On hyperspectral morphology by lattice auto-associative memories supervised orderings

Miguel A. Veganzones

Grupo de Inteligencia Computacional
Universidad del País Vasco (UPV/EHU)

Outline

1. Motivation
   - Mathematical Morphology & Lattice Computing
   - Multivariate Mathematical Morphology
2. Multivariate MM by LAAM-supervised orderings
   - LAAM supervised orderings
   - Experiments with hyperspectral images

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Mathematical Morphology

- *Mathematical Morphology* (MM) has been very successful defining image operators and fillters for grayscale and binary images.
- *Lattice Theory* gives the most general formal background for MM.
- We call *Lattice Computing* to an extension of MM encompassing general data mining, neural computing, and machine learning applications, encompassing developments such as the *Lattice Auto-Associative Memories* (LAAM).

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Lattice Theory

- A non-empty set L endowed with an order relation $\leq$, satisfying reflexivity, antisymmetry and transitivity properties, is a *partially-ordered set* or *poset*, denoted $\mathscr{L} = \langle L; \leq \rangle$.
- $\mathscr{L}$ is a *lattice* when an infimum ($\wedge$) and a supremum ($\vee$) exist for any pair of elements of L, $\langle L; \leq \rangle \equiv \langle L, \vee, \wedge \rangle$.
- $\mathscr{L}$ is a *complete lattice* when every finite non-empty subset $H \subseteq L$ has infimum $\bigwedge H$ and supremum $\bigvee H$.
- A complete lattice has both a smallest element called *bottom*, denoted as $\perp$, and a greatest element called *top*, denoted as $\top$.

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MM and Lattice Theory

- Morphological operations can be described as mappings between complete lattices.
- From now on, we denote complete lattices by the symbols $\mathbb{L}$ and $\mathbb{M}$.
- The *erosion* and *dilation* operators are mappings $\varepsilon : \mathbb{L} \to \mathbb{M}$ and $\delta : \mathbb{L} \to \mathbb{M}$ commuting with the infimum $\varepsilon\left(\bigwedge Y\right) = \bigwedge_{y \in Y} \varepsilon\left(y\right)$ and supremum operators $\delta\left(\bigvee Y\right) = \bigvee_{y \in Y} \delta\left(y\right)$, respectively.

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Multivariate MM

- The extension of MM to color and multivariate images is not straightforward since high dimensional pixels do not have an endowed total order.
- There are different strategies to define an order on a multivariate data space:
  - *Lexicographic ordering*: ranks the vector components so that the order of vector components is evaluated sequentially according to this rank, until any ambiguities are solved.
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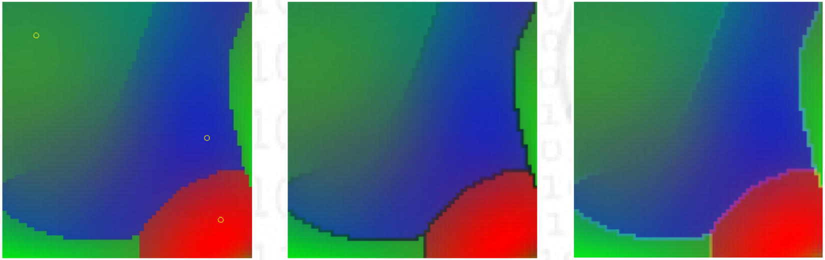

Original   Erosion   Dilation

Reduced orderings

- Reduced ordering (h-ordering): $\mathbf{x} \leq \mathbf{y} \Longleftrightarrow h(\mathbf{x}) \leq h(\mathbf{y})$ where $h : \mathbb{R}^n \to \mathbb{L}$.
- Supervised ordering: is a h-ordering that satisfies the conditions $h(\mathbf{b}) = \bot$, $\forall \mathbf{b} \in B$, and $h(\mathbf{f}) = \top$, $\forall \mathbf{f} \in F$, where $B, F \subset X$ such that $B \bigcap F = \emptyset$.
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Disambiguation

- h-functions are not necessarily injective.
- The induced h-ordering $\leq_h$ might be not a total order.
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- Given a training (foreground) set X, the LAAM h-mapping is defined as:

$$h_X(\mathbf{c}) = \zeta\left(\mathbf{x}^{\#}, \mathbf{c}\right)$$

where:

- Chebyshev distance: $\zeta(\mathbf{a}, \mathbf{b}) = \bigvee_{i=1}^{n} |a_i - b_i|$
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LAAM-supervised orderings (I)

- One-side LAAM-supervised ordering:
  $\forall \mathbf{x}, \mathbf{y} \in \mathbb{R}^n,\ \mathbf{x} \leq_X \mathbf{y} \Longleftrightarrow h_X(\mathbf{x}) \leq h_X(\mathbf{y})$.
  - The bottom element $\bot_X = 0$ corresponds to the set of fixed points of M_{XX} and W_{XX}, $h(\mathbf{x}) = \bot_X$ for $\mathbf{x} \in \mathscr{F}(X)$.
  - The top element is $\top_X = +\infty$.
- Relative LAAM-supervised ordering:
  $\forall \mathbf{x}, \mathbf{y} \in \mathbb{R}^n,\ \mathbf{x} \leq_r \mathbf{y} \Longleftrightarrow h_r(\mathbf{x}) \leq h_r(\mathbf{y})$ where $h_r(\mathbf{x}) = h_F(\mathbf{x}) - h_B(\mathbf{x})$.
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  - The mapping h_B maps the set of fixed points of the LAAM built with the background set into the bottom element: $\perp_a = h_B(\mathbf{b}); \mathbf{b} \in \mathscr{F}(B)$.
  - Conversely, h_F maps the set of fixed points of the LAAM built with the foreground set into the top element: $\top_a = h_F(\mathbf{f}); \mathbf{f} \in \mathscr{F}(F)$.

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Background/Foreground sets selection

- Assume that an Endmember Induction algorithm (EIA) provides a set of endmembers $E = \{\mathbf{e}_i\}_{i=1}^{p}$ from the image data.
- The matrix of distances $D = [d_{i,j}]_{i,j=1}^{p}$, where $d_{ij} = |\mathbf{e}_i, \mathbf{e}_j|$ is an appropriate distance between endmembers, i.e. the spectral angular mapping.
- One-side h-supervised ordering: the training data X consists of the endmember $\mathbf{e}_{k^*} \in E$ minimizing the average distance to the remaining endmembers: $k^* = \arg\min_k \left\{ \frac{1}{p-1} \sum_{i \neq k} d_{ik} \right\}_{i=1}^{p}$.
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Methodology

- Two well known benchmark hyperspectral scenes: Indian Pines and Pavia University.
- To compute Beucher gradients and ensuing watershed segmentations we have used:
  - The three proposed LAAM h-supervised orderings
  - A component-wise ordering
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Some results

- Global classification results of the Pavia University hyperspectral scene (disc $r = 3$): overall accuracy (OA), average accuracy (AA) and Kappa (κ) values.

Method		OA	AA	κ
Pixel-wise SVM		88.97	91.60	0.8565
SVM + NWHED	CW	92.87	94.83	0.9068
	LAAM_X	92.70	94.43	0.9045
	LAAM_a	92.81	94.46	0.9059
	LAAM_r	91.93	93.62	0.8944
SVM+WHED	CW	94.71	95.99	0.9306
	LAAM_X	94.90	96.27	0.9331
	LAAM_a	94.87	96.14	0.9326
	LAAM_r	94.69	95.83	0.9303

Conclusions

- We have introduced a Multivariate Mathematical Morphology using lattice computing techniques.
- Specifically, classification based on the LAAM reconstruction error measured by the Chebyshev distance induces an h-supervised ordering.
- Morphological operators and subsequent filters defined on them do not introduce false color results.
- The proposed spectral-spatial classification approach is comparable with the state of the art approaches in the literature.

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Further work

- Definition and application of multi-class h-supervised orderings, for better exploitation of the endmember information provided by EIAs.
- Other strategies for the selection of training sets, either in two-class foreground/background or multi-class approaches.
- Spectral-spatial segmentation can be further experimented and exploited.

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Thanks!

- Contact:
  - Dr. Miguel A. Veganzones.
  - E-mail: miguelangel.veganzones@ehu.es
  - Web: http://www.ehu.es/computationalintelligence