Applications of spin structures on complex algebraic curves

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Introduced with the name of theta characteristic by Riemann, a spin structure on a Riemann surface Σ is a line bundle L that is a square root of the holomorphic cotangent bundle of Σ. A spin structure often encodes important hidden geometric information. We discuss two examples and applications of these phenomena:

i) Let Σ be an oriented differentiable surface and let $F : \Sigma \to \mathbf{R}^3$ be a smooth immersion. The metric makes Σ into a Riemann surface endowed with a spin structure L and two sections s_1, s_2. This provides the *spin representation* of F:

$$F = \text{constant } + Re \int (s_1^2 - s_2^2, -i(s_1^2 + s_2^2), 2s_1 s_2).$$

When Γ is compact F also induces an algebraic structure on Σ, moreover $F(\Sigma)$ is minimal if the sections s_1 and s_2 are holomorphic. There are not minimal compact surfaces in $\mathbf{R}^3$, but one has a natural structure of algebraic curves in the case of minimal surfaces of bounded curvature, and the triply periodic case. As an application we proved the existence of triply periodic immersed minimal surfaces for any $g > 2$. The main ingredients are the study of the Shrödinger operator of Montiel and Ros and the variation of the periods. Recently in collaboration with Francesco Bastianelli, by means of an improvement in the theory of subcanonical Weierstrass points on an algebraic curve, we could extend the result to the *odd* triply-periodic minimal surfaces (the parity of a spin structure is the parity of the dimension of its holomorphic sections).

ii) The second application deals with Hurwitz schemes. In collaboration with Michela Artebani spin structures are used to show that any compact Riemann surface covers the Riemann sphere with only odd type of ramification points. Coupling ideas from algebraic geometry of Mumford and from the theory of minimal surfaces of Kusner and Schmitt, in collaboration with Juan Carlo Naranjo, a partial compactification of these coverings has been found. This allows to study new geometrical aspects related with the moduli spaces of such coverings.