $S^n(1)$, the first eigenvalue λ_1 of the Laplace operator is greater than or equal to the first eigenvalue of the sphere, $\lambda_1 \geq n$. Subsequently Obata proved that equality is achieved iff the Riemannian manifold is isometric to $S^n(1)$ observing that the trace-free part of the Riemannian Hessian of an eigenfunction f with eigenvalue $\lambda = n$ vanishes.

We prove quaternionic contact (qc)-versions of the Lichnerowicz's and Obata's sphere theorems.

On a $4n+3$-dimensional compact qc manifold we find a Lichnerowicz's type positivity condition in terms of the Ricci tensor and the torsion of the Biquard connection and prove that this implies that the first nonzero eigenvalue λ_1 of the sub-laplacian is greater than or equal to the first eigenvalue of the 3-Sasakian round unit sphere, $\lambda_1 \geq 4n$ ([1]). We also show that the equality is achieved if and only if the qc manifold is qc homothetic to the standard 3-Sasakian unit sphere ([2]).

To prove the inequality, we write on a qc manifold a Bochner-Weitzenbök-type formula for the sub-Hessian in terms of the curvature and torsion of the Biquard connection. We observe that in the equality case the trace-free part of the sub-Hessian of an extremal eigen-function vanishes and prove that this property can occur if and only if the complete (with respect to the natural Riemnnian metric) qc manifold is qc homothetic to the standard 3-Sasakian sphere provided the dimension is bigger than 7.

[1] Ivanov, S., Petkov, A., and Vassilev, D., The sharp lower bound of the first eigenvalue of the sub-Laplacian on a quaternionic contact manifold, *J. Geom. Anal.* **24** (2014), 756–778.

[2] Ivanov, S., Petkov, A., and Vassilev, D., The Obata sphere theorems on a quaternionic contact manifold of dimension bigger than seven, submitted; http://arxiv.org/abs/1303.0409.

Centro-affine geometry and curvature of the moduli space of G_2 metrics

Spiro Karigiannis

Department of Pure Mathematics, University of Waterloo, 200 University Avenue West, Waterloo, Ontario, N2L 3G1, Canada
karigiannis@uwaterloo.ca

I will discuss various aspects of the geometry of the moduli space of G_2 metrics. In particular I will discuss the centro-affine Hessian structure, the Yukawa coupling, and the curvatures of various natural metrics on this moduli space.

This is a combination of past work with Conan Leung and new work with Christopher Lin and John Loftin.

Ricci soliton metrics induced by closed G_2 forms and the Laplacian flow

Victor Manero

Department of Mathematics, University of the Basque Country UPV/EHU, Apartado 644, 48080 Bilbao, Spain
pitimanero@gmail.com

Bryant and Xu proved short time existence and uniqueness of solution for the Laplacian flow of any closed G_2-structure on a compact manifold. I will show some recent results obtained in collaboration with M. Fernández and A. Fino on the solutions to the Laplacian flow for left invariant closed G_2-structures determining a Ricci soliton metric on simply connected non-abelian nilpotent Lie groups. For each of these structures, it will be proved a long time existence and uniqueness of solution for the Laplacian flow on the noncompact manifold. Finally, it will be shown that the long time solution of the Laplacian flow converges to a flat G_2-structure.

Formality of Kähler orbifolds and Sasakian manifolds

Vicente Muñoz

Departmento de Geometría y Topología, Universidad Complutense de Madrid, Plaza de Ciencias 3, 28040 Madrid, Spain
vicente.munoz@mat.ucm.es

We prove that compact Káhler orbifolds are formal, and derive applications of it to the topology of compact Sasakian manifolds. In particular, answering questions raised by Boyer and Galicki, we prove that all higher Massey products on any simply connected Sasakian manifold vanish. Hence, higher Massey products do obstruct Sasakian structures. Using this we produce a method of constructing simply connected K-contact non-Sasakian manifolds. On the other hand, for every $n > 2$ we exhibit the first examples of simply connected compact regular Sasakian manifolds of dimension $2n + 1$ which are non-formal. They are non-formal because they have a non-zero triple Massey product.

These results were obtained in joint work with I. Biswas, M. Fernández and A. Tralle.

Disconnecting the G_2 moduli space

Johannes Nordström

Department of Mathematical Sciences, University of Bath, Claverton Down, Bath BA2 7AY, UK
j.nordstrom@bath.ac.uk

I will describe work in progress with Diarmuid Crowley and Sebastian Goette on exhibiting examples of closed 7-manifolds with disconnected G_2 moduli space, using η invariants to distinguish between the components.

Classical and Quantum Integrable Systems

Álvaro Pelayo

Department of Mathematics, Washington University in St. Louis, One Brookings Drive, Campus Box 1146, St. Louis, MO 63130-4899, USA
apelayo@math.wustl.edu

I will discuss the classification problem of finite dimensional integrable systems. Then I will explain how a solution to this problem will lead to substantial progress in solving the isospectrality problem for quantum integrable systems (via semiclassical analysis). The isospectrality problem is still open in most cases.

Twists and special holonomy

Andrew Swann

Department of Mathematics, Aarhus University, Ny Munkegade 118, Bldg 153, DK-8000 Aarhus C, Denmark, and CP^3*-Origins, Centre of Excellence for Cosmology and Particle Physics Phenomenology, University of Southern Denmark, Campusvej 55, DK-5230 Odense M, Denmark*
swann@imf.au.dk

The twist construction is a geometric version of T-duality. It provides a framework to translate results on nilmanifolds to more general geometric settings. It is good at producing manifolds with unusual (e.g. non Kähler) complex structures and geometries with torsion. This talk will described how the twist construction may be adapted to produce metrics of special holonomy (hyperKähler, quaternionic Kähler...) and it may be used to understand and invert constructions such as hyperKähler modification.

The Calabi-Yau equation on almost-Kähler torus fibrations

Luigi Vezzoni

Department of Mathematics, University of Torino, Via Carlo Alberto 10, 10123 Torino, Italy
luigi.vezzoni@unito.it

The Calabi-Yau equation is an elliptic partial differential equation whose study goes back to the celebrated Calabi-Yau theorem. The talk is about a modern reformulation of this equation in almost-Kähler manifolds whose study is manly motivated by a conjecture of Donaldson. In the talk there will be shown that the equation has always a unique solution when the manifold is the total space of a torus bundle over a torus and the initial data satisfy some constrains.

Symplectic harmonicity and primitive cohomologies

Raquel Villacampa

Department of Mathematics, Centro Universitario de la Defensa, Academia General Militar, Crta. de Huesca s/n, 50090 Zaragoza, Spain
raquelvg@unizar.es

In 1988 Brylinski [1] conjectured, by analogy with the Hodge theory, that any de Rham cohomology class admits a symplectically harmonic representative. This conjecture holds if and only if the manifold satisfies the Hard Lefschetz Condition [2], and therefore the number of de Rham classes admitting harmonic representative can vary if we consider different symplectic structures. When this occurs, the manifold is said *flexible*.

Recently, Tseng and Yau have introduced other cohomologies on symplectic manifolds that admit unique harmonic representative within each class and showed that there exist primitive cohomologies associated with them such that their dimensions can vary with the class of the symplectic form, giving rise another notion of flexibility, [3, 4].

In the present talk we relate both situations and we will give conditions in low dimensions to ensure that both flexibilities are equivalent.

[1] Brylinski, J.-L., A differential complex for Poisson manifolds, *J. Differential Geom.* **28** (1988), 93–114.
[2] Mathieu, O., Harmonic cohomology classes of symplectic manifolds, *Comment. Math. Helv.* **70** (1995), 1–9.
[3] Tseng, L.-S., Yau, S.-T., Cohomology and Hodge theory on symplectic manifolds: I, *J. Differential Geom.* **91** (2012), 383–416.
[4] Tseng, L.-S., Yau, S.-T., Cohomology and Hodge theory on symplectic manifolds: II, *J. Differential Geom.* **91** (2012), 417–444.