Nonlinear heat equations of fractional diffusion type

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Much recent research is taking place in the area of elliptic and parabolic equations, aimed at understanding the effect of replacing the Laplace operator, and its usual variants, by a fractional Laplacian operator or other similar nonlocal operators, which represent long distance interactions. Linear and nonlinear models are involved.

The lecture will describe some of the progress made by the author and collaborators on the topic of nonlinear fractional heat equations, in particular when the nonlinearities are of porous medium and fast diffusion type. The results cover existence and uniqueness of weak (sometimes classical) solutions, regularity and continuous dependence, positivity, estimates and symmetrization. Special attention is given to the construction of self-similar solutions and their asymptotic role and the positivity estimates.