Oscillatory motions in the restricted circular planar three body problem

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We consider the circular restricted three body problem which models the motion of a massless body under the influence of the Newtionan gravitational force caused by two other bodies, the primaries, which move along cicular planar Keplerian orbits.

The possible motions the massless body can perform were already known by Chazy. In 1922, he gave a complete classification of all possible states that the body $q(t)$ can approach as time tends to infinity. The possible final states are reduced to four:

- $H^{\pm}$ (hyperbolic): $\|q(t)\| \to \infty$ and $\|\dot{q}(t)\| \to c > 0$ as $t \to \pm\infty$.
- $P^{\pm}$ (parabolic): $\|q(t)\| \to \infty$ and $\|\dot{q}(t)\| \to 0$ as $t \to \pm\infty$.
- $B^{\pm}$ (bounded): $\limsup_{t\to\pm\infty} \|q\| < +\infty$.
- $OS^{\pm}$ (oscillatory): $\limsup_{t\to\pm\infty} \|q\| = +\infty$ and $\liminf_{t\to\pm\infty} \|q\| < +\infty$.

Examples of all these types of motion, except the oscillatory ones, were already known by Chazy.

In this talk, we prove the existence of oscillatory motions for any value of the masses of the primaries. We relate the existence of these motions with the existence of chaos in the dynamical system. To obtain such motions, we show that, for any value of the mass ratio and for big values of the Jacobi constant, there exist transversal intersections between the stable and unstable manifolds of infinity which guarantee the existence of a symbolic dynamics that creates the oscillatory orbits. The main achievement is to rigorously prove the existence of these orbits without assuming the mass ratio between the primaries small since then this transversality can not be checked by means of classical perturbation theory respect to the mass ratio. We also detect a curve in the parameter space, formed by the mass ratio and the Jacobi constant, where cubic homoclinic tangencies between the invariant manifolds of infinity appear.