Further Results on Swarms Solving Graph Coloring

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introduction

- Flock dynamics + friendliness relations allow the modelling and solution of Color Graph problems
- Supervised dynamics: spatial goals representing colors
- Experimental results prove the approach is competitive with other CGP solvers

Introduction

- Salient feature
  - Each individual boid only has local information
    - Enemies are boids corresponding to nodes connected to the own node
    - Friends are boids at distance 2 or greater in the underlying graph
  - Global problem is modeled over the set of boids,
    - boids are unaware of the problem being solved

introduction

- Salient feature
  - There is no explicit definition of the energy/ cost function
  - The boids only energy model is their satisfaction level

Introduction

- Existence of a parameter measuring the complexity of the problem
  – Hostility: the ratio between average friends and enemies in a graph

Introduction

- Definition: k-coloration of a graph is a partition of its nodes into k partitions such that no two connected nodes are in the same partition.

Definition of swarm for CGP

- Given a graph
  - Each node corresponds to a boid
  - Each edge in the graph means that the connected boids are enemies (graph distance equal to 1).
  - Nodes at graph distance greater than 1 are friends.

Definitions

- Dynamics of the boids: linear combination of
  - Repulsion from enemies
  - Attraction to friends,
  - Attraction to color spatial goals
  - Satisfaction level

$$v = V_m \mathcal{N}\left(\alpha_{E_s} v_{E_s} + \alpha_{A_c} v_{A_c} + \alpha_{E_a} v_{E_a} + \alpha_n v_n\right)$$

Definitions

- Satisfaction level
  - Boids outside a color goal increase their stress in time
  - Highly stressed boids can attack enemies pushing them out of the spatial goal
  - This mechanism is intended to escape local minima

Supervised territorial flock

- Defining the spatial goals corresponding to colors, the dynamics of the boids are increased with a "goal term"

$$v = V_m \mathcal{N}\left(\alpha_{E_s} v_{E_s} + \alpha_{A_c} v_{A_c} + \alpha_{E_a} v_{E_a} + \alpha_n v_n + \alpha_g v_g\right)$$

Supervised territorial flock

- Each boid is aware of a region of radius R around it.
  - Counting enemies and friends in this region
  - Perceiving the goal satisfaction
- Satisfaction and attack are modeled as a Finite State Machine

Supervised territorial flock

- Boid dynamics
  - Avoiding enemies

$$v_{s_i}^E = \mathcal{N}\left(-\sum_{b_j \in \partial_i^Z \cap E_i} (p_j - p_i)\right)$$

  - Attraction towards friends

$$v_{c_i}^A = \mathcal{N}\left(c_i - p_i \quad \text{where} \quad c_i = \frac{1}{|\partial_i^R \cap A_i|} \sum_{b_j \in \partial_i^R \cap A_i} p_j\right)$$

Supervised territorial flock

- Satisfaction control

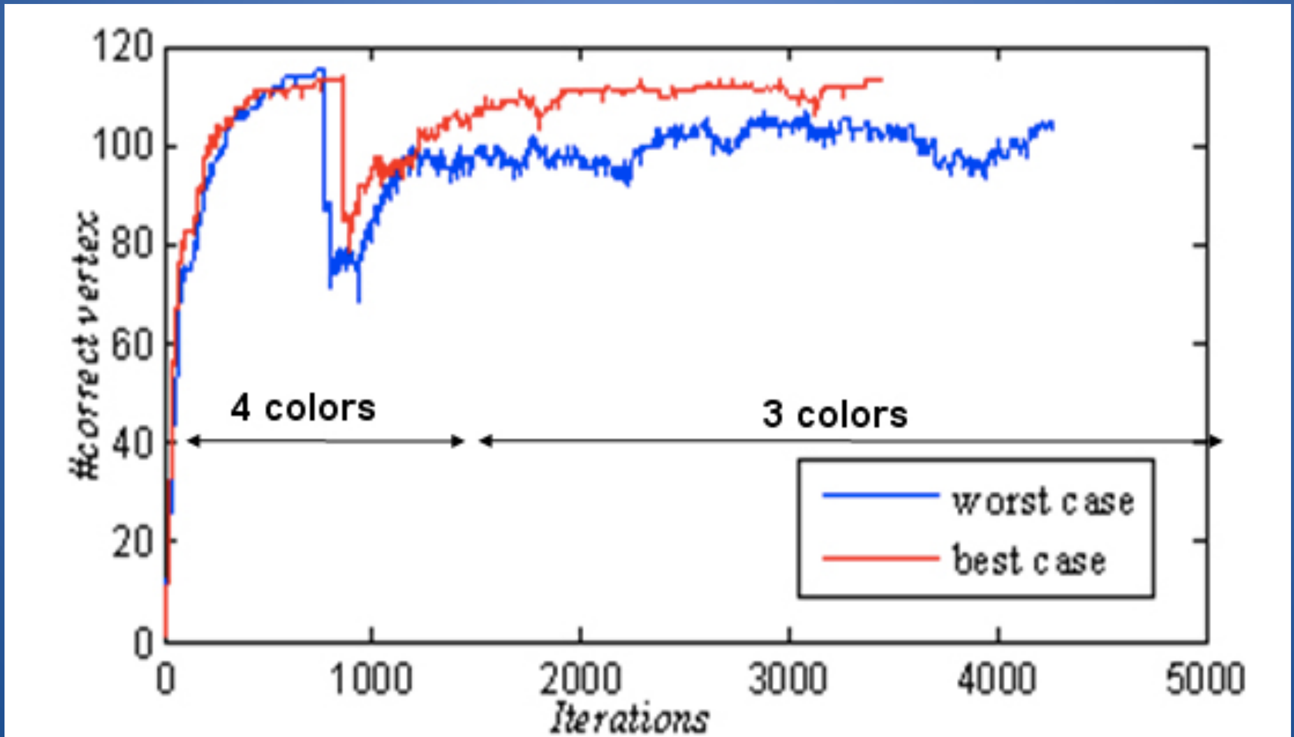
$$s_i\left(t+1\right)=\begin{cases} s_i\left(t\right)\dot{+}1 & in-goal-without-enemies\left(b_i\left(t\right)\right)\\ s_i\left(t\right)\dot{-}1 & in-goal-with-enemies\left(b_i\left(t\right)\right)\\ & \vee\, enemies-in-all-goals\left(b_i\left(t\right)\right)\\ s_i\left(t\right) & otherwise \end{cases}$$

Computational experiments

- 100 randomly generated trees
- Mizuno's generator of hard graphs for 3-coloratión by Brélaz heuristic,
  - 100 hard graphs with population P between 100 and 112 nodes.
- 100 random graphs generated by Kuratowski's theorem,
  - for which $e \leq 3P - 6$ being P the number of nodes and e the number of edges.
- Complete 100 node bipartite graphs

experiments

- Cascade coloration on hard graphs for Brelatz



experiments

- Average results

Mean values	Trees	Bipartite	Hard	Random
Number of friends	2.92	65.20	6.26	11.76
Number of enemies	2.00	33.83	3.48	3.54
Hostility	1.46	1.93	1.80	3.33
Final iteration	482.74	428.44	2345.70	1467.10
% successful runs	100.00	100.00	52.20	11.04
% well 3-colored nodes	100.00	100.00	99.21	92.16

experiments

- Correlation of hostility versus success
  - High negative correlation
  - Could be an indicator of difficulty for our approach

Pearson coefficient r	Final iteration	% successful runs	% well 3 colored
Hostility	0,26	-0,86	-0,97
Number of enemies	-0,52	0,50	0,35
Number of friends	-0,50	0,42	0,26

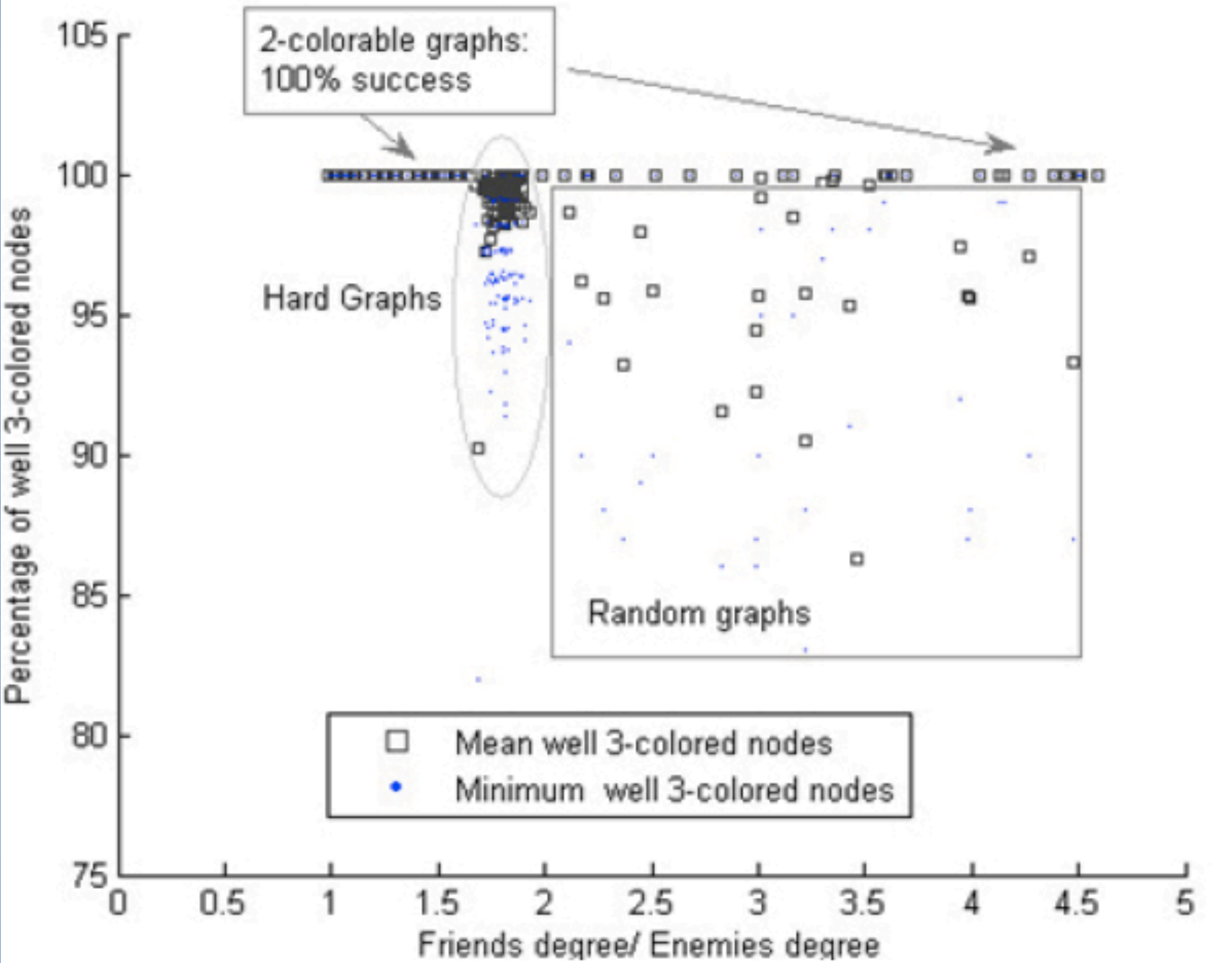


Fig. 2. Distribution of coloring success related to the Hostility quotient

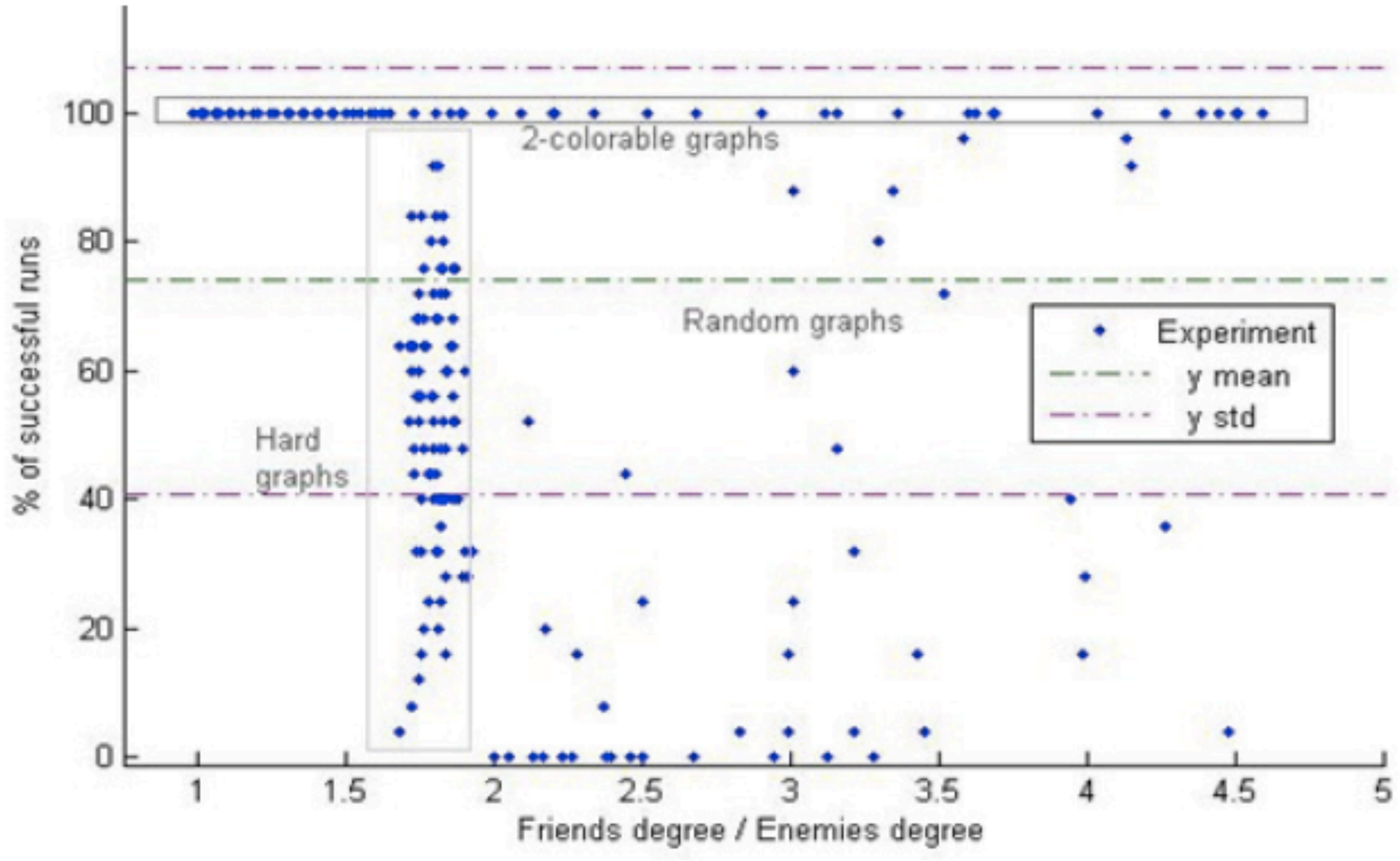


Fig. 3. Number of succesfull runs as a function of the Hostility quotient

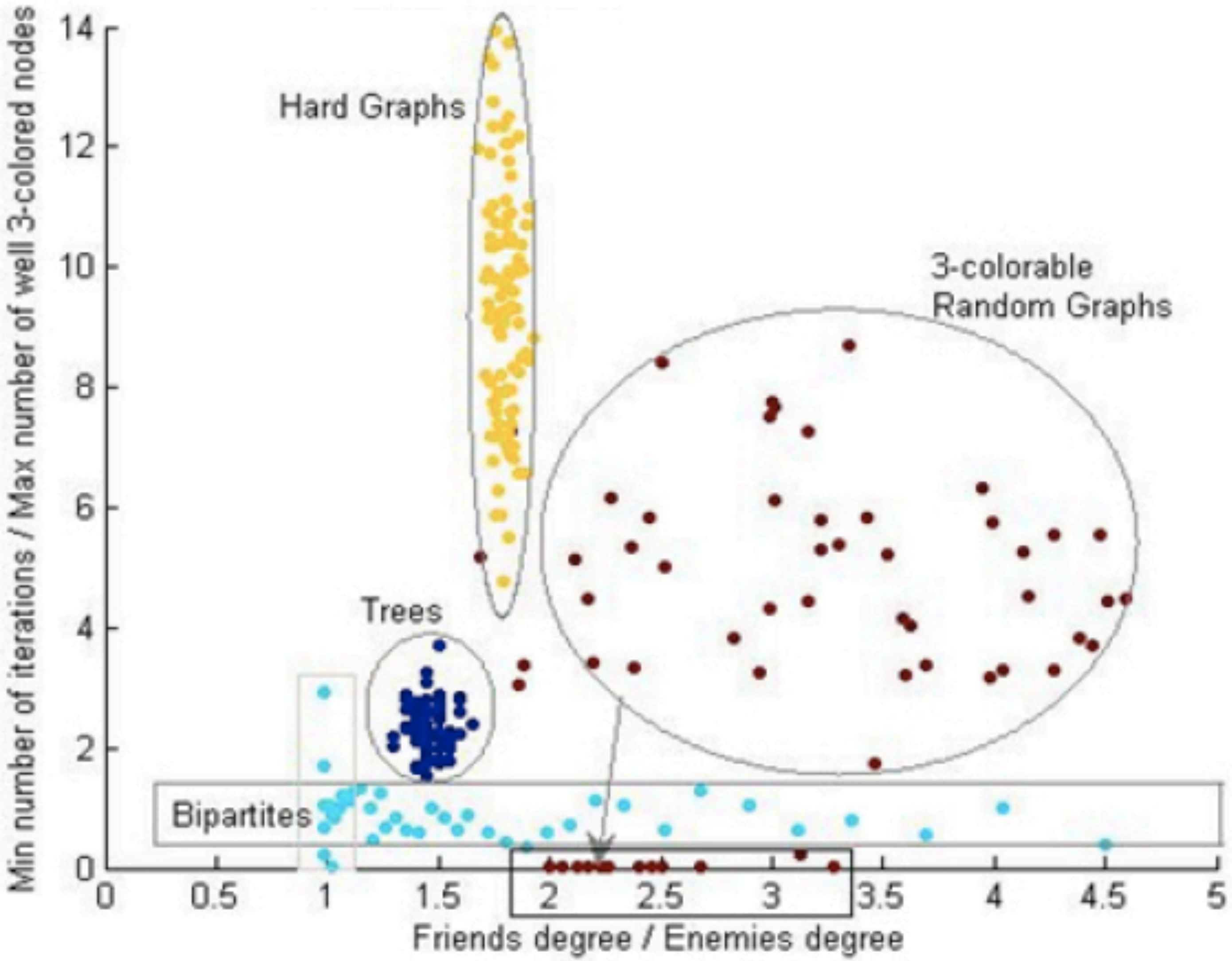


Fig. 4. Number of iterations versus the Hostility quotient

Conclusions

- AN approach to solve GCP modelled in terms of flocking birds that show territorial behavior.
- Extreme Territorial Flock Coloring (ETFC), based in goal velocity and attack is able to solve GCP.
- ETFC algorithm is able to color Mizuno's hard graphs for 3-coloration while Brélaz algorithm fail for all them.

- We found a global graph parameter, Hostility
  - the ratio of enemies to friends
- Hostility is a good characterization of the difficulty of solving GCP for each graph.
  - It has negative strong correlation with the success and time to solve the problem.