Conservation law models for traffic flow on a network of roads

Alberto Bressan

Department of Mathematics, Penn State University, University Park, Pa. 16802, USA
[bressan@math.psu.edu]

2010 Mathematics Subject Classification. 35L65, 49K20

Traffic patterns are the result of a large number of individual decisions, where each driver chooses an optimal departure time and an optimal route to reach destination. This talk will survey recent developments and open questions in the macroscopic modeling of traffic flow.

From a mathematical perspective, traffic flow can be modeled by a family of conservation laws, describing the density of cars along each road. These are supplemented by suitable boundary conditions, modeling flow at intersections. In this setting, the well-posedness of the Cauchy problem is a key mathematical issue, not yet fully resolved.

In addition, one can introduce a cost functional, accounting for the time that each driver spends on the road and a penalty for late arrival. For some models, one can prove the existence of a globally optimal solution, minimizing the sum of the costs to all drivers, and of a Nash equilibrium solution, where no driver can lower his individual cost by changing his own departure time or his route to reach destination.

The dynamic stability of Nash equilibria, and possible extensions of these results to more general models, will also be discussed.