S12. Geometric Topology

Organizers:

- Carlo Petronio (Università di Pisa, Italy)
- Joan Porti (Universitat Autònoma de Barcelona, Spain)

Speakers:

1. Javier Aramayona (Université Paul Sabatier, Toulouse, France)
   Homomorphisms from right-angled Artin groups to mapping class groups
2. Bruno Benedetti (Freie Universität Berlin, Germany)
   Embeddability of collapsible complexes
3. Michel Boileau (Université d'Aix-Marseille, France)
   Hidden symmetries and commensurability of hyperbolic knot complements
4. Antonio Costa González (Universidad Nacional de Educación a Distancia, Spain)
   Doubles of Klein surfaces and 3-manifolds
5. Raquel Díaz Sánchez (Universidad Complutense de Madrid, Spain)
   Schottky type uniformizations of stable Riemann orbifolds
6. Stefano Francaviglia (Università di Bologna, Italy)
   Moduli spaces of branched projective structures
7. Roberto Frigerio (Università di Pisa, Italy)
   Quasi-cocycles and bounded cohomology in higher degree
8. Ana García Lecuona (Université d'Aix-Marseille, France)
   The colored signature of a splice link
9. Yuya Koda (Tohoku University, Japan)
   Stable maps and branched shadows of 3*-manifolds*
10. Bruno Martelli (Università di Pisa, Italy)
    Some hyperbolic four-manifolds constructed using right-angled polytopes

11. Christoforos Neofytidis (Universität München, Germany)
The domination relation and branched covers

12. Cristina Pagliantini (ETH Zürich, Switzerland)
The simplicial volume of 3*-manifolds with boundary*

13. Alessandro Sisto (ETH Zürich, Switzerland)
On the bounded cohomology of acylindrically hyperbolic groups

14. Juan Souto (Université de Rennes 1, France)
Free vs. locally free Kleinian groups

15. Anastasiia Tsvietkova (UC Davis, USA)
Hyperbolic volume and the coefficients of the colored Jones polynomial of alternating links

16. Shicheng Wang (Peking University, China)
Embedding surfaces into S^3 *with maximum symmetry*

Homomorphisms from right-angled Artin groups to mapping class groups

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In this talk I will discuss injective homomorphisms from right-angled Artin groups into mapping class groups. I will explain a joint result with J. Souto that describes all such injections $A \to Mod(S)$, when the defining graph of A is a sufficiently thick *rigid subset* of the curve complex $C(S)$. As an application we obtain that if $\Gamma \subset Mod(S)$ is a subgroup that contains some power of every Dehn twist, then any injective homomorphism $\Gamma \to Mod(S)$ is a restriction of an automorphism of $Mod(S)$.

Finally, I will present a class of examples of **finite** rigid sets of $C(S)$, obtained in collaboration with C. J. Leininger, and describe some of their curious properties.

Embeddability of collapsible complexes

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All trees are planar graphs. How to extend this statement to higher dimensions? Already for $d = 2$, it is unknown whether all d-dimensional contractible simplicial complexes embed (linearly, or piecewise-linearly) in $\mathbb{R}^{2d}$. However, using Whitehead's simple homotopy theory, we show that all collapsible simplicial d-complexes linearly embed in $\mathbb{R}^{2d}$.

This is joint work with Karim Adiprasito, cf. http://arxiv.org/abs/1403.5217.

Hidden symmetries and commensurability of hyperbolic knot complements

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Two knot complements are commensurable if they share a finite sheeted cover. In the generic case of knots without hidden symmetries, commensurable knot complements are cyclically commensurable, which means that they have homeomorphic cyclic covers. This is no longer true for knot complements with hidden symmetries. To date, there are only three knots in S^3 which are known to admit hidden symmetries: the figure eight knot and the two commensurable dodecahedral knots. In this talk, I will discuss open questions and present new results in the case of small knots.

This is a joint work with Steve Boyer, Radu Cebanu and Genevieve Walsh.

Doubles of Klein surfaces and 3-manifolds

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A Klein surface is a surface with a dianalytic structure. A double of a Klein surface X is a Klein surface Y such that there is a degree-two morphism (of Klein surfaces) $Y \to X$. There are many doubles of a given Klein surface and among them the so-called natural doubles which are: the complex double, the Schottky double and the orienting double. We prove that if X is a non-orientable Klein surface with non-empty boundary, the three natural doubles, although distinct Klein surfaces, share a common double: "the double of doubles" denoted by DX. We describe how to use the double of doubles in the study of both moduli spaces and automorphisms of Klein surfaces. Furthermore, we show that the morphism from DX to X is not given by the action of an isometry group on classical surfaces. Finally we study the generalization of the concept of double of bordered 3-manifolds and how to represent these doubles using crystallizations.

The results of this talk are obtained in collaboration with P. Cristofori and A. M. Porto.

Schottky type uniformizations of stable Riemann orbifolds

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The retrosection theorem states that any Riemann surface can be uniformized by a Schottky group. This theorem has been generalized in two directions. On the one hand, for Riemann orbifolds, Reni-Zimmermann characterized which of them can be uniformized by a finite extension of a Schottky group. On the other hand, one can consider stable Riemann surfaces, that is, topological surfaces where some curves have been pinched to nodes and with a conformal structure outside the nodes. Hidalgo proved that any stable Riemann surface can be uniformized by a noded Schottky group. In this talk we will consider the same kind of problem for the case of stable Riemann orbifolds, and the uniformizing groups will be Kleinian groups which are finite extensions of noded Schottky groups.

Using a three dimensional language, these Schottky type uniformizations can be interpreted as characterizing which Riemann surfaces or 2-orbifolds with nodes can occur as the boundary of hyperbolic handlebodies or hyperbolic handlebody orbifolds with nodes.

This is a joint work with Rubén Hidalgo (U. T. Santa María, Valparaíso, Chile).

Moduli spaces of branched projective structures

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In this talk we will discuss some properties of the moduli spaces of branched $\mathbb{CP}^1$-structures on surfaces. Examples of such structures are hyperbolic metric with cone-singularities multiple of 2π and translation surfaces. We will focus in local/global surgeries that allow to travel in the moduli space of such structures.

Quasi-cocycles and bounded cohomology in higher degree

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Bounded cohomology of discrete groups is very hard to compute. For example, there is not a single group G whose bounded cohomology (with trivial coefficients) is known in every degree, unless it is known to vanish in all positive degrees (this is the case, for example, of amenable groups).

In degree 2, bounded cohomology has been extensively studied via the analysis of *quasi-morphisms*. In this talk I will describe how *quasi-cocycles*, which are the higher-dimensional analogue of quasi-morphisms, can be exploited to prove non-vanishing results for bounded cohomology in degree 3. Namely, I will sketch the proof of the fact that, for any acylindrically hyperbolic group G (whence for any non-elementary relative hyperbolic group and any mapping class group of a closed hyperbolic surface), the bounded cohomology module $H_b^3(G,\mathbb{R})$ is infinite-dimensional (and, in fact, it contains an infinite-dimensional subspace of elements with vanishing seminorm).

This is a joint work with M. B. Pozzetti and A. Sisto.

The colored signature of a splice link

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The splice of two links is an operation defined by Eisenbund and Neumann which generalizes several other operations on links, such as the connected sum, the cabling or the disjoint union. The precise definition will be given in the talk but the rough idea goes as follows: the splice of the links L' and L'' along the components K' and K'' is the link $(L' \setminus K') \cup (L'' \setminus K'')$ obtained by identifying the exterior of K' with the exterior of K''. There has been much interest to understand the behavior of different link invariants under the splice operation (genus, fiberability, Conway polynomial, Heegaard-Floer homology among others) and the goal of this talk is to present a formula relating the colored signature of the splice of two oriented links to the colored signatures of its two constituent links.

This is a joint work with Vincent Florens.

Stable maps and branched shadows of 3-manifolds

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In the early 1990s, Turaev introduced the notion of shadows as a combinatorial presentation of both 4 and 3-manifolds. Later, Costantino-Thurston revealed a strong relation between the Stein factorizations of stable maps of 3-manifolds into the real plane and the shadows of the manifolds. In fact, a shadow can be seen locally as the Stein factorization of a stable map. In this talk, we define the notion of stable map complexity for a closed orientable 3-manifold counting, with some weights, the minimal number of codimension 2 singular fibers of stable maps into the real plane. Then we show that this number equals the minimal number of vertices of its branched shadows. In consequence, we give a complete characterization of hyperbolic 3-manifolds admitting stable maps with the stable map complexity 1 in terms of Dehn surgeries. We also discuss an estimation of the hyperbolic volume by this complexity.

This is joint work with Masaharu Ishikawa (Tohoku University).

Some hyperbolic four-manifolds constructed using right-angled polytopes

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We expose some constructions of finite-volume hyperbolic four-manifolds via right-angled regular polytopes. We will look for "small" examples: manifolds having low volume, few cusps, and/or few geodesic boundary components.

The domination relation and branched covers

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The existence of a map of non-zero degree defines a transitive relation, called domination relation, on the homotopy types of closed oriented manifolds of the same dimension. A fundamental question in this field is that of identifying simple homotopy representatives for a dominant map between two manifolds. Among the most prominent examples are branched coverings, whose study goes back to Alexander's classical theorem on the realization of smooth n-manifolds as branched covers of S^n and to pioneer works by Edmonds on the deformation of non-zero degree maps to branched coverings in dimensions two and three. In this talk, we show that large classes of connected sums of bundles over a sphere admit 2-fold branched coverings by products of type $S^1 \times N$. As a consequence we show that every simply connected manifold in dimensions four and five is dominated by a non-trivial product, answering a question of Kotschick and Löh up to dimension five.

The simplicial volume of 3-manifolds with boundary

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The simplicial volume is a homotopy invariant of compact manifolds introduced by Gromov in 1982. For a connected compact oriented manifold it is the ℓ^1-seminorm of the real fundamental class of the manifold itself. Even if the simplicial volume depends only on the homotopy type of a manifold, it is deeply related to the geometric structures that a manifold can carry. For example, closed manifolds which support negatively curved Riemannian metrics have non-vanishing simplicial volume, while the simplicial volume of flat or spherical manifolds is null. Several vanishing and non-vanishing results for the simplicial volume are available by now, but the exact value of non-vanishing simplicial volumes is known only in very few cases. In this talk we provide sharp lower bounds for the simplicial volume of compact 3-manifolds in terms of the simplicial volume of their boundaries. As an application, we compute the simplicial volume of several classes of 3-manifolds, including handlebodies and products of surfaces with the interval.

On the bounded cohomology of acylindrically hyperbolic groups

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Examples of acylindrically hyperbolic groups include non-elementary (relatively) hyperbolic groups, mapping class groups, $Out(F_n)$, many $CAT(0)$ groups, and so on.

Acylindrically hyperbolic groups contain certain subgroups, called hyperbolically embedded subgroups, that are useful for several applications. For example, if H is hyperbolically embedded in G then the restriction map $H_b^*(G) \to H_b^*(H)$ in bounded cohomology is surjective.

I will discuss the following result:

Theorem. *Let G be acylindrically hyperbolic. Then $H_b^2(G)$ naturally embeds in the product of the bounded cohomologies of its virtually free, hyperbolically embedded subgroups.*

Time permitting, I will say a few words about the image of the embedding as well.

This is joint work with Tobias Hartnick.

Free vs. locally free Kleinian groups

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Suppose that $k \geq 3$ and identify $\mathrm{SO}_+(k,1)$ with the orientation preserving isometry group of hyperbolic k-space. It is well-known that a finitely generated discrete and torsion free subgroup Γ of $\mathrm{SO}_+(k,1)$ whose limit set Λ_Γ is a Cantor set is the free product of abelian subgroups. In particular, in the absence of higher rank abelian subgroups, any such Γ is isomorphic to a free group. It is remarkable, but also relatively well-known, that all this fails if Γ is not finitely generated. In fact, we prove:

Theorem 1. *For all ϵ there are discrete and torsion free subgroups Γ of the isometry group $\mathrm{SO}_+(k,1)$ of hyperbolic k-space without higher rank abelian groups, whose limit set Λ_Γ is a Cantor set of Hausdorff dimension less than $1+\epsilon$, but which are not free.*

On the other hand, using a result by Besson, Courtois and Gallot we get:

Theorem 2. *Any non-elementary discrete and torsion free subgroup Γ of $\mathrm{SO}_+(k,1)$ whose limit set has Hausdorff dimension less than 1 is a free group.*

Hyperbolic volume and the coefficients of the colored Jones polynomial of alternating links

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Since quantum invariants were introduced into knot theory, there has been a strong interest in relating them to the intrinsic geometry of a link complement. This is for example reflected in the Volume Conjecture, which claims that the hyperbolic volume of a link complement in S^3 is determined by the colored Jones polynomial.

In the work of M. Lackenby, and of I. Agol and D. Thurston, an upper bound for volume of a hyperbolic link complement in terms of the number of twists of a link diagram is obtained. We will discuss how to refine this bound for alternating links, and how to express the refined bound in terms of the three first and three last coefficients of the colored Jones polynomial.

Embedding surfaces into S^3 with maximum symmetry

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We restrict our discussion to the orientable category. For $g > 1$, let OE_g be the maximum order of a finite group G acting on the closed surface Σ_g of genus g which extends over (S^3, Σ_g), for all possible embeddings $\Sigma_g \hookrightarrow S^3$. We will determine OE_g for each g, indeed the action realizing OE_g. In particular, with 23 exceptions, OE_g is $4(g+1)$ if $g \neq k^2$ or $4(\sqrt{g}+1)^2$ if $g = k^2$, and moreover OE_g can be realized by unknotted embeddings for all g except for $g = 21$ and 481.

This is a joint work with C. Wang, Y. Zhang and B. Zimmermann.