Entropies and nonlinear diffusion equations

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In recent years, entropies (Lyapunov functionals) have proven to be a very useful tool for the study of the asymptotic behavior of nonlinear diffusion equations. This feature is mainly due to their monotonicity properties when evaluated on the solution of these equations. However, the connections between nonlinear diffusion equations and entropies are more deep. Not only entropies can be used to obtain results on the large-time behavior of the solution to nonlinear diffusions, but, on the reverse, nonlinear diffusions allow to obtain a number of results for entropies, in the form of inequalities in sharp form.

In this lecture, we focus our attention on Renyi entropies, which are mainly popular in the framework of information theory. In particular, we outline the relationships between Renyi entropies and the nonlinear diffusion equations, by showing in details the most important consequences. These results have been partially obtained in collaboration with José Antonio Carrillo (Imperial College, London), and Giuseppe Savaré (University of Pavia).