JUDDER VIBRATION IN DISC BRAKES EXCITED BY THERMOELASTIC INSTABILITY

Abstract: Friction vibrations and noises which are common in brakes, have attracted a great deal of attention lately. This paper analyses low frequency vibrations in disc brakes excited at high car speed. This vibration, called judder, has a frequency in the range 10 to 300 Hz and usually comes in association with hum noises. The dynamic phenomenon shows two principal components, one normal and the other one tangential to the disc brake surface. It is explained how variations of friction coefficient, and thermoelastic instability caused by the tangential component, contribute to the appearance of judder. A numerical analysis in 3D using the finite element method has been implemented combining both tangential and normal components, and solving the thermoelastic process. Special attention is dedicated to the simulation of the thermoelastic process showing the correlation with experiments.

Keywords: Judder, Thermoelastic Instability, Rotor Thickness Variation, CAE Design, Hot spots.

Introduction

Lately, brakes, being a significant source of noise and vibration, have been the subjects of many studies where safety and comfort are concerned. Low-frequency vibrations, collectively referred to as *hum*, are defined as those up to 1000 Hz. When frequency is higher, the vibration is classed as high frequency and is subsumed under the term *squeal*. Squeal is discussed in an extensive bibliography (Nossier 1998, Suzuki 1997, Hu 1997), while hum has received less attention (Nack 1995, Ishihara 1996, Abdelhamid 1997). These vibrations and noises are induced in the contact process. Authors have pointed out several mechanisms to justify the role of friction in the dynamic phenomenon but there is not a unique theory that can justify every experimental observation.

Recently, customer annoyance problems are being reported to occur in vehicles from all manufacturers due to these vibrations. Hum noises in the interior are a usual cause of guarantee claims. And in addition to these direct problems, vibrations can also induce added wear and prompt fracture of elements involved.

This paper has a base in a study (Avilés et al. 1995) commissioned by the Renault Research Department in Rueil-Malmaison. The aim was to determine the origin of a strong vibration that occurs in cars with disc brakes when, with certain friction pads, braking is effected at 160 Km/hr or more. The dominant frequency in question is about 210 Hz, and gives rise to hum noise in the interior of the car.

After a thorough analysis of the empirical data supplied by Renault, one can see how two vibration components appear on the friction pads. One of these components, tangent to the disc surface, exhibits a dominant frequency of about 200 Hz. The other, which is normal to the disc surface, exhibits a frequency that is variable but has a constant harmonic around the tenth i.e. there are approximately ten maxims for each revolution of the disc. Whilst the former study proved to be a verified solution to the question on the source of the tangential component of judder, there was not a clear cause to the origin of the normal one.

A usual characteristic observed in the tests is a corresponding phenomenon to the normal component of judder, that is the geometric distortions and patches that appear on the disc surface, usually from nine to thirteen in number. While most authors agree that these distortions are directly related to judder, there is little agreement as to how distortion arises. (Hideo 1986, Dweib 1990 a/b)

This work exams the hypothesis that the tangential component of the vibration affecting sliding velocity triggers the phenomenon of thermoelastic instability, this being the cause of hot spotting responsible of the normal component of the vibration. The analysis implemented is to couple dynamic, contact and thermal studies, and the software application developed will be incorporated to the CAE design cycle.

Thermoelastic instability as a vibration exciter.

Judder is a highly complex dynamic process involving friction, velocity, pressure, temperature, wear, modes and frequencies of vibration, and more. In Avilés et al. (1995a), it has been shown that another decisive factor is the type of pad used, and that there is a relation between friction coefficient and judder level. Friction coefficient depends basically on velocity, temperature and pressure. If we then assume a more or less constant pressure on brake-circuit during braking the relative velocity between disc and pad falls with time, while the temperature of the contact surface rises. For this reason, the force of friction varies with time. In acting upon a structural system, in this case the car suspension, it gives rise to certain dynamic phenomena. These for the most part compose the vibration that is called judder.

As it is shown in Avilés et al. (1995a), judder presents two vibration components, one tangential to the disc surface and the other one perpendicular to that plane. The friction pad sliding on the undulated surface of the disc produces the normal one. This deformation of the disc is due to hot spotting; a phenomenon well explained by Barber (1967) and called thermoelastic instability. This phenomenon arises as a consequence of the perturbation of the sliding, in this case, of the friction pad on the disc.

The dynamic problem previously described brings about irregularities on the disc-pad contact surface, these in turn showing a relationship between the thermal and dynamic processes. Those irregularities appear as blue stains on disc, severe band or circle wear, and material transformation in the contact surface (Kennedy 1984).

British Rail, concerned with unexpected wear, carried out several studies that allowed J.R. Barber to notice that under determined sliding conditions thermal expansion exceeded wear leading to an unstable change in contact area (Barber 1967). The coupled phenomenon involving contact pressure, frictional heat, and thermal expansion that leads to a change from a uniform contact between sliding solids to a disperse contact at a macroscopic level on areas called hot spots is defined as thermoelastic instability.

Regarding automotive disc brakes, the most prominent effect has been the excitation of low frequency vibrations and noises when braking from high speeds. Its appearance with few kilometres and the corresponding guarantee costs has led the companies to work over this occurrence (Burton 1978, Lee 1994, and Ruiz Ayala 1996).

The phenomenon arises when a uniform distribution of contact pressure is modified by some factor in sliding, this in turn inducing a chain of processes. First, frictional heat is changed along the pressure variation, altering the uniform temperature distribution of sliding solids and producing an anomalous thermal expansion. This expansion of the contact surface alters the contact pressure again concentrating on protuberances.

The effect of wear being a function of temperature is added here, spots where temperature is higher are subject to more abrasion. Therefore wear could regularise surface deformation putting an end to the concentration of contact. No doubt that there is a dependence on sliding velocity and the necessity of some time under instability conditions for the phenomenon to arise. The instability evolves when the value of sliding velocity is above one called critical velocity.

In this situation hot spots appear on contact surface with very high temperatures (700ºC) and important gradients. High temperatures speed up oxides, what alters the friction coefficient, adding wear and reducing the life cycle of the component. In addition to this, thermal cracks are produced by thermal stresses located nearby hot spots.

In disc brakes, thermal expansion of hot spots produces rotor thickness variation and the appearance of a vibration normal to the surface with a constant harmonic. Due to the magnitude of the rotating velocity, the vibration generated has a low frequency (around 200 Hz) and it is transferred to the vehicle through the suspension causing the so-called roughness.

Two different approaches to analyse thermoelastic instability are being currently used. Analytical studies based in the basic equations developed by Lee and Barber (1993, 1995) to determine the effect of various factors on the value of critical speed (Hartsock 1999). Or numerical studies using FE techniques that provide the determination of the critical speed solving an eigenvalue analysis based in Burton's perturbation method (Du 2000).

Here a more realistic analysis will be implemented, not to get the value of critical velocity but to solve the actual thermoelastic process in braking in combination with the dynamic analysis. Whether there is already some study in this line employing an axisymetric 3D model (Kao 2000), ours uses solid elements, the same model for all the thermoelastic analyses, and is linked to the dynamic analysis.

Hypothesis

A negative slope in the curve of variation of friction coefficient as a function of sliding velocity is the cause of tangential vibration but it does not account for normal component. In the other hand, non-uniform frictional heat, hot spotting and wear are a consequence of thermoelastic instability.

Experimental tests carried out by Renault (Avilés 1995 a) show the existence of the tangential vibration for some time before the appearance of the normal one on new discs, while they are simultaneous with used discs. In addition to this, the normal component has a constant harmonic coincident in number with the hot spots recorded. Therefore it is logical to believe that normal vibration is related to thermoelastic instability.

The hypothesis proposed is that tangential vibration is the cause of a non-uniform heating of the disc due to its effect on sliding velocity, and triggers the thermoelastic instability phenomenon that produces protuberances on disc surface. When friction pads slide over these pimps a normal vibration is induced. This process takes some time where normal vibration does not exist, but when discs have been previously used a residual warping excites normal component.

Analysis methodology

The hypothesis proposed exposes the diversity of mechanical analyses needed in the resolution and simulation of the dynamic phenomenon called judder. This vibration has two components clearly identifiable in tangential and normal directions. Both are independent in their manifestation, however in collateral phenomena such as temperature distribution and wear on disc there is a cause- effect relationship. In figure 1 it is shown the coupling of analyses needed to determine the judder vibration.

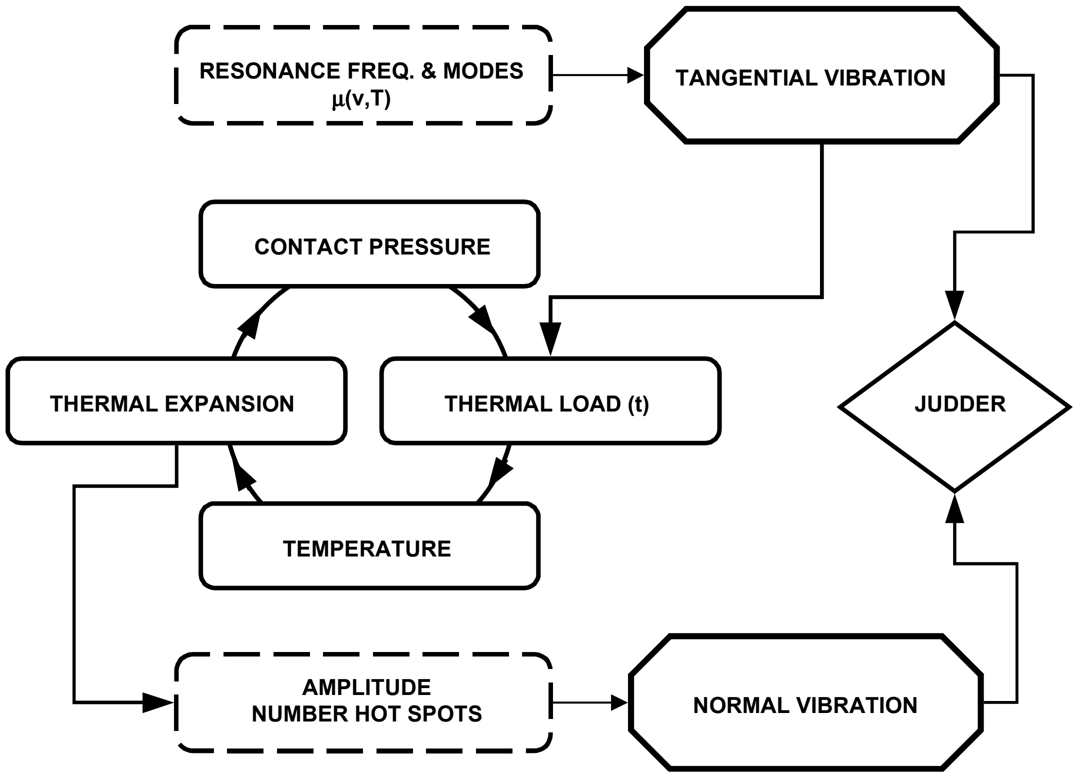


Figure 1: Algorithm of the complete analysis of judder.

The analysis of the tangential component, friction induced, is solved using modal uncoupling techniques on a FEA model of the suspension and a mathematical model developed for Renault (Avilés et al, 1995a/b) as mentioned before.

Normal component study is based on protuberances that appear on disc surface and its rotation. Temperature field on the disc, as a consequence of tangential vibration, has a non-uniform distribution where hot spots appear and thermal expansion distorts contact surface. Normal vibration is induced as friction pads slide on the undulated disc surface. Working out pimp number, position and magnitude along time is a previous task to simulate the vibration.

Thermoelastic instability has been identified as the cause of non-uniform deformation of the disc. Therefore it is compulsory to analyse the thermoelastic process in the contact between disc and pad.

Thermal load due to friction between disc and pads is figured out from the equation

$$Q = \mu PV = \mu(v,T)\cdot P(r,\theta,\varphi(t))\cdot V(r,t) \qquad \textbf{(1)}$$

where μ is the coefficient of friction (as a function of sliding velocity and temperature), P is the contact pressure (dependent on radial and polar position of disc points, and the position of friction pads), and V is the sliding velocity (function of radial position and time). As observed, these magnitudes are functions of the radius and angular position of contact area as well as braking time.

Friction coefficient is a variable magnitude with some factors such as sliding velocity or temperature, and it is precisely this variation what causes the dynamic problems observed in the tangential direction to disc surface. Its calculation as a function of those factors has to be done experimentally.

The contact pressure analysis needed to obtain frictional heat is affected by the thermal distortion produced by variation of the temperature field. Uncoupling of these phenomena requires a recurrent strategy. Contact pressure distribution is evaluated along a certain number of disc revolutions, frictional heat produced is then calculated allowing temperature field to be computed. Hence, variation of pressure distribution due to thermal expansion is obtained and a new cycle of analysis can be studied recursively to the end of braking.

Analysis procedure starts with the static analysis of contact pressure. When there is no uniformity in the thermal expansion of the disc surface or in the wear in the circunferential direction, obtaining the pressure distribution in different positions of the pad on the disc has a great importance. The number of positions depends on the disc and pad FEA models.

For the contact boundary condition to be introduced, the finite element method provides two alternatives. First one is to employ contact elements, which is the most general but implies the nonlinearly of the analysis. The second one is the usage of a boundary condition that imposes the equality of displacements between two nodes of the contact surfaces in the normal direction to them. As a result the force transmitted through that boundary condition is obtained and that will provide the contact pressure distribution. Nevertheless this linkage is bilateral i.e. the boundary condition will persist under tension. Actually surfaces will separate when the force reduces to null and this can be detected evaluating those forces in each step of the analysis cycle. In this case the boundary condition between nodes in tension is eliminated and the analysis is done again. This modification of the model is correct in subsequent steps of the thermoelastic analysis. This second alternative is linear, provides a better control and a higher accessibility to results, in addition to this it takes advantage from the fact that separations are not reversible.

Disc and pad temperature effect is included in the analysis as a boundary condition depending on the thermal expansion coefficient. In figure 2 it is shown the complete finite element model of the disc brake.

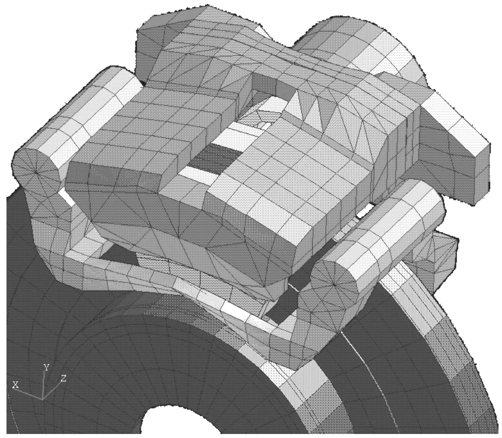
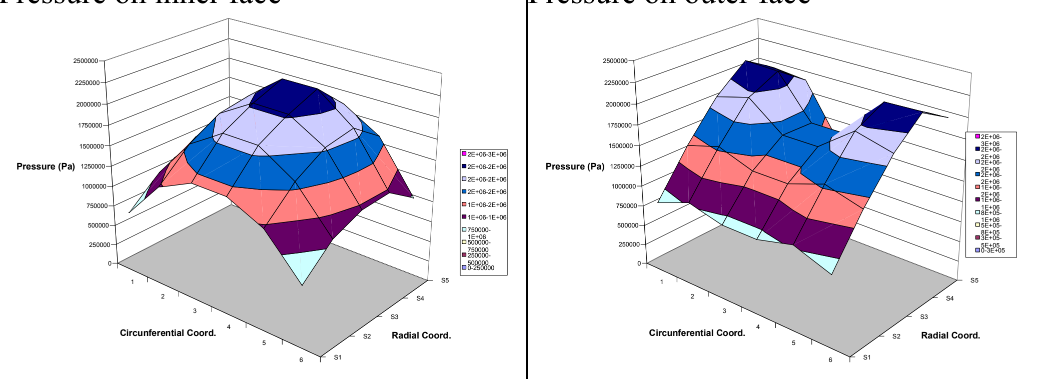


Figure 2: FEA model of the disc brake.

In order to find out the contact pressure, forces previously obtained have to be distributed on the area where applied. This transformation could be done through shape functions. In figure 3 it can be observed the asymmetry of contact on both faces of the disc, already noticed on wear results of experimental tests.

Pressure on inner face | Pressure on outer face



Max. P. 2.086 MPa | Max. P. 2.004 MPa

Figure 3: Contact pressure distribution under friction pads on both faces of the disc.

For each point on the contact surface, pressure is dependent on whether the pad is on it or no, and when in contact, on which part of the pad is on it. Hence pressure distribution can be expressed as a function of the radius **r** and the circumferential position of the point θ, and the pad position φ. In figure 4 it is shown the pressure distribution on the first sector of the FEA model along time.

To end, sliding velocity influences the friction coefficient through the Stribeck curve and the thermal flux directly through equation 1.

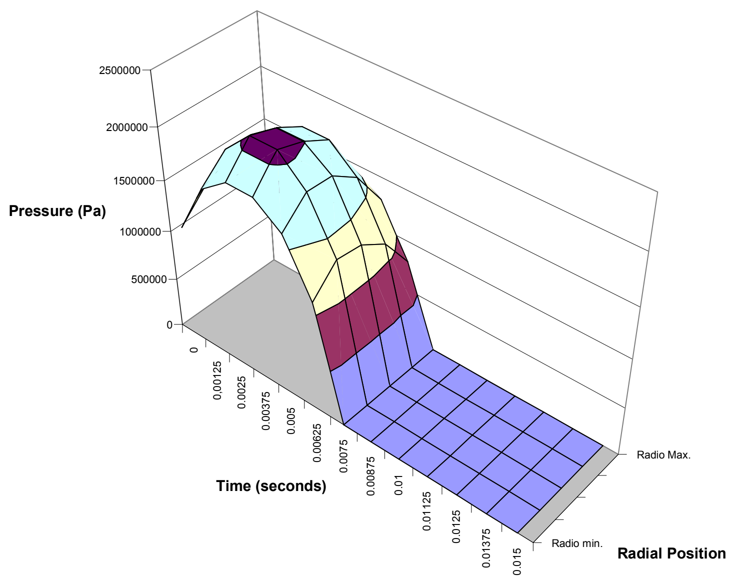


Figure 4: Pressure distribution on points θ=0 on disc along time.

The tangential vibration, arising in the contact due to the variation of friction coefficient with sliding velocity, has also an influence on the frictional heat. Whilst the vibration movement of the pad is on the sense of the rotation of the disc, the sliding velocity that generates frictional heat decreases and vice versa. Then heat generation is not homogeneous but sinusoidal. This variation in heat flux is a factor that contributes to the perturbation of the uniform contact, which initiates thermoelastic instability. (Barber 1987, Burton 1978)

Thermal load is applied on every sector of the disc during the time that the pad takes in covering that angle. This heat flux can be expressed from equation (1) in the form

$$Q = \mu PV = \mu(v,T) \cdot P\big(r,\theta,\varphi(t)\big) \cdot r \cdot \omega(t) \quad \textbf{(2)}$$

where ω is the angular velocity in the contact coming from the difference between rotational velocity of the disc and the one corresponding to the friction pad due to the tangential vibration. Therefore, thermal load can be considered as the product of two factors as it can be observed in figure 5; one magnitude dependent on the position of load, and the relative angular velocity $\omega(t)$ dependent on time.

$$Q = \mu \cdot P(r,\theta,\varphi) \cdot r \cdot \omega(t) = F(r) \cdot \omega(t) \quad \textbf{(3)}$$

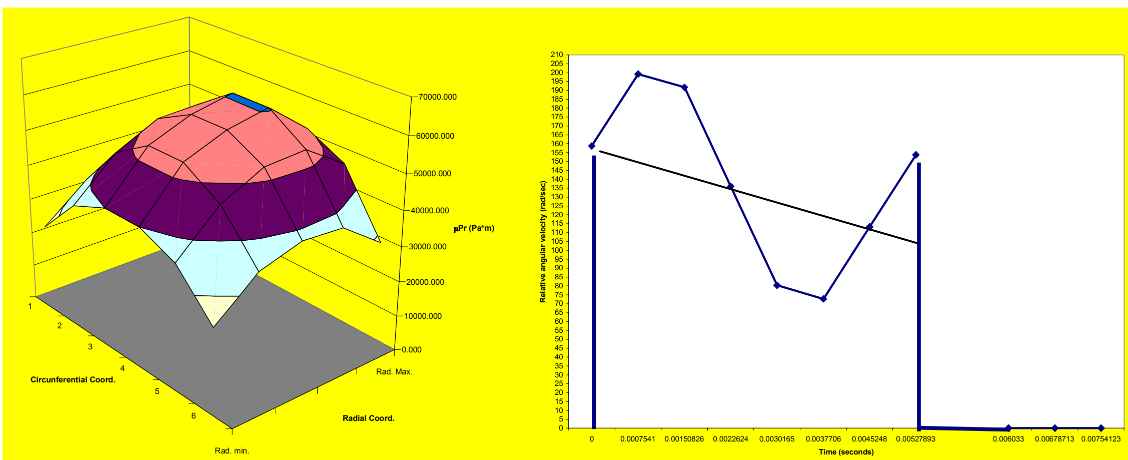


Figure 5: Components of thermal load: functions of position and time respectively.

If the pressure distribution in the central line of the disc- pad contact is considered as the most representative and variation along time and disc surface is taken into consideration, a representation as shown in figure 6 can be obtained.

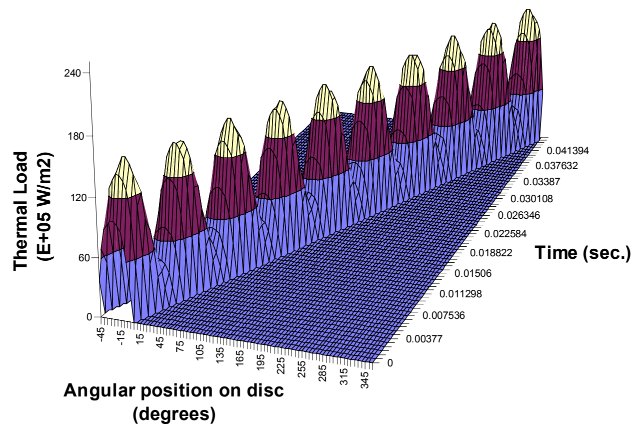


Figure 6: Thermal load on disc.

Another fact that shows the complexity of analysis is the effect of the temperature field generated by friction on magnitudes contributing to frictional heat itself. This feedback of the phenomenon compels the recurrent schedule and the braking thermal analysis being done in consecutive steps. Some magnitudes have to be recalculated from one step to the next one.

Regarding boundary conditions, in the first step of the thermal analysis the initial temperature is considered 20ºC in all nodes but in subsequent cycles previously calculated temperature field is used. To include convection, additional calculations have to be done having in mind the model of the disc rotating in airflow. Once the temperature field on disc is worked out, this is included in the static analysis for thermal expansion to be studied.

Having in mind the characteristics of the mechanical device considered is possible to solve a cycle of analysis every a certain number of disc revolutions because the magnitudes involved do not vary greatly in the time of a revolution of the disc. Thermal load is calculated for cycle, and transient thermal analysis is solved along the time employed in those revolutions.

Temporal discretization

Due to discretization of the FEA model of the disc, continuous analysis along time is not possible being necessary to discretizise time as well. Time step chosen should allow a correct consideration of the variation of factors affecting thermal load and an adequate transient thermal analysis.

Firstly, time step size election is conditioned by element size and heat transfer properties so that temperature field varies in an adequate manner over the model. Secondly, load and boundary conditions variation has to be considered. In this case, thermal load due to friction is variable in a periodic manner and the time step chosen should represent it in detail. It is selected the fraction 1/5 of the period of the tangential vibration as shown in figure 7, that in the end it is the cause of modification of frictional heat.

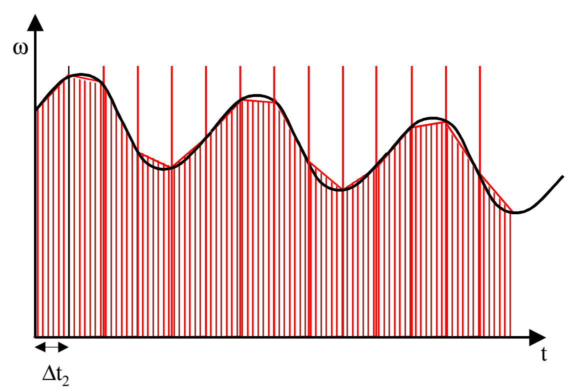


Figure 7: Circunferential velocity discretizised with time steps.

Thirdly, it has to be borne in mind the way thermal load is applied on finite elements. When disc is divided in 36 sectors, the load is applied on 36 different positions of the pad, and is held over the time employed in moving from one to another. In figure 8, variation of the relative angular velocity on each sector is shown, being its value null along time not in contact with pad. The following sector starts receiving heat when previous one ends. This fact provides another factor of influence in the election of time step. For the thermal load to be implemented correctly in the transient analysis it has to be applied in successive steps over every sector, and in each one the same time. In this manner every sector receives heat flux the same number of times and continuity of the load is asserted.

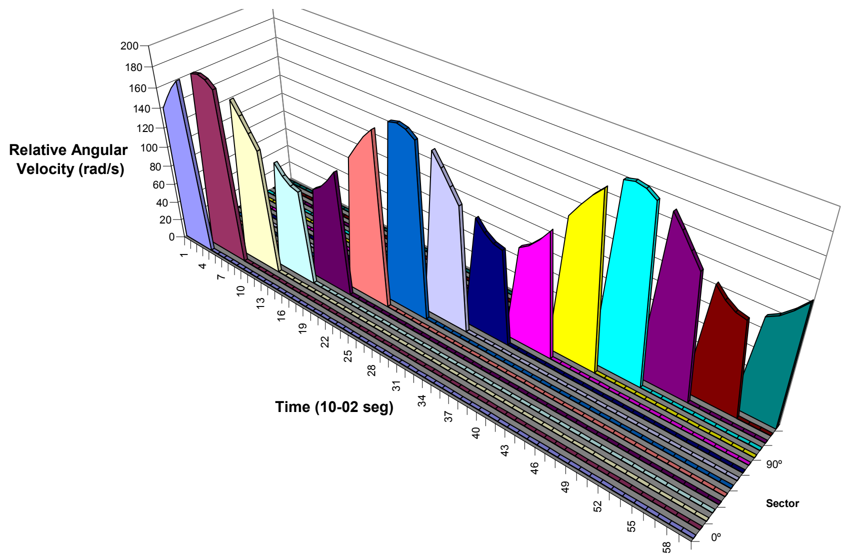


Figure 8: Variation of relative angular velocity over time and applied sector.

Braking effect causes that time employed in covering one sector enlarges while the time step remains constant. This implies a risk so that in some time the analysis is carried out in one sector more times than in the rest. However, the cyclic schedule of the procedure minimises this handicap. A different time step size can be defined in every cycle, eluding the problem described.

Time step size will be a part of the contact time on the first sector, but that election should allow an equal number of steps applied in every sector. Therefore, if n is the number of time steps, the $(35 \times \boldsymbol{n} + 1)^{\text{th}}$ step should be in the contact time of the 36[th] sector (FEA model has 36 sectors). Hence, bearing in mind that time step size is obtained from

$$\Delta t = \frac{t_{1/2} - t_{1/1}}{n} \tag{4}$$

where $\mathbf{t_{1/1}}$ is the first instant of contact in the first sector, $\mathbf{t_{1/2}}$ is the ending time of that contact, and n the number of steps in that sector, the following expression should be verified:

$$t_{1/1} + (35 \times n + 1) \cdot \Delta t > t_{36/1} \tag{5}$$

where $\mathbf{t_{36/1}}$ is the starting time for the contact in the last sector. Solving adequately the equation (3) is possible to obtain the number of steps applicable on each sector avoiding the previous problem:

$$n < \frac{t_{1/2} - t_{1/1}}{t_{36/1} - \left(t_{1/1} + 35 \cdot \left(t_{1/2} - t_{1/1}\right)\right)} \tag{6}$$

Using this value the total number of time steps and the step size can be computed.

In the case that in every thermoelastic analysis cycle is studied more than a disc revolution the procedure to be used is analogous getting the equation:

$$n < \frac{t_{1/2} - t_{1/1}}{t_{(36 \cdot v)/1} - \left(t_{1/1} + (35 \cdot v) \cdot \left(t_{1/2} - t_{1/1}\right)\right)} \tag{7}$$

where $\boldsymbol{\nu}$, is the number of revolutions of the disc analysed in every cycle.

Results

The mathematical model described in Avilés et al. (1995) is applied to the FEM model and provides the dynamic response of the calliper in the tangential direction. In the current study a more detailed FEM model of a McPherson type suspension was employed, it is shown in figure 9. Results show that judder is excited when braking from a velocity of 140 Km/hr if friction pads with a negative slope in the curve of variation of friction coefficient versus sliding velocity are used.

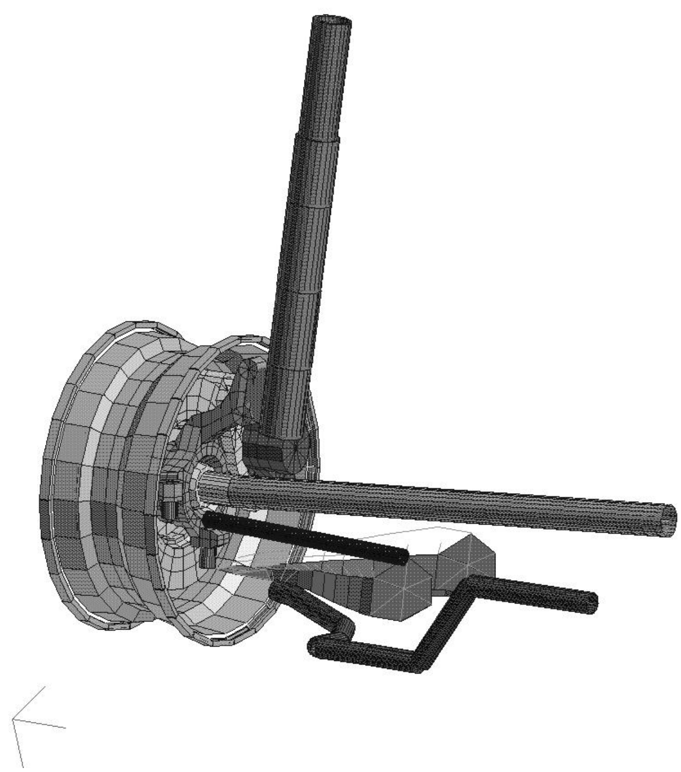
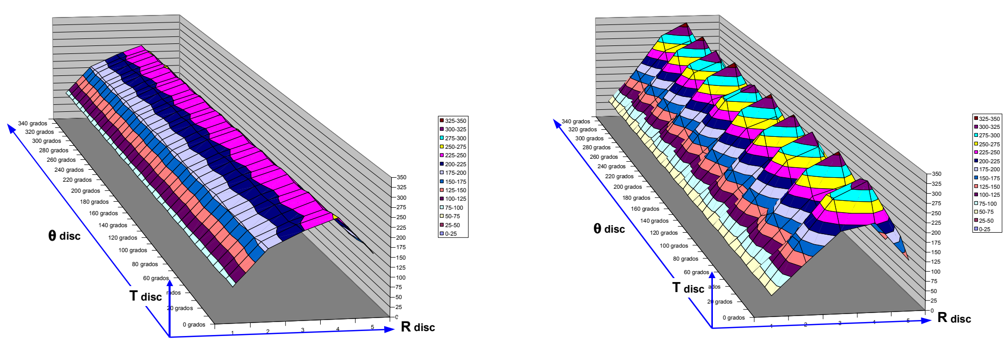


Figure 9: FEA model of the suspension.

These results showed a good correlation with the experimental observations in the tangential direction but simulation of the normal component needs the global approach described in the procedure. To validate that procedure, and having in mind that the normal vibration is easily simulated once the thickness variation of the disc is worked out, several tests were carried out focusing in the thermal behaviour along braking to verify the correlation of simulation figures to experimental results.

To start, the first experiment considers a constant drag velocity and a tangential vibration of a constant frequency of 221 Hz. This initial test seeks the verification of hot spots appearance due to a perturbation of the contact produced by a tangential vibration. The test represents a sliding of a vehicle at a speed of 140 Km/hr lasting for 4.6 seconds. Relating temperature field, ten hot spots were obtained on each face of the disc. This fact is shown on figure 10 were distributions of temperature on disc surface at the last revolution are shown on a rectangular area that represents the circular annulus of contact on disc.



Inner face Outer face

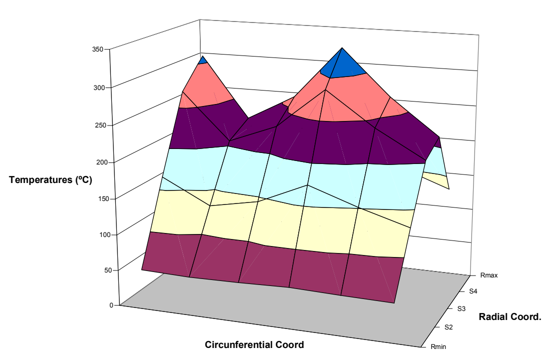
Figure 10: Temperatures after 4.6 seconds.

Hot spots are the result of a lack of homogeneity in temperature distribution on radial and circunferential directions. The least favourable situation is that in which the frequency of the tangential vibration is an exact multiple of disc rotation. In that case, maximum values of heat generation appear in the same positions on disc in every revolution and produce high values of temperature quickly. In the simulation, frequency of the vibration is ten times the rotational one, so heat generation has ten maxims, and as a consequence produces ten hot spots. As a result of hot spotting, contact pressure increases on them. With a higher pressure frictional heat is superior and hot spots receive even more heat flux, leading to thermoelastic instability.

In addition to this, it is observed an asymmetry in the magnitude of hot spots on both surfaces. Thermal buckling of the disc is the cause of that effect. The difference between both faces of the disc becomes more evident along braking because thermoelastic instability accelerates all thermal effects.

Pressure distributions under pads depend on the temperature field on disc. The difference between pressure with and without vibration represents the modification in the contact produced by hot spotting. In figure 11 it can be observed the distribution of temperatures in an area of the disc surface and the variation in contact pressure produced from a distribution under uniform conditions due to the appearance of hot spots. As a consequence it can be stated that hot spots condition contact pressure not only because of their position but also in their magnitude.

Temperature field



Pressure variation from uniform conditions

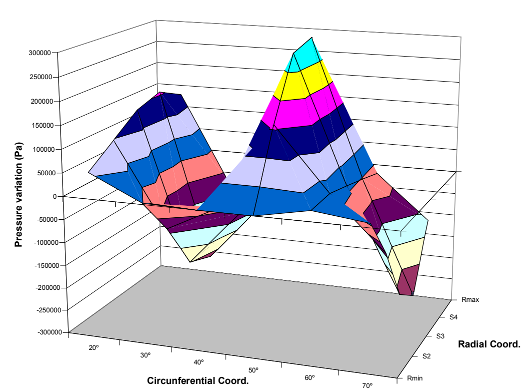


Figure 11: Effect of temperature on contact pressure.

To simulate braking effectively, deceleration of disc rotation has to be introduced. Results obtained in tests of a braking from a vehicle speed of 160 Km/hr to 90 Km/hr subject to tangential vibration are presented. The number of hot spots varies along braking as it is shown in figures 12 and 13. From the beginning of braking, when the number of hot spots is 9, there is a change in circunferential distribution to 10 hot spots after 40 revolutions being the speed at that instant 135 km/hr.. Towards the 80th round there is another transition to 11 hot spots being the vehicle speed 110 Km/hr.

This change in the number of hot spots corresponds to experimental results on new discs where the vibration level appears later in the braking and harmonic of the normal vibration is higher than with used discs. This is because the first revolutions take so little time and the magnitude of the thermal expansion is so low that the vibration reaches a noticeable magnitude when the number of hot spots is already high. However, for used discs the normal vibration has a noticeable magnitude from the beginning of braking and the harmonic has a lower number and remains constant. This is due to the residual effect of wear and thermal expansion of previous braking on the disc surface.

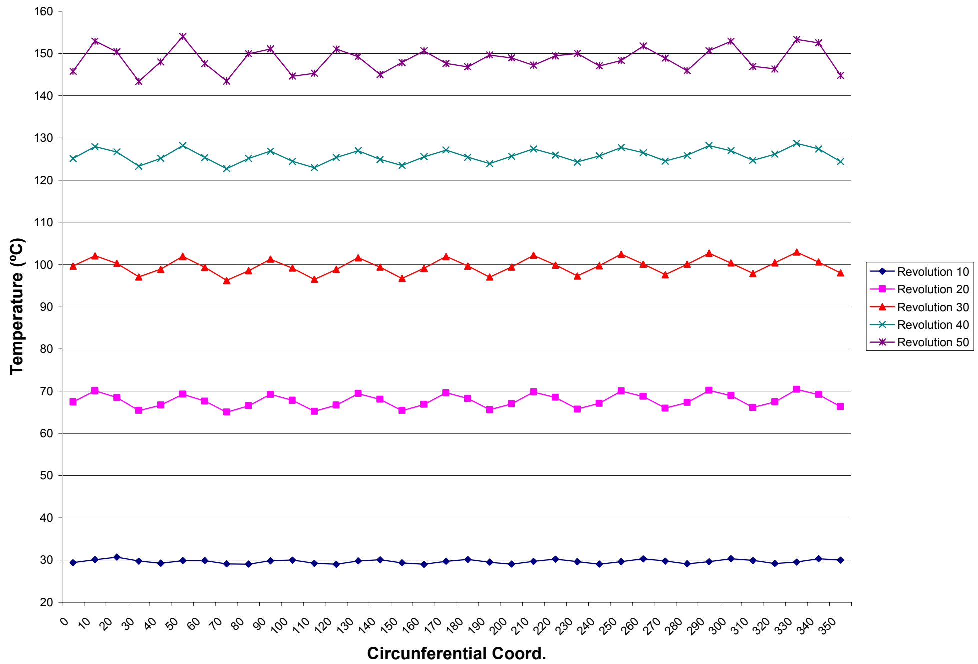


Figure 12: Temperature on outer disc surface every 10 revolutions.

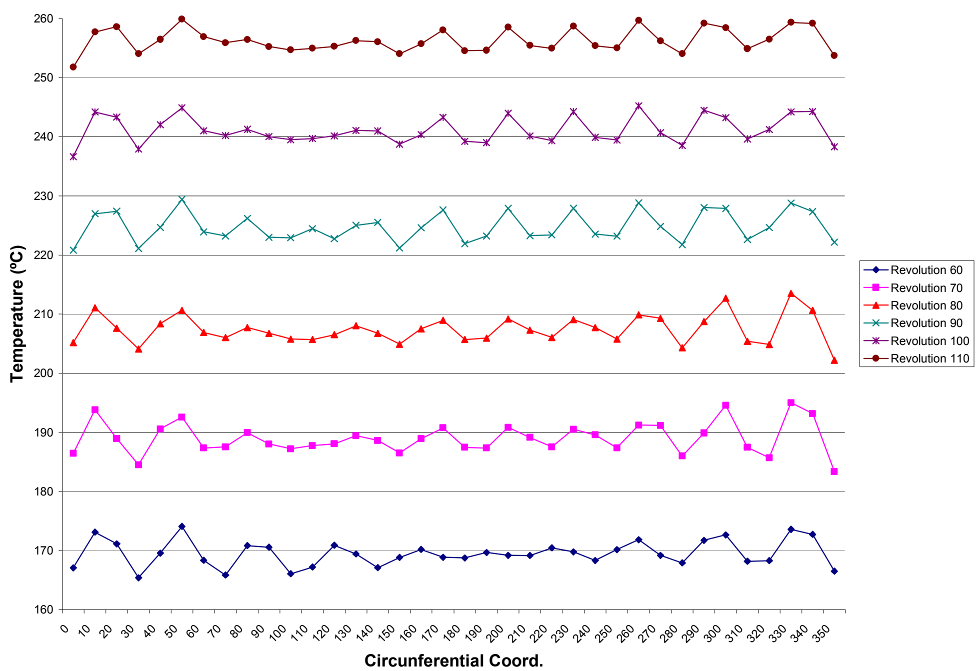


Figure 13: Temperature on outer surface every ten rounds from 50th revolution to 100th.

Conclusions

The procedure proposed provides a simulation of braking based in some factors such as, friction pad characteristics, speed, deceleration and tangential vibration. That simulation produces the temperature field on the disc along time subject to a judder vibration. The study of the magnitude of the critical velocity for hot spotting to arise is not the object of the procedure.

The mathematical model exposed in Avilés et al. (1995) produces a simulation of the tangential component of the vibration that shows a good correlation with experimental results. The origin of that vibration, which is characterised by a near constant frequency, is in the variation of the coefficient of friction versus sliding velocity with negative slope.

The normal component of the vibration, which has a constant harmonic, is produced by a rotor thickness variation due to the appearance of hot spotting. The procedure proves that the following hypothesis is correct: tangential vibration is the cause of a non-uniform heating of the disc, due to its effect on sliding velocity, that triggers the thermoelastic instability phenomenon which produces protuberances on disc surface. Nevertheless, thermoelastic instability is a phenomenon much more complicated than what it could be included in the procedure so long, so there may be other factors triggering or promoting the instability in addition to the tangential vibration.

The number of hot spots produced by thermoelastic instability phenomenon is a consequence of the relation between frequencies of the tangential vibration and rotation of the disc. An integer proportion ratio between them is the situation that promotes more the instability.

The magnitudes of the temperature on hot spots are validated against experimental data on models of new discs. Simulation of braking on used discs needs the inclusion of a wear mechanism and the consideration of residual stress to modify the FEA model.

The asymmetry in the distribution of hot spots on both faces of the disc observed in some experimental observations is not clearly detected, in spite of the fact that pressure distributions on both faces of the disc are asymmetric. A more detailed FEA model should be used in order to clarify the distribution of temperatures on the inner face but that would increase the computational cost of the procedure.

The finite element model discretization has a great influence in the number of hot spots found. The number of sectors chosen to discretizise the disc marks the number of nodes and hence the number of different temperature values detectable. With a disc disecretised in 36 sectors the maximum number of hot spots ideally detachable is half that number, that is 18. However, in practise the proportion reduces to a third for results with an acceptable quality, that is 12 hot spots at the most.

The difference in the dynamic behaviour of the braking system under judder conditions when new or used disc were mounted can be explained on the basis of this procedure. When hot spotting appears, friction pads slide over those pimps and a normal vibration is induced. This process takes some time where normal vibration does not exist, but when discs have been previously used a residual warping excites normal component from the beginning. A change in the number of hot spots was recorded in experiments when the discs employed were new. In the simulation, the model was that of a new disc, and the results show a good correlation to reality. In order to analyse the situation with a used disc, it is needed an approach that could consider the change in the profile of the contact surface due to wear and residual thermal stress modifying the FEA model. Current research is being carried out in that direction.

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References.

Abdelhamid M. K. (1997) "Brake judder analysis: case studies" *Sae Technical Paper* 972027 pp. 1225- 1230.

Avilés R., Hennequet G., Hernández A., Llorente J.I. (1995) "Low frequency vibrations in disc brakes at high car speed. Part I: experimental approach". *Int. J. of Vehicle Design*, Vol. 16, Nº6, pp. 542-555.

Avilés R., Hennequet G., Amezua E., Vallejo J. (1995) "Low frequency vibrations in disc brakes at high car speed. Part II: mathematical model and simulation". *Int. J. of Vehicle Design*, Vol. 16, Nº6, pp. 556-569.

Barber J.R. (1967) "The influence of thermal expansion on the friction and wear process". *Wear*, Vol. 10, pp. 155-159.

Barber J.R. (1987) "Stability of thermoelastic contact" *Proceedings of the Institution of Mechanical Engineers*. C157/87 pp. 981- 986.

Bill R.C., Ludwing L.P. (1980) "Wear of seal materials used in aircraft propulsion systems" *Wear*, Vol 59, pp. 165-189.

Bryant M., Burton R.A. (1982) "Thermal deformation in brushes for current passage through a steadily moving patch". *Wear*, Vol. 79, pp. 83-92.

Burton R.A. (1978) "Thermomechanical effects in sliding wear". *Proceedings of the Internatinal Conference on the fundamentals of tribology*. pp. 519-640.

Comninou M., Barber J.R. (1989) “Thermoelastic contact problems”. *Thermal stresses* Vol. III. Elsevier.

Dweib A.H., D´Souza A.F. (1990) “Self- excited vibrations induced by dry friction: part 1: experimental study”. *Journal of Sound and Vibration.* Vol. 137 (2). pp. 163-175.

Dweib A.H., D´Souza A.F. (1990) “Self- excited vibrations induced by dry friction: part 2: stability and limit-cycle analysis”. *Journal of Sound and Vibration*. Vol. 137 (2). pp. 177- 190.

Dow, T.H., Burton, R.A. (1972) “Thermoelastic instability of the sliding contact in the absence of wear”. *Wear*, Vol. 19, pp. 315-328.

Du, S. And Fash, J.W. (2000) “Finite element analysis of frictionally-excited thermoelastic instability in 3D annular disk”, *Int. J. Of Vehicle Design*, Vol. 23, Nos. 3,4; pp. 203-217.

Hartsock, D.L., Hecht, R.L. and Fash, J.W. (1999) “Parametric analyses of thermoelastic instability in disc brakes”, *Int. J. Of Vehicle Design*, Vol. 21, Nos 4/5 (Special Issue), pp. 510-526.

Hideo I. (1986) “Analysis of brake judder caused by thermal deformation of brake disc rotors” *SAE* 865131 pp. 2213- 2219.

Hu Y., Nagy L. (1997) “Brake squeal analysis by using nonlinear transient finite element method” *Sae Technical Paper* 971510. pp. 1- 8

Ishihara N., Nishiwaki M., Shimizu H. (1996) “Experimental analysis of low-frecuency brake squeal noise” *Sae Technical Paper* 962128. pp. 1- 6.

Kao, T.K., Richmond, J.W. and Douarre, A. (2000) “Brake disc hot spotting and thermal judder: an experimental and finite element study”, *Int. J. Of Vehicle Design*, Vol. 23, Nos. ¾, pp. 276-296.

Kennedy, F.E. (1984) “Thermal and thermomechanical effects in dry sliding”. *Wear*, Vol. 100, pp. 453-476.

Lawson, D.K. and Dow, T.A. (1985) “The sparking and wear of high current density electrical brushes”. *Wear*, Vol. 102, pp. 105-125.

Lee, K., Barber, J.R. (1993) “Frictionally excited thermoelastic instability in automotive disk brakes”. *Journal of Tribology* Vol. 115, pp. 607- 614.

Lee, K., Barber, J.R. (1994) “An experimental investigation of frictionally excited thermoelastic instability in automotive disk brakes under a drag brake application”. *Journal of Tribology* Vol. 116, pp. 409- 414.

Nack W., Joshi A.M. (1995) “Friction induced vibration: brake moan” *Sae Technical Paper* 951095. pp. 127- 133.

Nosseir T.A., Said M.A.R., El Nahas N.S., Abu el Fetouh G. (1998) “Significance of squeal in disc brake design”. *Int. J. of Vehicle Design* Vol. 19, No 1, pp 124-133

Ruiz Ayala, J.R., Lee, K., Rahman, M., Barber, J.R. (1996) “The effect of the intermittent contact on tha stability of thermoelastic sliding contact”. *Journal of Tribology* Vol. 118, pp. 102- 108.

Suzuki T., Kumemura Y., Okudai N. (1997) “Analysis of disc brake squeal” *Sae Technical Paper* 971038. pp. 89- 93