



# A Novel Lattice Associative Memory Based on Dendritic Computing

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# Introduction

- **Associative memory** seems to be one of the primary functions of the brain
- In classical **pattern recognition**, patterns are viewed as column vectors in Euclidean space.

$$\mathbf{x} = (x_1 \dots x_n)' \in R^n$$

One **goal** in the theory of associative memories is for the memory to **recall** a stored pattern  $\mathbf{y} \in R^m$  when presented a pattern  $\mathbf{x} \in R^n$

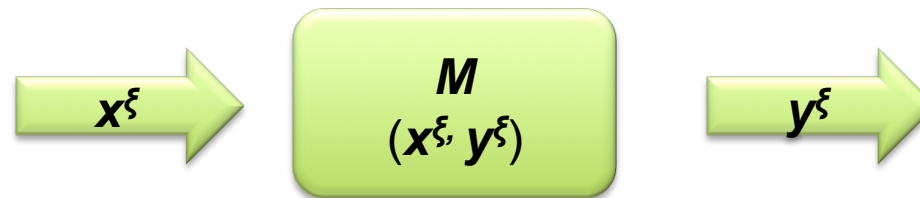
# Introduction

Suppose  $X = \{x^1, \dots, x^K\} \subset R^n$        $Y = \{y^1, \dots, y^K\} \subset R^m$

are **two sets** of pattern vectors with desired **association** given by the diagonal

$$\{(x^\xi, y^\xi) : \xi = 1, \dots, K\}$$

The **goal** is to **store** these pattern pairs  $(\mathbf{x}^\xi, \mathbf{y}^\xi)$  in some memory  $\mathbf{M}$  such that  $\mathbf{M}$  recalls  $\mathbf{y}^\xi$  when presented with the pattern  $\mathbf{x}^\xi$ .



If  $\mathbf{X}=\mathbf{Y}$  , then the memory  $\mathbf{M}$  is called an **auto-associative** memory, otherwise it is called a **hetero-associative** memory or simply an associative memory.

# The Dendritic Lattice Based Model of ANNs

A **lattice based neural network** is an ANN in which the basic neural **computations** are based on the **operations** of a **lattice ordered group**.

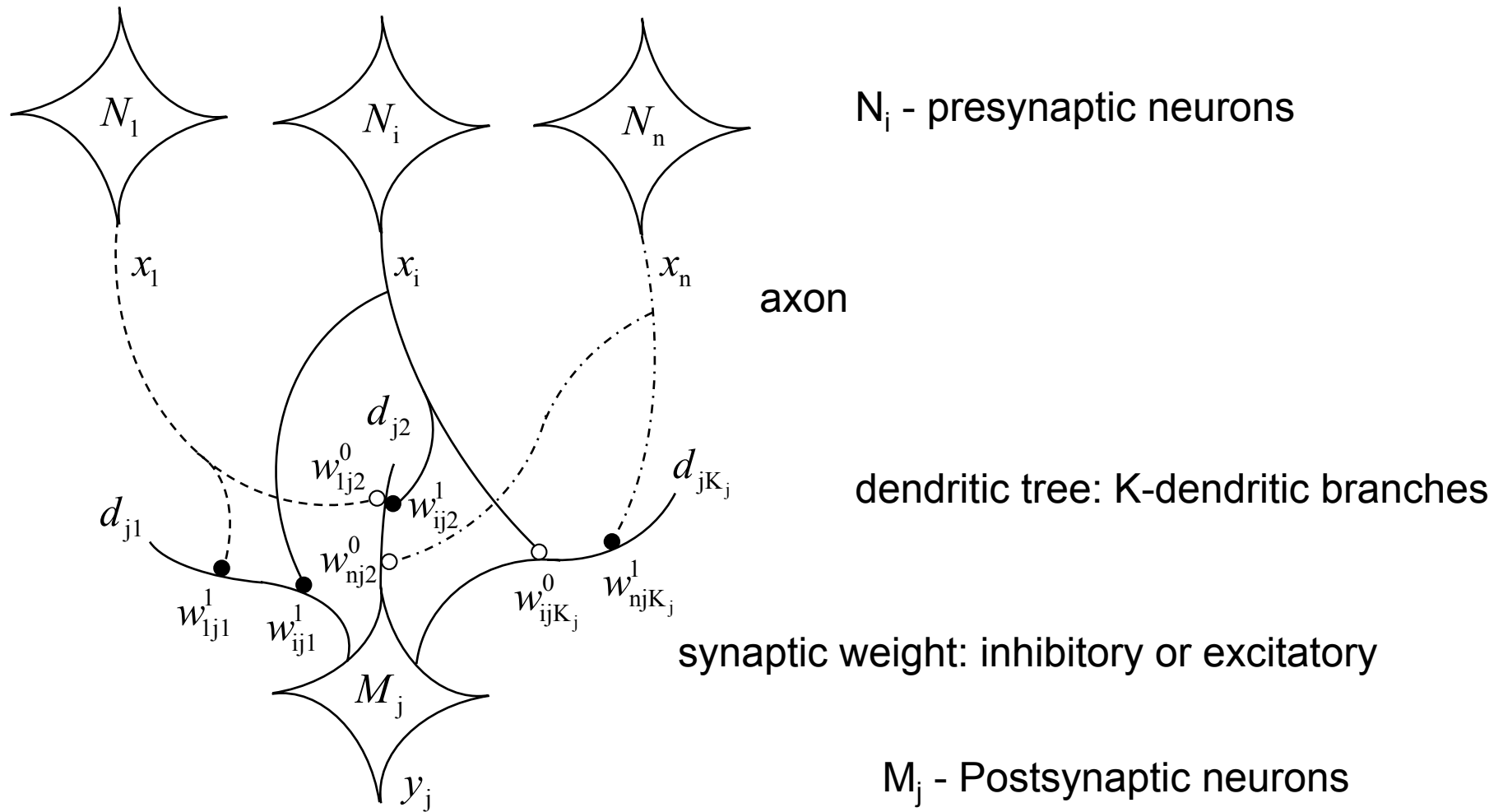
Lattice ordered group: a set  $L$  with an associated algebraic structure

$$(L, \vee, \wedge, +)$$

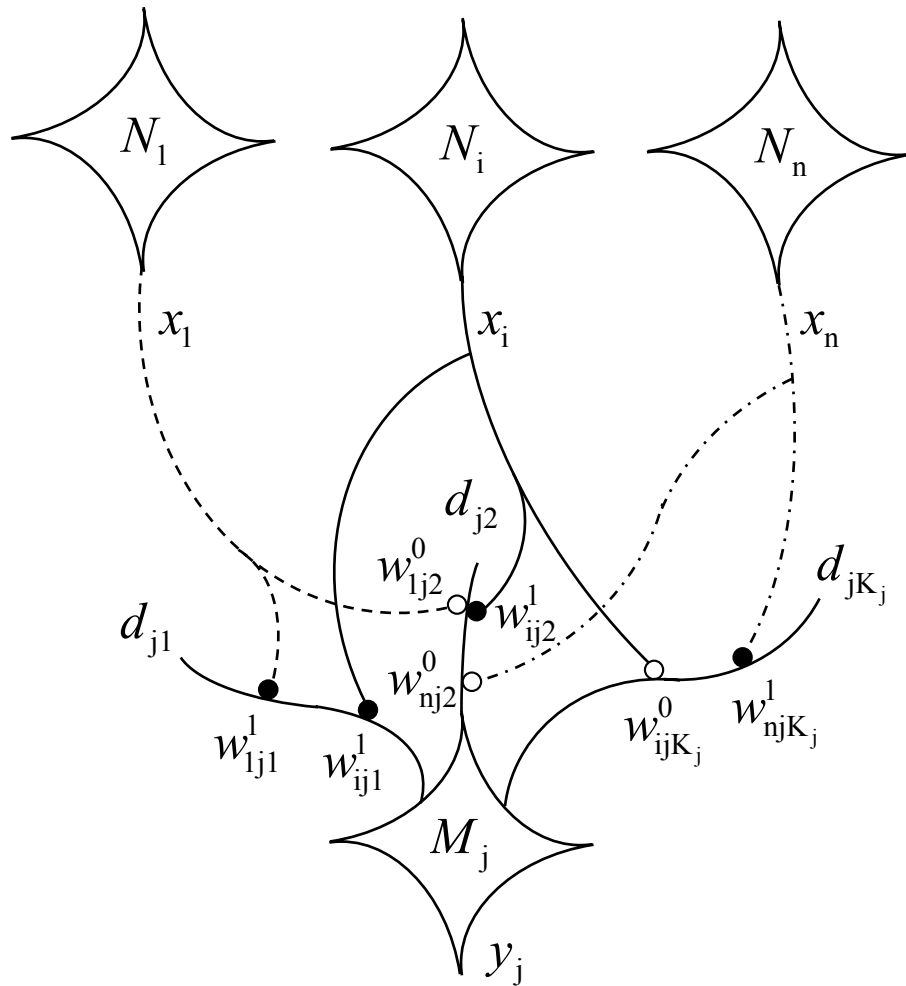
Where  $(L, \vee, \wedge)$  is a lattice and  $(L, +)$  is a group with the property that every group translation is isotone:

$$\text{if } x \leq y, \text{ then } a + x + b \leq a + y + b, \forall a, b \in L$$

# The neural pathways from the presynaptic neurons to the postsynaptic neuron



# The Dendritic Lattice Based Model of ANNs



The total **response** (or output) of **dendritic branch** to the received input at its synaptic sites is given by

$$\tau_k^j(\mathbf{x}) = p_{jk} \prod_{i \in I(k)} \prod_{l \in L(i)} (-1)^{1-l} (x_i + \omega_{ijk}^l)$$

The state of postsynaptic neurons  $M_j$

$$\tau^j(\mathbf{x}) = p_j \sum_{k=1}^{K_j} \tau_k^j(\mathbf{x})$$

dendritic tree:  $K$ -dendritic branches

$M_j$  - Postsynaptic neurons

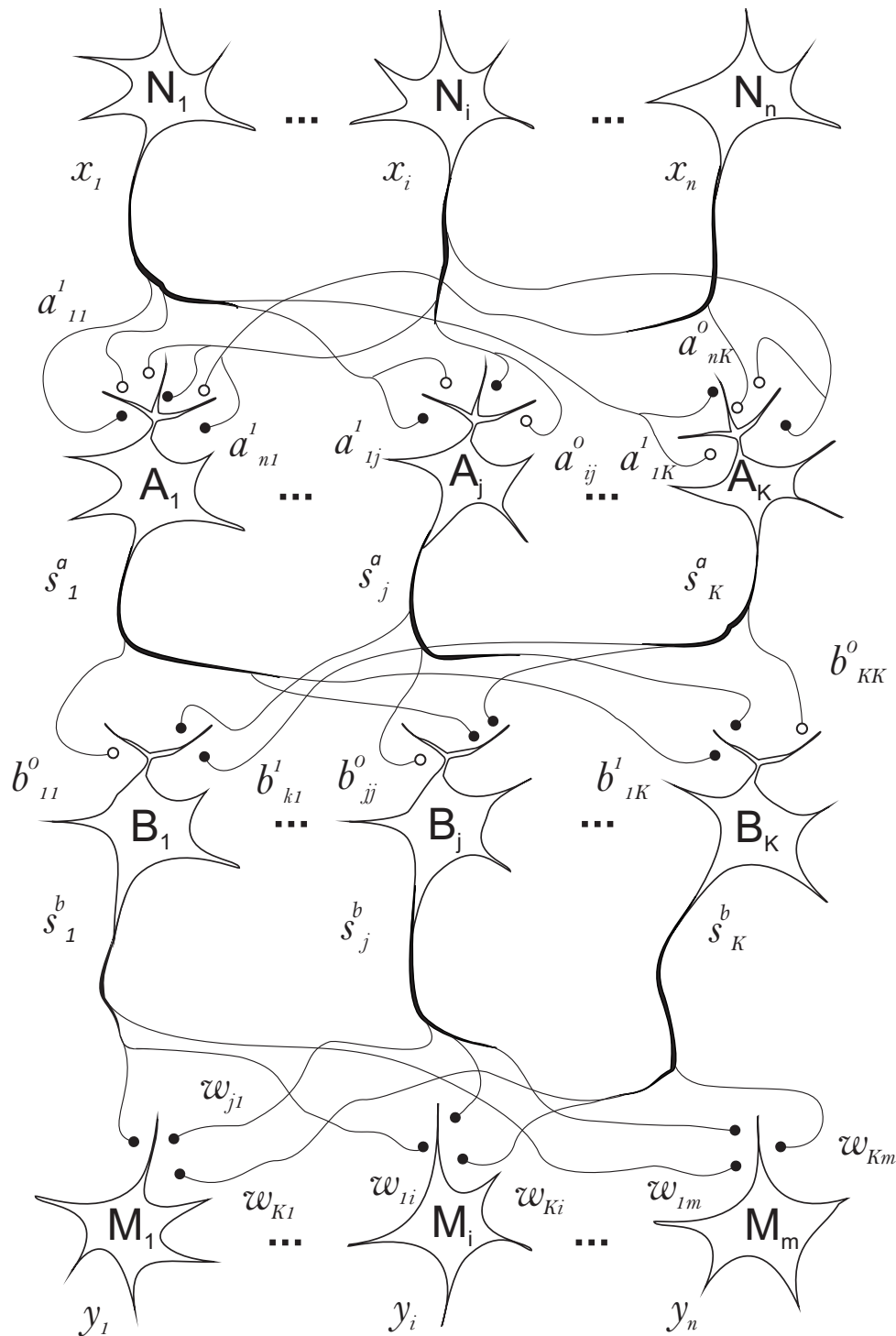
# A dendritic network

1.  $N_j$  - an input layer

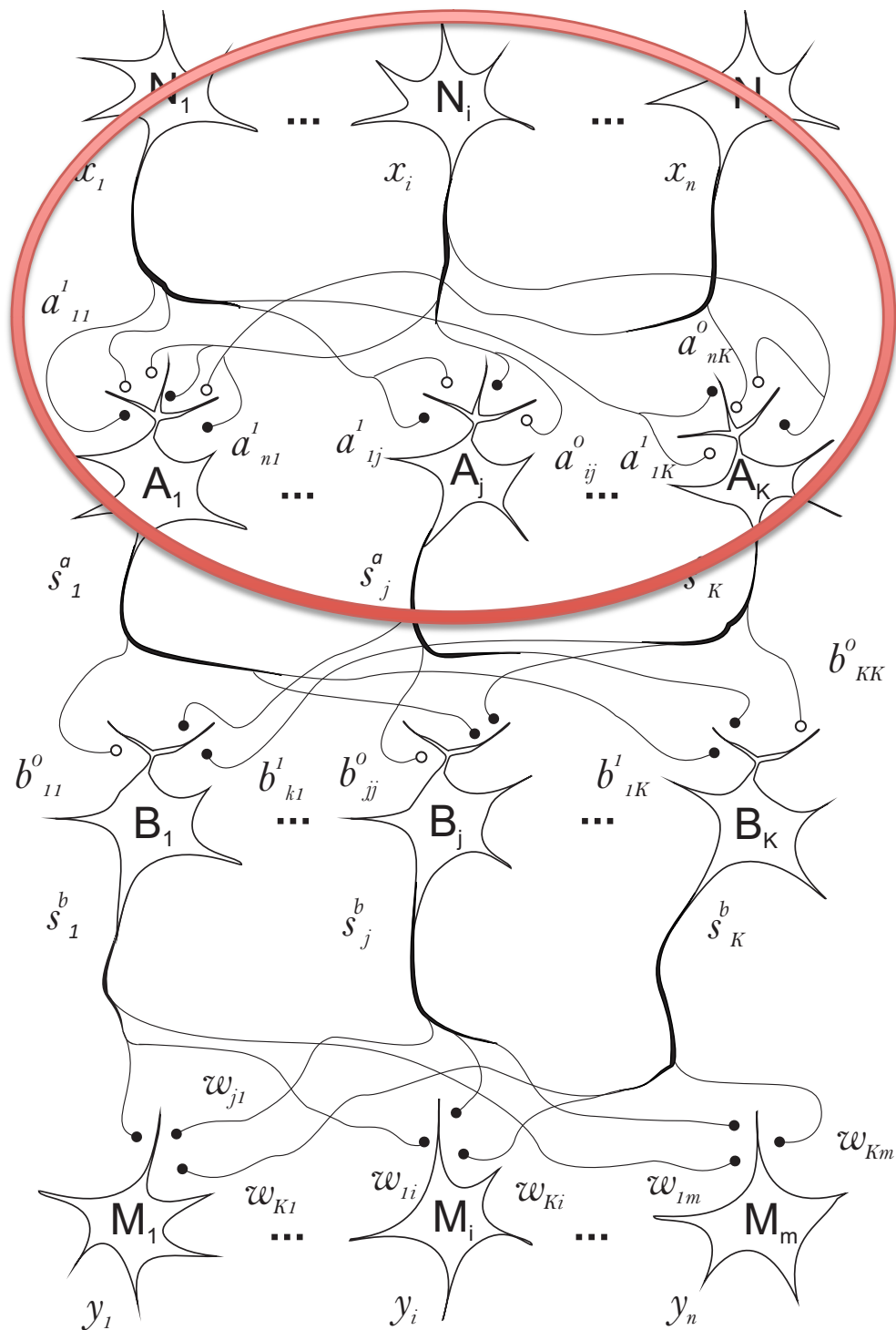
2.  $A_j$  - the first hidden layer

3.  $B_j$  - the second hidden layer

4.  $M_j$  - an output layer







## 2. $A_j$ - the first hidden layer

The synaptic weights:  $a_{ij}^l = -x_i$

$$\begin{aligned} \tau_i^j(\mathbf{x}) &= - \bigwedge_{l=0}^1 (-1)^{1-l} (x_i + a_{ij}^l) = \\ &= (x_i - x_i^j) \vee (x_i^j - x_i) \end{aligned}$$

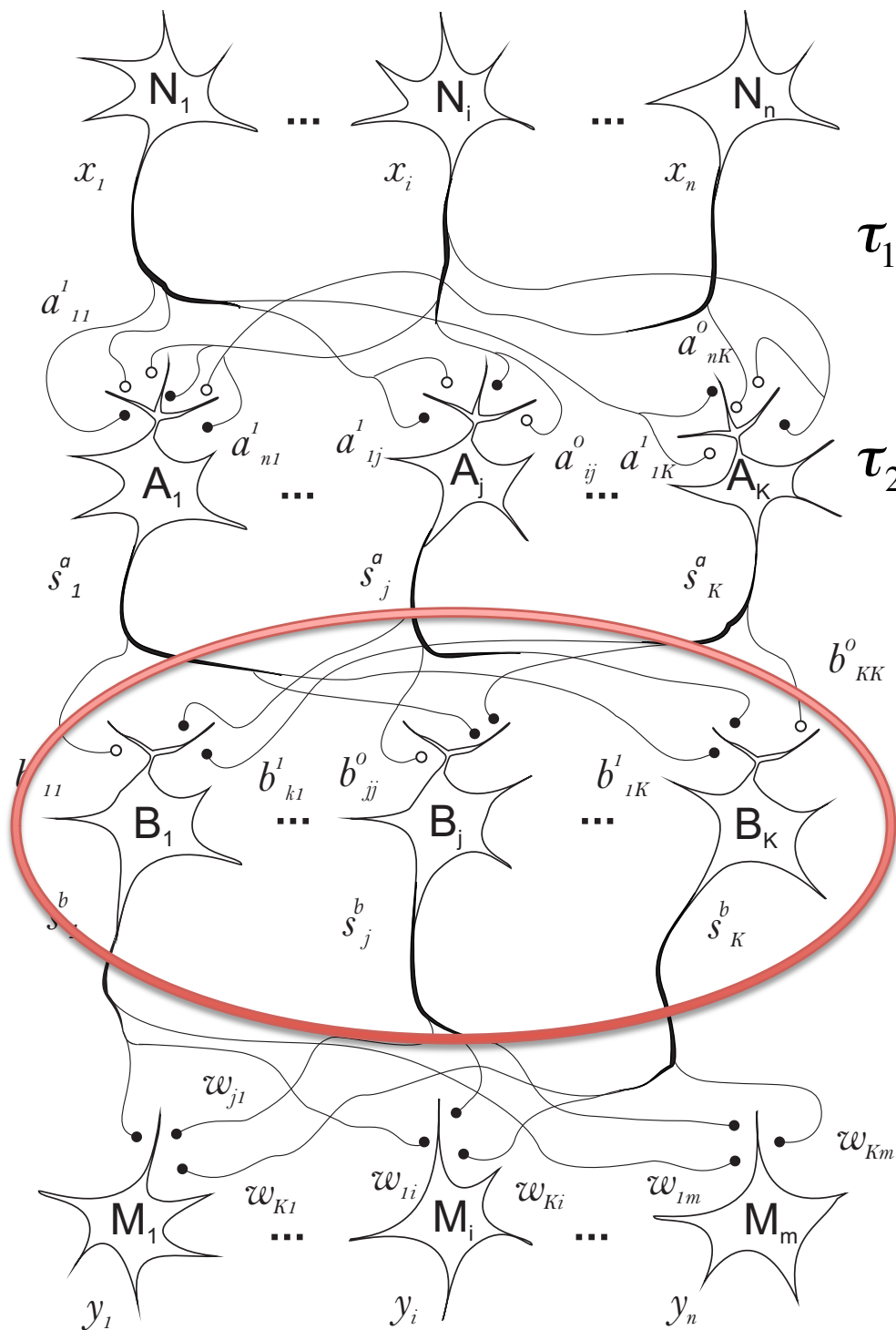
The state of neuron  $A_j$ :

$$\begin{aligned} \tau_A^j(\mathbf{x}) &= \sum_{i=1}^n (x_i - x_i^j) \vee (x_i^j - x_i) = \\ &= \sum_{i=1}^n |x_i - x_i^j| \quad \mathbf{L_1\text{-distance}} \end{aligned}$$

The identity function for A-layer neurons

$$f_A(z) = \begin{cases} z & \text{if } z \leq T \\ \infty & \text{if } z > T \end{cases}$$

The output  $s_A^j = f_A(\tau_A^j(\mathbf{x}))$



### 3. $B_j$ - the second hidden layer

The synaptic weights:  $b_{jj}^l = 0$

$$\tau_1^j(s_A) = \bigwedge_{i \in I(k)} \bigwedge_{l \in L(i)} (-1)^{1-l} (s_A^j + b_{jj}^l) = -s_A^j$$

$$b_{rj}^l = 0$$

$$\tau_2^j(s_A) = \bigwedge_{i \in I(k)} \bigwedge_{l \in L(i)} (-1)^{1-l} (s_A^r + b_{rj}^l) = -s_A^j$$

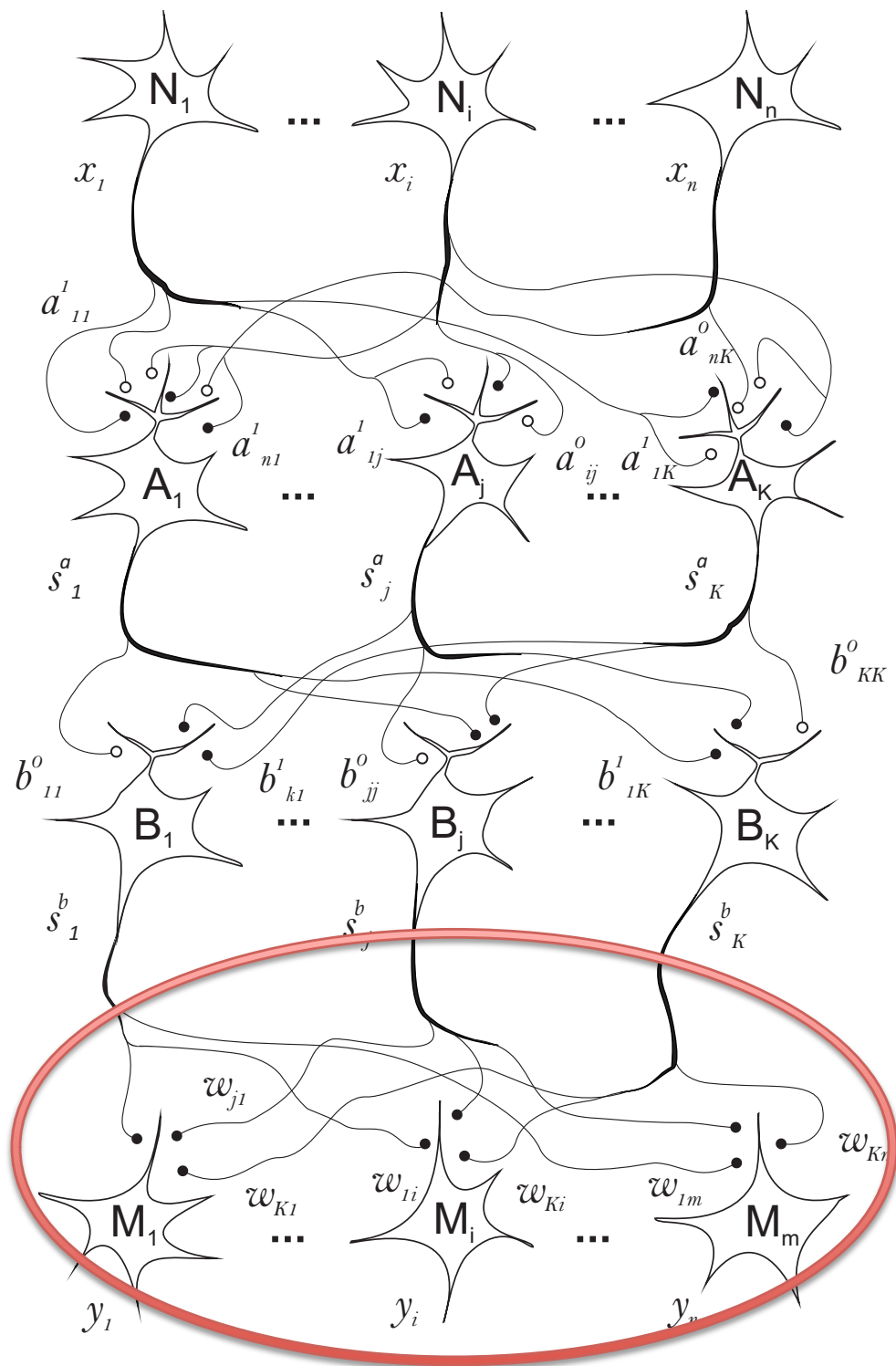
The state of neuron  $B_j$ :

$$\tau_B^j(\mathbf{x}) = \sum_{k=1}^2 \tau_k^j(s_A) = \bigwedge_{r \neq j} s_A^r - s_A^j$$

The identity function for B-layer neurons

$$f_B(z) = \begin{cases} 0 & \text{if } z > 0 \\ -\infty & \text{if } z \leq 0 \end{cases}$$

The output  $s_B^j = f_B(\tau_B^j(\mathbf{x}))$



4.  $M_i$  - an output layer

The synaptic weights:  $w^l_{ji} = y_i^j$

The state of neuron  $M_j$ :

$$\tau_1^j(s_B) = \mathbf{V}_{i=1}^K (s_B^j + w_{rj}^1) = \mathbf{V}_{i=1}^K (s_B^j + y_i^j)$$

The identity function for A-layer neurons

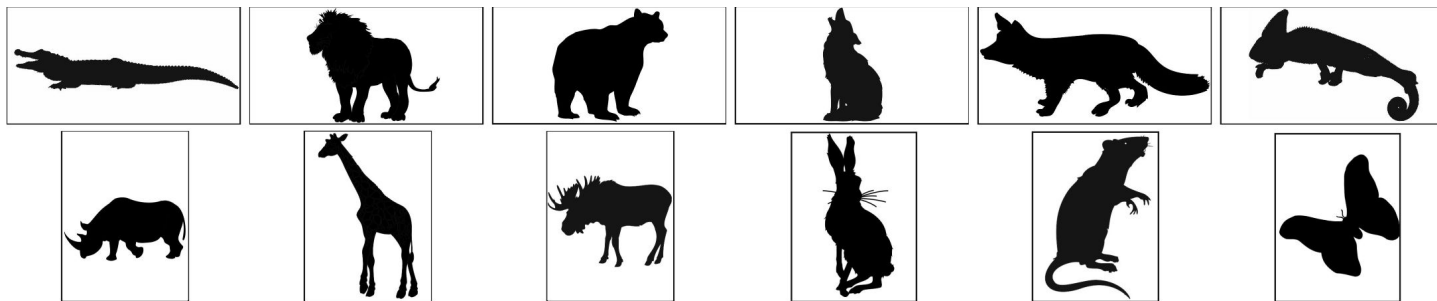
$$f_M(z) = z$$

The output  $y_i = \tau^i(s_B)$

# Experiments with Noisy and Corrupted Inputs

- Experiment 1

In this experiment, each of the sets X and Y consists of six **Boolean** exemplar patterns. The set X is derived from the set of six **700 × 350** with the set of **associated** output patterns is derived from the six **380 × 500**



# Experiment 1

Every pattern image was corrupted adding “salt and pepper” **noise**. Each noisy pixel of corrupted image is rounded to either 0 or 1 to preserve the **Boolean** character of the images. The range of the noise levels varied from 1% to 99% and was tested on all the images. **The DLAM shows perfect recall.**

50%

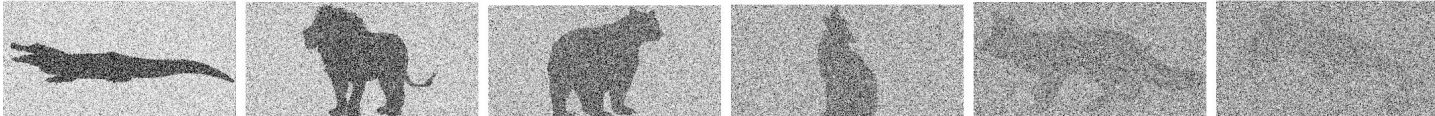
60%

70%

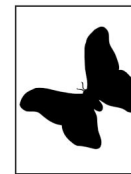
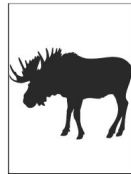
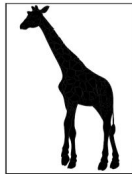
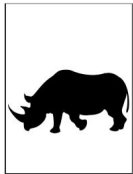
80%

90%

94%



**DLAM**



# Experiment 2

In this example we use a database of **grayscale** images. Both **predator** and **prey** images are of size **265×265**.



# Experiment 2

- We **simulate noise** pattern acquisition and **tested** image corruption changes: camera motion, Gaussian noise, the application of a circular averaging filter, a morphological erosion with a line as structuring elements and a morphological dilation with elipsoid as structuring elements.



The DLAMs perfect recall

# Experiments with Noisy and Corrupted Inputs

In the Experiment 1 and 2, the threshold  $T$  for the activation function given by

$$f_A(z) = \begin{cases} z & \text{if } z \leq T \\ \infty & \text{if } z > T \end{cases}$$

was set to  $T = \infty$

With this threshold, the **DLAM** performance is very **impressive** in that associations can be recalled even at **99% random noise** levels of the input data.

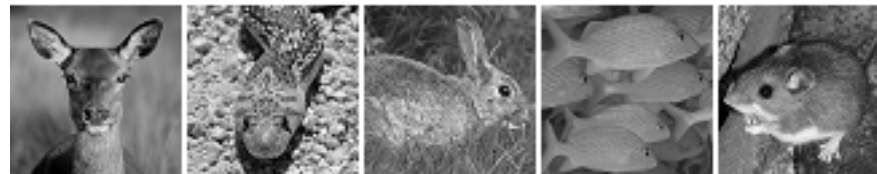
However, images with such high and even lower noise levels of corruption **can not be identified by a human** observer when not first shown the original pattern images.



# Experiment 3



DLAM  
 $L_1$ -distance



To **avoid misclassification** of intruders, a threshold  $T$  is determined as  $T < \infty$

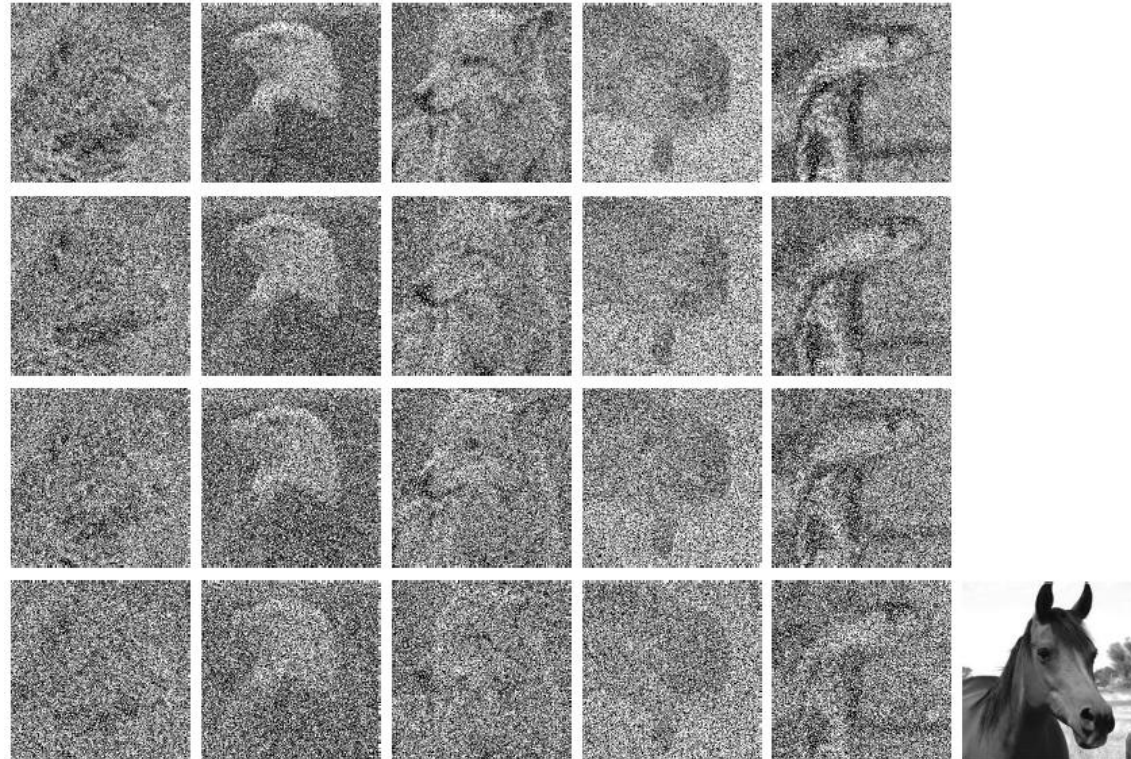
# Experiment 3

Noise	0%	50%	60%	63%	65%	70%	80%	90%	100%	Horse
Leopard	0	4470	5374	5634	5813	6297	7158	8066	8932	<b>5667</b>
Eagle	0	4492	5348	5626	5844	6252	7154	8080	8947	6293
Wolf	0	4484	5396	5663	5832	6265	7177	8051	8965	6367
Dolphin	0	4452	5385	5640	5816	6281	7162	8059	8952	6713
Cobra	0	4487	5277	5621	5801	6292	7147	8052	8946	6189
<b>Avarage</b>	0	4477	5276	<b>5637</b>	5821	6277	7160	8062	8948	6246

The **nearest** predator is the **leopard**.

Thus, the **deer** will be associated with the horse when the **horse** is used as **input** to the DLAM.

# Experiment 3



Computing  $T_j = d_1(x^j, \bar{x}^j)$  for each  $j$  and each noise level as well as  $d_1(x^1, x) = 5667$ , where  $x^1 = leopard$  and  $x = horse$ , and  $T = \frac{1}{5} \sum_{j=1}^5 T^j = 5637$  when  $\bar{x}^j$  represents as 63% corruption of  $x^j$

Thus,  $T$  eliminates  $x$  as an intruder.

# Conclusions

- We present a new **hetero**-associative **lattice** memory based on **dendritic computing**.
- We report experimental results showing that this memory exhibits extreme **robustness** in the presence of various types of **noise**.



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