The symmetric Baker–Campbell–Hausdorff formula up to terms of degree 20 written in a Hall basis or a Lyndon basis

We write the symmetric version of the BCH formula

$$\log(\exp(X/2)\exp(Y)\exp(X/2)) = X + Y - \frac{1}{24}[X,[X,Y]] - \frac{1}{12}[Y,[X,Y]] + \cdots$$

in the form

$$\log(\exp(X/2)\exp(Y)\exp(x/2)) = \sum_{i\geq 1} z_i\, E_i, \tag{1}$$

where $z_i \in \mathbb{Q}$ ($i \geq 1$) and $\{E_i \ : \ i = 1,2,3,\ldots\}$ is a basis of the free Lie algebra $\mathcal{L}(X,Y)$ generated by the symbols X and Y, with elements E_i of the form

$$E_1 = X, \quad E_2 = Y, \quad \text{and} \quad E_i = [E_{i'}, E_{i''}] \quad \text{for each} \quad i \geq 3 \tag{2}$$

for appropriate values of the integers $i', i'' < i$ (for $i = 3, 4, \ldots$). We denote as $|i|$ the homogeneous degree of the element E_i of the basis, that is, $|1| = |2| = 1$, and $|i| = |i'| + |i''|$ for $i \geq 3$.

For the symmetric BCH formula, $z_i = 0$ for terms of even degree $|i|$.

In the file 'sbchHall19.dat', there is a table with five columns with integer entries that can be used to obtain the symmetric BCH formula up to terms of degree 19 in a classical Hall basis. The first three columns of the table determines the 58636 elements of a Hall basis E_i with homogeneous degree $|i| \leq 19$. The first column is simply $i = 1, 2, 3 \ldots, 58636$. The second and third column give the values (for each i) of i' and i'' respectively. The fourth and fifth columns contain integers p_i and q_i such that (1) holds with $z_i = p_i/q_i$.

In the file 'sbchLyndon20.dat', a table with five columns with integer entries is provided that similarly can be used to obtain the symmetric BCH formula up to terms of degree 19 in the Lyndon basis of the free Lie algebra $\mathcal{L}(X,Y)$ generated by the symbols X and Y.

In both cases, the elements of the basis are ordered (in the table) in such a way that $i < j$ if $|i| < |j|$. Thus, in order to identify the terms of homogeneous degree $n \leq 19$, it is enough to know the number l_n of elements E_i in the basis with homegeneous degree $|i| = n$ ($1 \leq n \leq 20$). The values of l_n for $n = 1, 2 \ldots, 19$ are: 2, 1, 2, 3, 6, 9, 18, 30, 56, 99, 186, 335, 630, 1161, 2182, 4080, 7710, 14532, 27594.