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# Lattice Independent Component Analysis for mobile robot localization

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HAIS 2010

San Sebastián, June 24th, 2010



# Outline



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- Introduction.
- Independent Component Analysis.
- Linear Mixing Model and Lattice Independent Component Analysis.
- Experimental validation.
- Conclusions.



# Objectives



- Previous work: Test the usefulness of the convex coordinates extracted with LAMs as feature vectors for view classification in a robotic mapping context.
- Approach has been generalized as Lattice Independent Component Analysis.
- Compare the approach with similar ones:
  - Independent Component Analysis (ICA).



- Appearance based topological maps.
  - Locations represented by one image.
  - No metric spatial information stored.
  - Localization by image matching.
  - Images stored using global descriptors.
    - Color histograms
    - Edge density
    - PCA
    - ICA



# Basic idea



- The data is generated as a convex combination of a set of endmembers which are the vertices of a convex polytope covering some region of the input data.
- We can extract these endmembers as a Strong Lattice Independent set with the ELHA.
  - Equivalent to the sources of the ICA.
- Unmixing of the input data based on the endmembers to obtain the feature vectors.



# Summary of the approach

- Appearance based robot localization.
- Obtain a descriptor of the images representing each location.
  - Using LICA.
  - Using ICA.
- Based on those descriptors:
  - Build the topological map.
  - Perform localizacion as a classification problem.



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# Independent Component Analysis



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- Assumes that the data is a linear combination of non Gaussian, mutually independent latent variables with an unknown mixing matrix:

$$x = As$$

- The ICA searches for the linear transformation of the data:

$$Wx = s$$





# Independent Component Analysis



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- The model is completely identifiable if:
  - Sources are statistically independent.
  - At least  $M - 1$  of them are non Gaussian.
- Two implementations used:
  - Mean Field ICA (MF ICA).
  - Molgedey and Schouster ICA (MS ICA).



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  - Endmember Induction Heuristic Algorithm.
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# Linear Mixing Model

- Assumes that the data follows a linear model:

$$x = \sum_{i=1}^M a_i s_i + w = Sa + w$$

- **S** are the vertices of the convex region enclosing the data (endmembers).
- **a** is abundance vector.
- **w** is the observation noise.



# Linear Mixing Model



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- Each item is the combination of several pure items.
- Abundance coefficients correspond to the fraction of the contribution of each endmember.
- Two constraints:

$$a_i \geq 0$$

$$i = 1, \dots, M$$

$$\sum_{i=1}^M a_i = 1$$



# Linear Mixing Model

- The mixing inversion process consists in the estimation of the abundance coefficients.
- Unconstrained Least Squared Error.

$$\hat{a} = (S^T S)^{-1} S^T x$$

- Does not necessarily fulfill the constraints.
- Compromise solution for speed.



# Lattice Independent Component Analysis



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1. Induce from the given data a set of Strongly Lattice Independent vectors.

- EIHA.
- Does not impose any statistical assumption.
- One pass and very fast.
- Unsupervised and incremental.
- It detects naturally the number of endmembers.

2. Apply the unconstrained least squares estimation to obtain the mixing matrix.



# Endmember Induction Heuristic Algorithm



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- Tool for extraction of an affine independent set of vectors from a dataset as a Strong Lattice Independent set.
- Incremental algorithm.
- New data is presented to a LAAM built on the already detected endmembers.
  - If there is not recall, a new endmember has been detected.



# Endmember Induction Heuristic Algorithm



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1. Shift the data sample to zero mean  
 $\{\mathbf{f}^c(i) = \mathbf{f}(i) - \overline{\boldsymbol{\mu}}; i = 1, \dots, n\}$ .
2. Initialize the set of endmembers  $E = \{\mathbf{e}^1 = \mathbf{f}^c(i^*)\}$  where  $i^*$  is a randomly picked sample index. Initialize the set of lattice independent binary signatures  $X = \{\mathbf{x}^1\}$  where  $\mathbf{x}^1 = \mathbf{b}(\mathbf{e}^1)$ . The initial set of endmember sample indices is  $I = \{i^*\}$ .
3. Construct the LAAM's based on the lattice independent binary signatures:  $M_{XX}$  and  $W_{XX}$ .





# Endmember Induction Heuristic Algorithm



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4. For each incoming image feature vector  $\mathbf{f}^c(i)$ 
  - (a) Compute the noise corrections sign vectors  $\mathbf{f}^+(i) = \mathbf{b}(\mathbf{f}^c(i) + \alpha \vec{\sigma})$  and  $\mathbf{f}^-(i) = \mathbf{b}(\mathbf{f}^c(i) - \alpha \vec{\sigma})$
  - (b) Compute  $y^+ = M_{XX} \boxtimes \mathbf{f}^+(i)$
  - (c) Compute  $y^- = W_{XX} \boxtimes \mathbf{f}^-(i)$
  - (d) If  $y^+ \notin X$  or  $y^- \notin X$  then  $\mathbf{f}^c(i)$  is a new endmember to be added to  $E$ , execute once 3 with the new  $E$  and resume the exploration of the data sample. Add  $i$  to the set of indices  $I$ .
  - (e) If  $y^+ \in X$ , let  $k$  be the index in  $E$  of the corresponding endmember. If  $\mathbf{f}^c(i) > \mathbf{e}^k$  then execute step 4g.
  - (f) If  $y^- \in X$ , let  $k$  be the index in  $E$  of the corresponding endmember. If  $\mathbf{f}^c(i) < \mathbf{e}^k$  then execute step 4g.
  - (g) The new data sample is more extreme than the stored endmember, then substitute  $\mathbf{e}^k$  in  $E$  with  $\mathbf{f}^c(i)$ . Index  $i$  substitutes the corresponding index in  $I$ .
5. The output set of endmembers is the set of original data vectors  $\{\mathbf{f}(i) : i \in I\}$  corresponding to the sign vectors selected as members of  $E$ .



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# Experimental validation



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- We try to perform the visual recognition of designed landmark positions by a supervisedly built classifier.
- Feature vectors: convex coordinates computed with LICA and ICA.
  - Induce the endmembers (sources) from the dataset.
    - EIHA.
    - MF and MS ICA.
  - Unmix the dataset to obtain the convex coordinates.



# Map building

- Off-line mapping: The dataset (the images acquired by the robot along the path) is recorded in a first training step.
- Several positions selected as landmarks.
- Compute the convex coordinates of the images.
- Each map position formed with several images close to its reference landmark.



# Localization

- Localization:
  - Images are classified on the regions.
  - Feature vectors: convex coordinates obtained by an unmixing process from the training set's endmembers.
  - k-NN classifier.



# Sample path

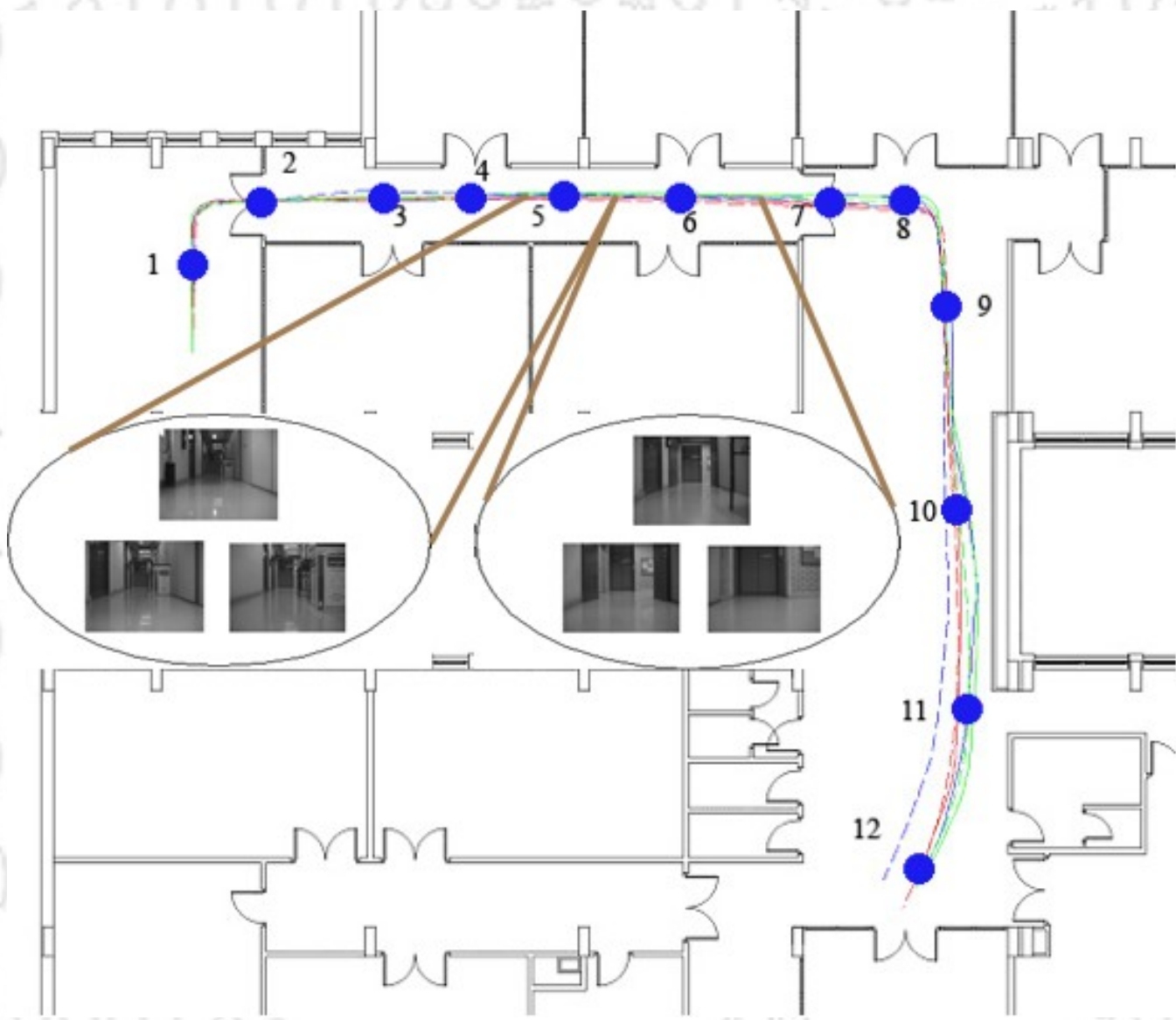
- Pre-recorded data sets:
  - 6 walks over the same path.
  - 1<sup>st</sup> used as training set.
- Landmarks selected as places of practical relevancy.
- Odometry used for validation.



# Sample path



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# Results LICA

#Endmembers	Pass 1	Pass 2	Pass 3	Pass 4	Pass 5	Average
19	0.82	0.70	0.65	0.71	0.75	0.72
20	0.78	0.75	0.65	0.71	0.71	0.72
16	0.78	0.67	0.62	0.68	0.73	0.70
20	0.80	0.75	0.65	0.72	0.71	0.72
18	0.80	0.74	0.66	0.73	0.74	0.74
<b>Average</b>	0.80	0.72	0.65	0.71	0.73	0.72

Table 1. Classification results using LICA ( $\alpha = 7$ ) and 3-nn.

#Endmembers	Pass 1	Pass 2	Pass 3	Pass 4	Pass 5	Average
8	0.63	0.59	0.56	0.63	0.60	0.60
9	0.59	0.54	0.46	0.53	0.61	0.55
8	0.67	0.61	0.54	0.60	0.57	0.60
10	0.65	0.55	0.48	0.60	0.57	0.57
8	0.54	0.54	0.43	0.50	0.41	0.48
<b>Average</b>	0.62	0.57	0.49	0.57	0.55	0.56

Table 2. Classification results using LICA ( $\alpha = 8$ ) and 3-nn.





# Results ICA

#Indep. Comp.	Pass 1	Pass 2	Pass 3	Pass 4	Pass 5	Average
5	0.32	0.31	0.31	0.30	0.24	0.30
10	0.27	0.30	0.26	0.23	0.24	0.26
15	0.36	0.33	0.32	0.34	0.32	0.33
20	0.27	0.26	0.21	0.25	0.21	0.24
25	0.69	0.62	0.54	0.65	0.53	0.61
<b>Average</b>	0.38	0.36	0.33	0.35	0.31	0.35

**Table 3.** Classification results using Mean Field ICA and 3-nn.

#Indep. Comp.	Pass 1	Pass 2	Pass 3	Pass 4	Pass 5	Average
5	0.48	0.49	0.50	0.42	0.39	0.45
10	0.70	0.57	0.54	0.55	0.58	0.59
15	0.76	0.61	0.57	0.64	0.62	0.64
20	0.81	0.69	0.62	0.74	0.69	0.71
25	0.82	0.69	0.62	0.73	0.67	0.71
<b>Average</b>	0.71	0.61	0.57	0.62	0.59	0.62

**Table 4.** Classification results using Molgedey and Schouster ICA and 3-nn.



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# Conclusions

- Proposed and applied a Lattice Independent Component Analysis (LICA) to the appearance based mobile robot localization.
  - Endmembers extracted with EIHA.
  - Feature vectors unmixed from the endmembers.
- Similarities with ICA.
- Comparison shows that improves the MF ICA and performs similarly to MS ICA.



**Thank you for your attention.**