Stochastic Stability Analysis of the Linear Continuous and Discrete PSO Models

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<u>Análisis estadístico de la atracción central estocástica</u>

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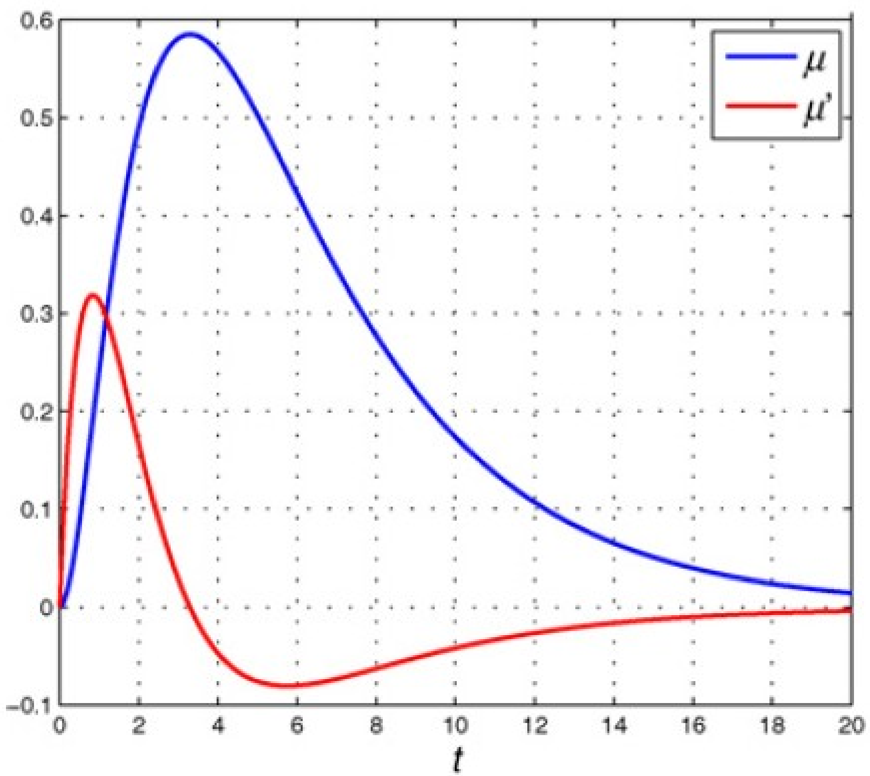
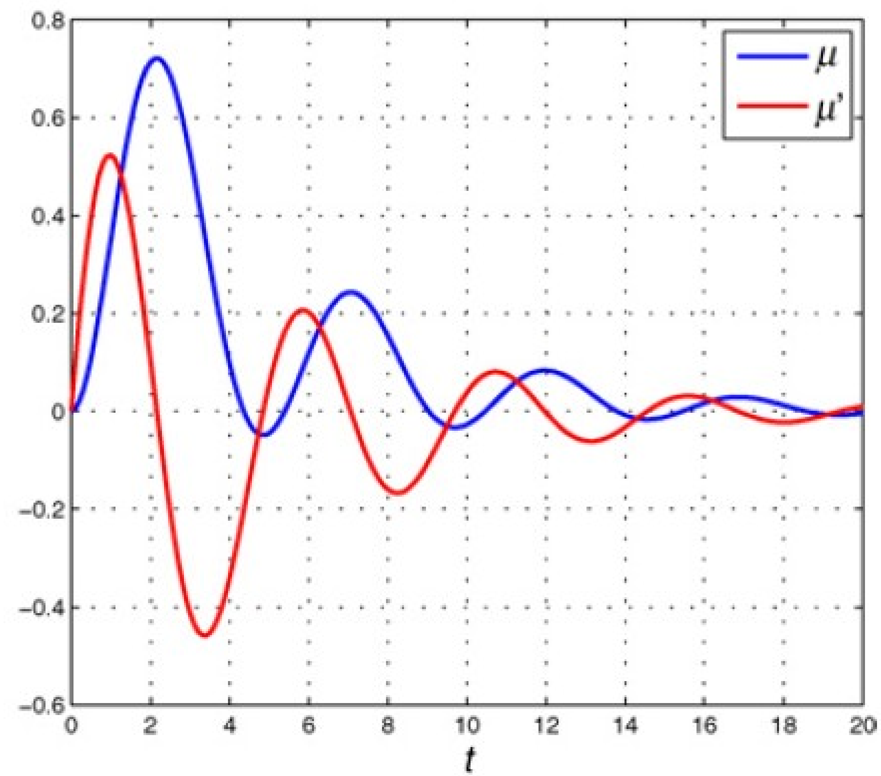
Particle Swarm Optimization

- Este artículo trata de encontrar un analisis estiocastico lineal de una curva atraida por un centro de atracción estocastico.





El eje x representa el tiempo, el eje y representa la posición

PSO Basics

La formulación es sencilla, se tiene una velocidad, con la cual se obtiene la posición. Se evalua la posición de cada individuo con una función de coste $c(x_i)$.

$$\mathbf{v}_i^{k+1} = \omega \mathbf{v}_i^k + \phi_1(\mathbf{g}^k - \mathbf{x}_i^k) + \phi_2(\mathbf{l}_i^k - \mathbf{x}_i^k)$$

$$\mathbf{x}_i^{k+1} = \mathbf{x}_i^k + \mathbf{v}_i^{k+1}$$

Con $\phi_1 = r_1 a_g \quad \phi_2 = r_2 a_l \quad r_1, r_2 \to U(0,1) \quad \omega, a_l, a_g \in \mathbb{R}.$

Una region de estabilidad de primer orden puede ser:

$$S_D = \left\{ (\omega, \overline{\phi}) : |\omega| < 1,\ 0 < \overline{\phi} < 2(\omega + 1) \right\}$$


Punto Negro

La siguiente diferencia vectorial involucra a cada uno de los nidividuos del Swarm

$$\begin{cases} \mathbf{x}_i^{k+1} + (\phi - \omega - 1)\mathbf{x}_i^k + \omega \mathbf{x}_i^{k-1} = \phi \mathbf{o}_i^k = \phi_1 \mathbf{g}^k + \phi_2 \mathbf{l}_i^k \\ \mathbf{x}_i^0 = \mathbf{x}_{i0} \\ \mathbf{x}_i^1 = \varphi\left(\mathbf{x}_i^0, \mathbf{v}_{i0}\right) \end{cases}$$

Las trayectorias se estabilizan en torno a :

$$\mathbf{o}_i^k = \frac{a_g \mathbf{g}^k + a_l \mathbf{l}_i^k}{a_g + a_l}$$

a centered discretization in acceleration

$$\mathbf{x}_i''(\mathbf{t}) \simeq \frac{\mathbf{x}_i(t+\Delta t) - 2\mathbf{x}_i(t) + \mathbf{x}_i(t-\Delta t)}{\Delta t^2}$$

and a regressive schema in velocity

$$\mathbf{x}_i'(t) \simeq \frac{\mathbf{x}_i(t) - \mathbf{x}_i(t-\Delta t)}{\Delta t}$$

Difference equation (4) can be considered the result of a centered discretization in acceleration

$$\mathbf{x}_i''(\mathbf{t}) \simeq \frac{\mathbf{x}_i(t+\Delta t) - 2\mathbf{x}_i(t) + \mathbf{x}_i(t-\Delta t)}{\Delta t^2} \tag{5}$$

and a regressive schema in velocity

$$\mathbf{x}_i'(t) \simeq \frac{\mathbf{x}_i(t) - \mathbf{x}_i(t-\Delta t)}{\Delta t} \tag{6}$$

in time $t = k \in \mathbb{N}$, applied to the following system of stochastic differential equations:

$$\begin{cases} \mathbf{x}_i''(t) + (1-\omega)\,\mathbf{x}_i'(t) + \phi \mathbf{x}_i(t) = \phi_1 \mathbf{g}\,(t) + \phi_2 \mathbf{l}_i\,(t) \quad t \in \mathbb{R} \\ \mathbf{x}_i(\mathbf{0}) = \mathbf{x}_{i0} \\ \mathbf{x}_i'(\mathbf{0}) = \mathbf{v}_{i0} \end{cases}$$

$$\begin{aligned} &\mathbf{v}_i(t+\Delta t) = (1-(1-\omega)\,\Delta t)\,\mathbf{v}_i\,(t) + \phi_1 \Delta t\,(\mathbf{g}\,(t) - \mathbf{x}_i(t)) \\ &+\phi_2 \Delta t\,(\mathbf{l}_i\,(t) - \mathbf{x}_i(t))\,,\, \mathbf{x}_i(t+\Delta t) = \mathbf{x}_i(t) + \Delta t.\mathbf{v}_i(t+\Delta t) \\ &\ t,\ \ \Delta t \in \mathbb{R} \\ &\mathbf{x}_i\,(0) = \mathbf{x}_{i0}\ \ \mathbf{v}_i\,(0) = \mathbf{v}_{i0}. \end{aligned}$$

Stochastic analysis

$$\frac{d\mathbf{Y}(t)}{dt} = A\mathbf{Y}(t) + \mathbf{b}(t) \tag{9}$$
$$\mathbf{Y}(0) = \begin{pmatrix} x(0) \\ x'(0) \end{pmatrix}$$

where

$$\mathbf{Y}(t) = \begin{pmatrix} x(t) \\ x'(t) \end{pmatrix} \quad A = \begin{pmatrix} 0 & 1 \\ -\phi & \omega - 1 \end{pmatrix}$$
$$\mathbf{b}(t) = \begin{pmatrix} 0 \\ \phi_1 g(t) + \phi_2 l(t) \end{pmatrix}.$$

Interpreted in the mean square sense[1] [19], the mean of the stochastic process $x(t)$, $\mu(t) = E(x(t))$, fulfills the following ordinary differential equation:

$$\begin{aligned} &\mu''(t) + (1-\omega)\mu'(t) + \overline{\phi}\mu(t) \\ &= E(\phi o(t)) = \frac{a_g E(g(t)) + a_l E(l(t))}{2} \quad t \in \mathbb{R} \\ &\mu(0) = E(x(0)) \\ &\mu'(0) = E(x'(0)). \end{aligned} \tag{10}$$

$$\mu(t) = \mu_h(t) + \mu_p(t) \tag{11}$$

where $\mu_h(t)$ is the general solution of the corresponding homogeneous differential equation, and

$$\mu_p(t) = \frac{a_g E\left(g\left(t\right)\right) + a_l E\left(l\left(t\right)\right)}{a_g + a_l}.$$

This last expression for $\mu_p(t)$ turns out to be $E\left(o\left(t\right)\right)$, where

$$o(t) = \frac{a_g g\left(t\right) + a_l l\left(t\right)}{a_g + a_l}.$$

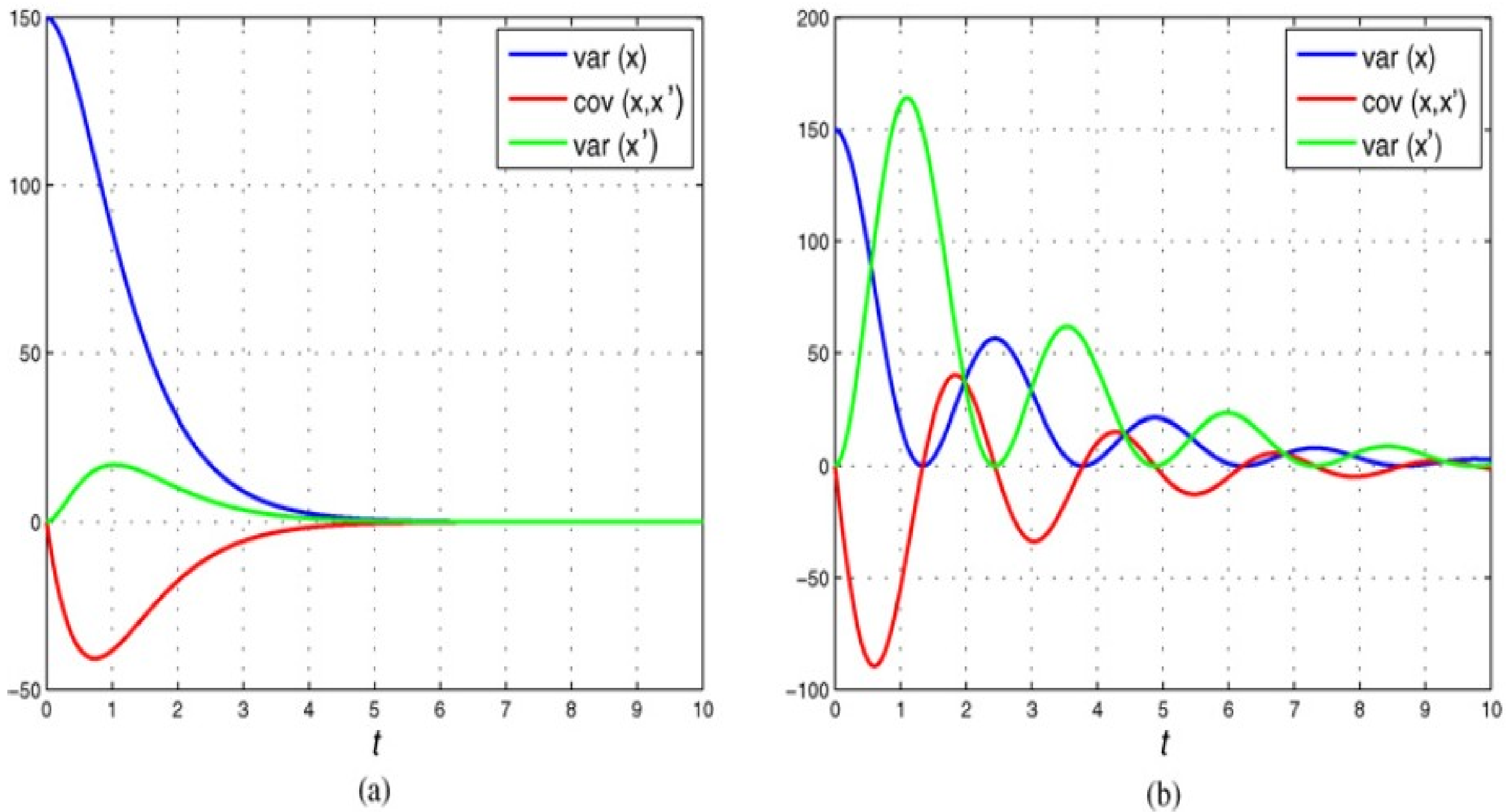


Fig. 2. Homogeneous solution of the covariance equation for the same $(\omega, \overline{\phi})$ points as in Fig. 1, which located on the complex and real zones of the second order stability region. (a) Real zone. (b) Complex zone.