Gravitational Swarm approach for Graph Coloring

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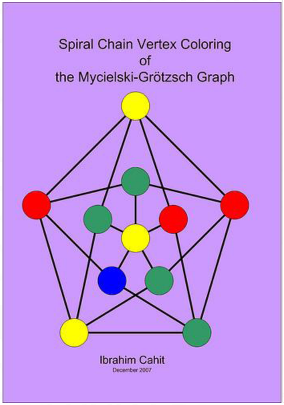
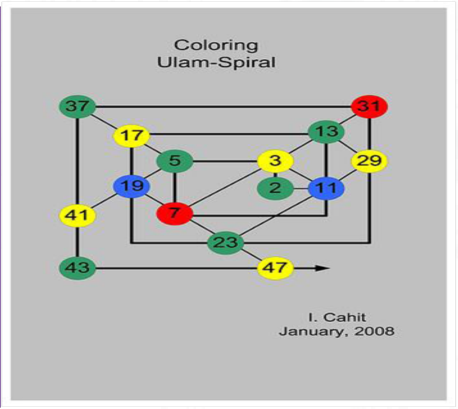
Nature Inspired Cooperative Strategies for Optimization,2011

Outline

Graph Coloring Problem

- The graph coloring problem GCP: consist in assigning a color to the vertices of a graph with the limitation that a pair of vertices that are linked cannot have the same color.





Swarm Intelligence.

- Swarm Intelligence: is a model where the emergent collective behavior is the outcome of a process of self-organization, where the agents evolve autonomously following a set of internal rules for its motion and interaction with the environment and the other agents.
  - There is no leader.
  - Has a high level of scalability.
  - The failure of some agents would not alter too much the overall system.

The model.

- A Swarm of agents move through a toric world.
- The agents are attracted by the goals, each goal represent a color.
- The agents have no information about the global problem, they only know the relationship friend or foe between them.
- If an agent arrives into a goal then it gets that color and stop moving.

The model.

- Let be $G = (V, E)$ a graph with V vertices and E edges.
- Let have $F = \{B, \overrightarrow{v}, C, \overrightarrow{v_g}\}$ where:
  - $B = \{b_1, b_2, ..., b_n\}$ is the group of SI agents.
  - $\overrightarrow{v}$ the speed vector in the instant t.
  - $C = \{1, 2, ..., k\}$ the number of colors.
  - $\overrightarrow{v_g}$ the attraction to the goal.

Fact

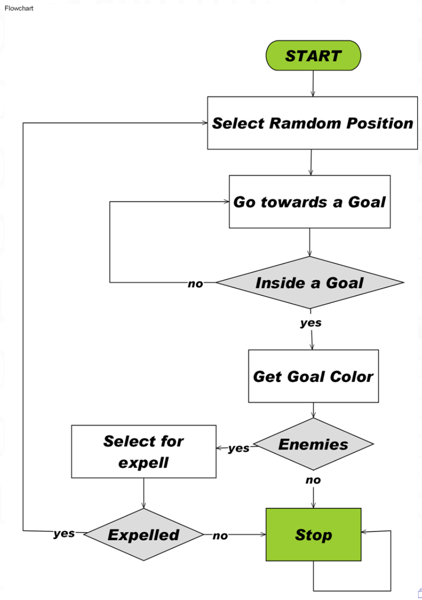
$$f = |\forall i, Cb_i \in C \text{ } and \text{ } \nexists j \mid enemy(b_i, b_j) \wedge Cb_i = Cb_j|$$

- This energy is the count of the graph nodes which have a color assigned.
- The predicate enemy is true if a pair of SI agents have an edge between them, so they cannot have the same color.
- Cb_i denotes the color of the SI agent associated with the target goal.

The dinamic of the system.

- The agents move through the space with a speed v.
- The goals attract the agents with a force proportionally to the euclidean distance.
- When an agent reaches to a goal:
  - There are no enemies, so the agent take the goal color and stops.
  - There are enemies, so the agent try to expell the enemies of the goal.
- When all the agents ara inside goals without enemies, the problem is solved.

Agent's Dinamic Flowchart.



Algorithm.

1. Supervised Coloring: the chromatic number is given to the algorithm.
2. Unsupervised Coloring: the goals are romeved and the agent move through the space creating clusters, the number of clusters is the chromatic number of the graph.
3. Coloring Waterfall

Algorithm 1 Coloring waterfall

```
Input: $G$ the graph to color.
Upper_bound = unsupervised_coloring($G$)
while Upper_bound > 1 do
   let Upper_bound = Upper_bound - 1
   solution = supervised_coloring($G$,(Upper_bound),max_time)
   if not solution then
      chromatic_number = (Upper_bound+1)
      exit while
   end if
   end while
```

Convergence issues.

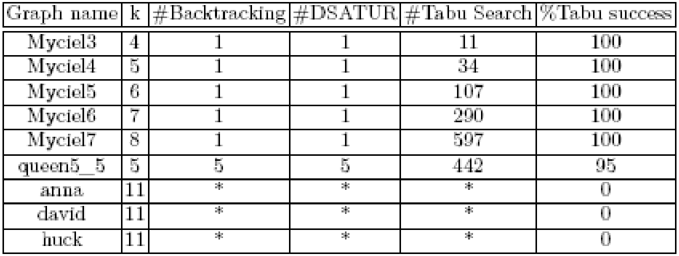
- The gravitational fields cover all the space, so all the agents moves towards a goal.
- If an agents arrive to a goal and can go inside then stoped.
- If all the agents speed is zero, then the system has converged to some fixed state.
- This state must be a solution of the problem, because:
  - An agent only stops if it is inside a goal without enemies.
  - If one agent never stops it means that the initial chromatic number is not a solution of the system.

Experimental results.

1. We have used DIMACS well known graphs.
2. We implement our GSI algorithm, and also three more algorithm to compare with:
   1. Backtraking.
   2. Brèlaz DSATUR
   3. Tabu Search
3. We let the algorithms a maximum number of steps or cicles to find a solution.
   1. And also a maximun time.
4. We have also compare our results with tests and bechmarks appearing in the bibliography.

Previous methos.

Graph coloring problem solved with Backtraking, DSATUR and Tabu Search:



Graph name	k	#Backtracking	#DSATUR	#Tabu Search	%Tabu success
Myciel3	4	1	1	11	100
Myciel4	5	1	1	34	100
Myciel5	6	1	1	107	100
Myciel6	7	1	1	290	100
Myciel7	8	1	1	597	100
queen5_5	5	5	5	442	95
anna	11	*	*	*	0
david	11	*	*	*	0
huck	11	*	*	*	0

Gravitational Swarm Intelligence.

Graph name	#Vertices	#Edges	Density	K	Success %	Average #Steps
anna	138	986	0.10	11	100	300
david	87	812	0.21	11	100	209
huck	74	662	0.22	11	100	84
jean	80	508	0.16	10	100	165
myciel3	11	20	0.36	4	100	21
myciel4	23	71	0.28	5	100	25
myciel5	47	236	0.21	6	100	97
myciel6	95	755	0.17	7	100	92
myciel7	191	2360	0.13	8	100	417
queen 5x5	25	160	0.53	5	100	302
1_fullins_3	30	100	0.23	4	100	37
1_fullins_4	93	593	0.14	5	100	76
1_fullins_5	282	3247	0.08	6	100	222
2_fullins_3	52	201	0.15	5	100	67
2_fullins_4	212	1621	0.07	6	100	176
miles250	128	387	0.04	8	100	317

Conclusions.

- We proposed a new algorithm for the Graph Coloring Problem using Swarm Intelligence.
- We have modeled the problem as a collection of agents trying to reach some of a set of goals.
- We have solved the GCP using a parallel evolution of the agents in the space.
- We have argued the convergence of the system, and we have demonstrated empirically that it provides effective solutions in terms of precision and computational time.

Future work.

- We will continue to test our algorithm on an extensive collection of graphs, comparing its results with state of the art heuristic algorithms.
- For future work, we have to improve our implementation of the algorithm to make it faster.
- Even though our algorithm finds the global optimum of the problem, we will search for new nature inspired behavior rules to improve the algorithm.

The End

Thanks for your attention.

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