Lattice Independent Component Analysis for fMRI analysis

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Contents

- Introduction and motivation
- Description of the approach
- Some theoretical background
- The endmember induction algorithm
- Results on a case study

Introduction

- Current techniques for fMRI analysis
  - SPM: statistical parametric maps
    - General Linear Model
    - Statistical inference (t-test, F-test)
    - Random Field Theory to set the test threshold
  - ICA: linear source deconvolution
    - Statistically independent sources
    - Mixing Matrix

Introduction

- SPM is a kind of supervised approach
  - Experimental settings are included in the GLM design matrix.
  - Suited for block design experiments
  - Not suited for event driven experiments
  - The aim is to discover voxel sites that show correlations of BOLD signal and the experimental design.

Introduction

- ICA is a kind of unsupervised approach
  - Linear approach
  - The sources correspond to an unsupervisedly discovered design matrix
  - The mixing matrix corresponds to the correlations
  - Suited
    - to discover patterns in the voxels activations
    - for event driven experiments
    - for the study of brain connectivity

Introduction

- The Lattice Independent Component Analysis
  - Is a mixture of a linear and non-linear approach
    - Linear Mixing Model
    - Lattice Autoassociative Memories
  - Endmembers are equivalent to ICA's independent sources and the GLM's design matrix

Introduction

- Lattice Independent Component Analysis can be suited
  - to discover patterns in the voxel's activations
  - for event driven experiments
  - for the study of brain connectivity

General description

Algorithm 4.1 Lattice Independent Component Analysis

Given a fMRI data organized as a set of time series $X \in \mathbb{R}^{N \times T}$, where N is the number of voxels and T the time duration

1. Apply EIHA to obtain endmembers $E = \mathbb{R}^{c \times T}$
2. For each voxel compute the endmember abundance coefficients by ULSE, obtaining $A = \mathbb{R}^{N \times c}$.
3. For each abundance volume $A(.,k) = \mathbb{R}^{N}$ detect the statistical significant voxels as follows:
   (a) Compute the empirical distribution of the abundance values
   (b) Set the significance threshold to the 99% percentil value.

Some theoretical background

- Linear Mixing Model
- Lattice Autoassociative Memories
- Strong Lattice Independence

Linear Mixing Model

$$\mathbf{x} = \sum_{i=1}^{M} a_i \mathbf{s}_i + \mathbf{w} = \mathbf{S}\mathbf{a} + \mathbf{w},$$

Convex mixture

$$a_i \geq 0, i = 1, .., M$$

$$\sum_{i=1}^{M} a_i = 1.$$

Linear unmixing by Unconstrained Least Squares estimation

$$\widehat{\mathbf{a}} = \left(\mathbf{S}^T \mathbf{S}\right)^{-1} \mathbf{S}^T \mathbf{x}.$$

Lattice Associative Memories

- Early Morphological Associative Memories
- LAMs are associative memories built by Lattice Matrix products

$$W_{XY} = \bigwedge_{\xi=1}^{k} \left[\mathbf{y}^{\xi} \times \left(-\mathbf{x}^{\xi}\right)' \right] \text{ and } M_{XY} = \bigvee_{\xi=1}^{k} \left[\mathbf{y}^{\xi} \times \left(-\mathbf{x}^{\xi}\right)' \right],$$

where $\times$ is any of the $\boxtimes$ or $\boxtimes$ operators.

$$C = A \boxtimes B = [c_{ij}] \Leftrightarrow c_{ij} = \bigvee_{k=1,\ldots,n} \{a_{ik} + b_{kj}\},$$

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Latttice Autoassociative Memories

- When X=Y we have Lattice Autoassociative Memories (LAM).
- Appealing property: Perfect recall

$$W_{XX} \boxtimes X = X = M_{XX} \boxtimes X, \text{ for any } X.$$

- Only for noise-free patterns…

Strong Lattice Independence

Definition 1. *Given a set of vectors* $\{\mathbf{x}^1, ..., \mathbf{x}^k\} \subset \mathbb{R}^n$ *a* linear minimax combination *of vectors from this set is any vector* $\mathbf{x} \in \mathbb{R}^n_{\pm\infty}$ *which is a* linear minimax sum *of these vectors:* $x = \mathcal{L}(\mathbf{x}^1, ..., \mathbf{x}^k) = \bigvee_{j \in J} \bigwedge_{\xi=1}^{k} (a_{\xi j} + \mathbf{x}^{\xi})$ *, where J is a finite set of indices and* $a_{\xi j} \in \mathbb{R}_{\pm\infty}$ $\forall j \in J$ *and* $\forall \xi = 1, ..., k$.

Definition 2. *The* linear minimax span *of vectors* $\{\mathbf{x}^1, ..., \mathbf{x}^k\} = X \subset \mathbb{R}^n$ *is the set of all linear minimax sums of subsets of* X*, denoted* $LMS(\mathbf{x}^1, ..., \mathbf{x}^k)$.

Strong Lattice Independence

Definition 3. *Given a set of vectors* $X = \{\mathbf{x}^1, ..., \mathbf{x}^k\} \subset \mathbb{R}^n$, *a vector* $\mathbf{x} \in \mathbb{R}^n_{\pm\infty}$ *is* lattice dependent *if and only if* $x \in LMS\left(\mathbf{x}^1, ..., \mathbf{x}^k\right)$. *The vector* $\mathbf{x}$ *is* lattice independent *if and only if it is not lattice dependent on* X. *The set* X *is said to be* lattice independent *if and only if* $\forall \lambda \in \{1, ..., k\}$, $\mathbf{x}^\lambda$ *is lattice independent of* $X \setminus \{\mathbf{x}^\lambda\} = \{\mathbf{x}^\xi \in X : \xi \neq \lambda\}$.

Strong Lattice Independence

Definition 4. *A set of vectors* $X = \{\mathbf{x}^1, ..., \mathbf{x}^k\} \subset \mathbb{R}^n$ *is said to be* max dominant *if and only if for every* $\lambda \in \{1, ..., k\}$ *there exists and index* $j_\lambda \in \{1, ..., n\}$ *such that*

$$x_{j_\lambda}^\lambda - x_i^\lambda = \bigvee_{\xi=1}^{k} \left(x_{j_\lambda}^\xi - x_i^\xi \right) \forall i \in \{1, ..., n\} .$$

Similarly, X *is said to be* min dominant *if and only if for every* $\lambda \in \{1, ..., k\}$ *there exists and index* $j_\lambda \in \{1, ..., n\}$ *such that*

$$x_{j_\lambda}^\lambda - x_i^\lambda = \bigwedge_{\xi=1}^{k} \left(x_{j_\lambda}^\xi - x_i^\xi \right) \forall i \in \{1, ..., n\} .$$

Definition 5. *A set of lattice independent vectors* $\{\mathbf{x}^1, ..., \mathbf{x}^k\} \subset \mathbb{R}^n$ *is said to be* strongly lattice independent *(SLI) if and only if* X *is max dominant or min dominant or both.*

Conjecture 1. [17] If $X = \{\mathbf{x}^1, ..., \mathbf{x}^k\} \subset \mathbb{R}^n$ is strongly lattice independent then X is affinely independent.

Algorithm 4.2 Endmember Induction Heuristic Algorithm (EIHA)

1. Shift the data sample to zero mean $\{\mathbf{f}^c(i) = \mathbf{f}(i) - \overrightarrow{\boldsymbol{\mu}}; i = 1, .., n\}$.

2. Initialize the set of endmembers $E = \{\mathbf{e}^1 = \mathbf{f}^c(i^*)\}$ where i^* is a randomly picked sample index. Initialize the set of lattice independent binary signatures $X = \{\mathbf{x}^1\}$ where $\mathbf{x}^1 = \mathbf{b}(\mathbf{e}^1)$. The initial set of endmember sample indices is $I = \{i^*\}$.

3. Construct the LAM's based on the lattice independent binary signatures: M_{XX} and W_{XX}.

4. For each pixel $\mathbf{f}^c(i)$

   (a) Compute the noise corrections sign vectors$\mathbf{f}^+(i) = \mathbf{b}\left(\mathbf{f}^c(i) + \alpha\overrightarrow{\boldsymbol{\sigma}}\right)$ and $\mathbf{f}^-(i) = \mathbf{b}\left(\mathbf{f}^c(i) - \alpha\overrightarrow{\boldsymbol{\sigma}}\right)$

   (b) Compute $y^+ = M_{XX} \boxtimes \mathbf{f}^+(i)$

   (c) Compute $y^- = W_{XX} \boxtimes \mathbf{f}^-(i)$

   (d) If $y^+ \notin X$ or $y^- \notin X$ then $\mathbf{f}^c(i)$ is a new endmember to be added to E, execute once 3 with the new E and resume the exploration of the data sample. Add i to the set of indices I.

   (e) If $y^+ \in X$, let k be the index in E of the corresponding endmember. If $\mathbf{f}^c(i) > \mathbf{e}^k$ then execute step 4g.

   (f) If $y^- \in X$, let k be the index in E of the corresponding endmember. If $\mathbf{f}^c(i) < \mathbf{e}^k$ then execute step 4g.

   (g) The new data sample is more extreme than the stored endmember, then substitute $\mathbf{e}^k$ in E with $\mathbf{f}^c(i)$. Index i substitutes the corresponding index in I.

5. The output set of endmembers is the set of original data vectors $\{\mathbf{f}(i) : i \in I\}$ corresponding to the sign vectors selected as members of E.

A Case Study

- Data noise is removed by adequate preprocessing
- Whole brain BOLD/EPI images were acquired on a modified 2T Siemens MAGNETOM Vision system.
  - There are 64x64x64 voxels of size 3mm x 3mm x 3mm.
  - The data acquisition took 6.05s, with the scan-to-scan repeat time (RT) set arbitrarily to 7s., 96 acquisitions were made (RT=7s) in blocks of 6, i.e., 16 blocks of 42s duration.
- Successive blocks alternated between rest and auditory stimulation, starting with rest.
  - Auditory stimulation was bi-syllabic words presented binaurally at a rate of 60 per minute.
- We have discarded the first 10 scans.

Case study

- We have computed
  - An standard SPM study
  - A fastICA
    - Activation is detected by 99% percentil thresholding
  - Our Lattice Independent Component Analysis
- Aim
  - Test that our approach behaves camparably stablished approaches in well known datasets

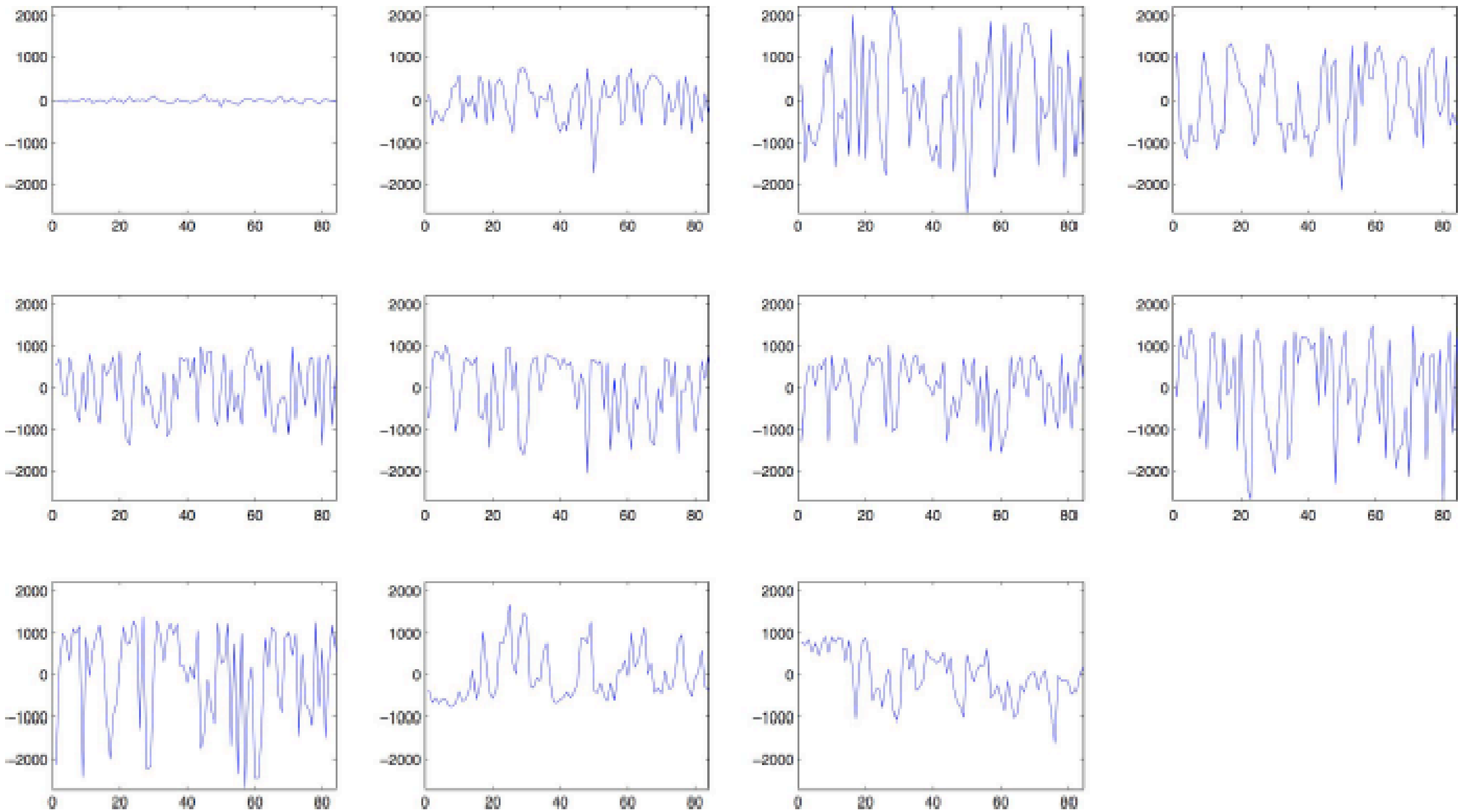


Fig. 1. Eleven endmbers detected by EIHA over the lattice normalized time series of the whole 3D volume.

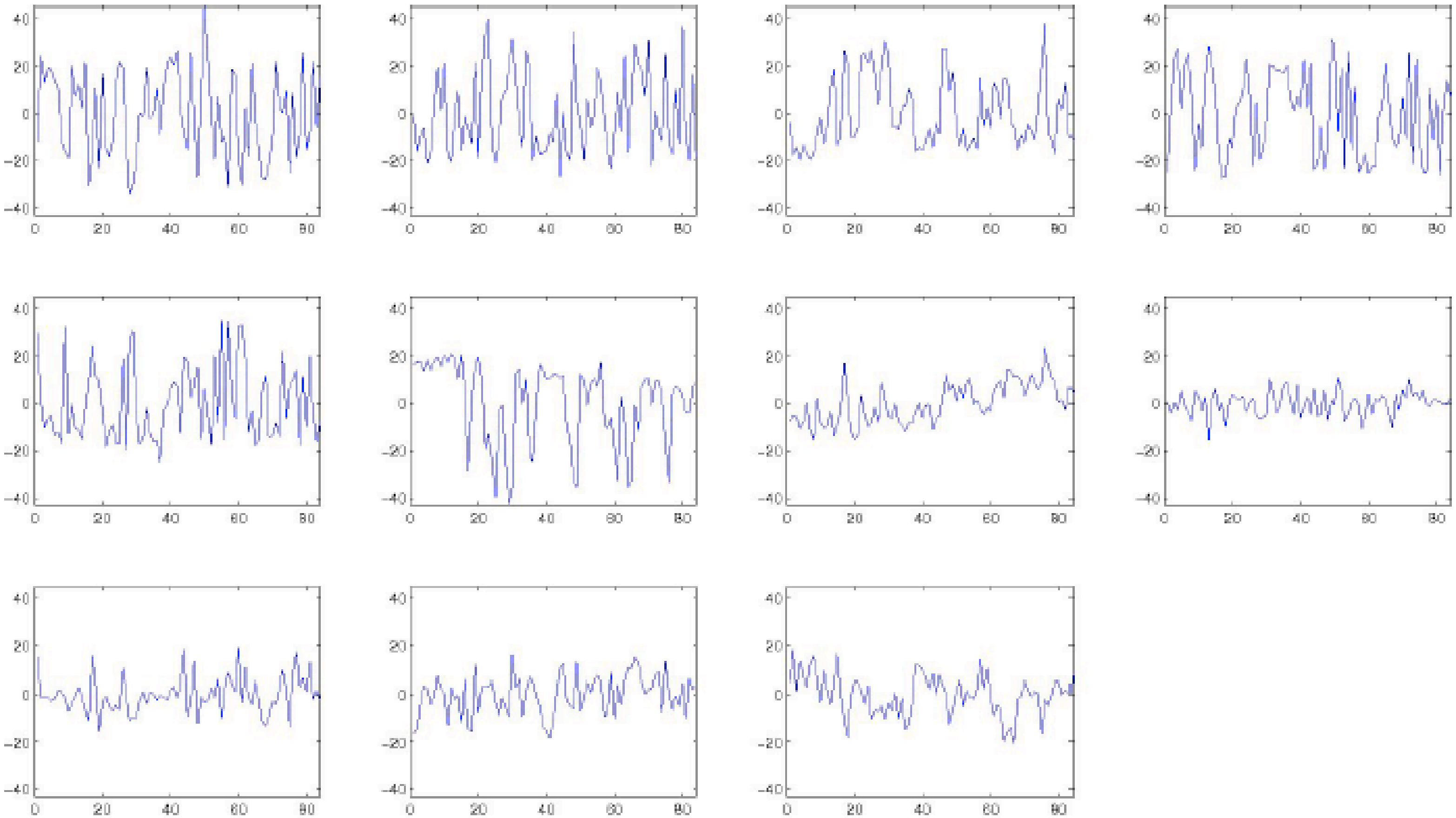
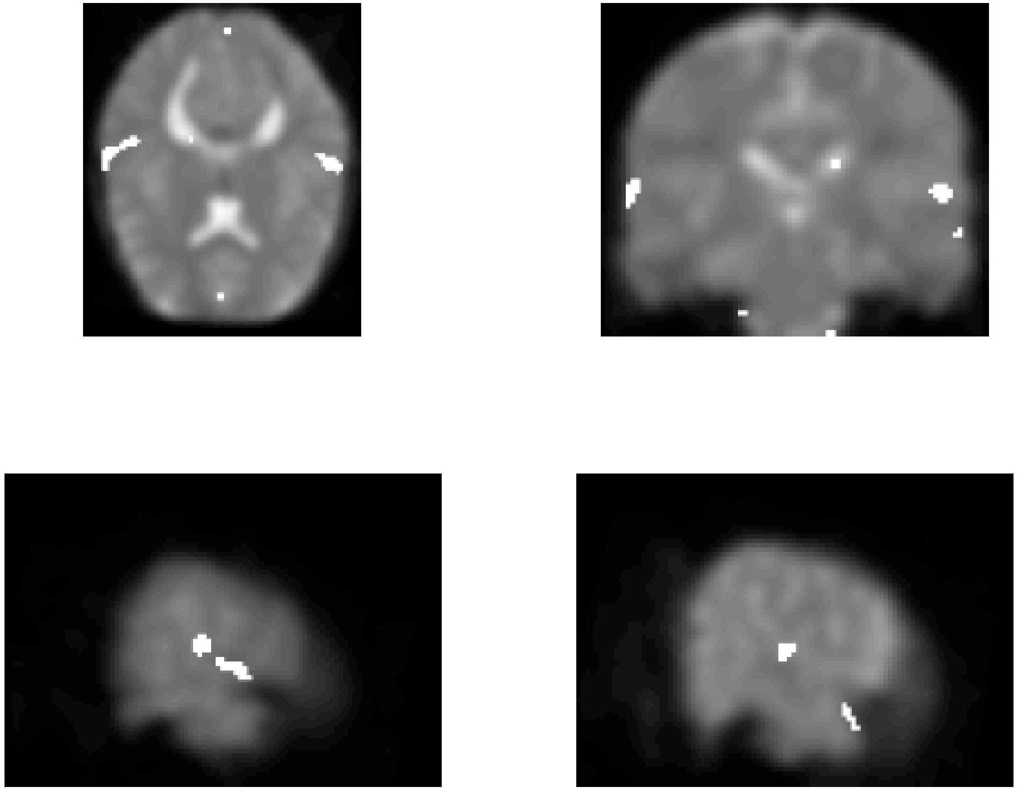
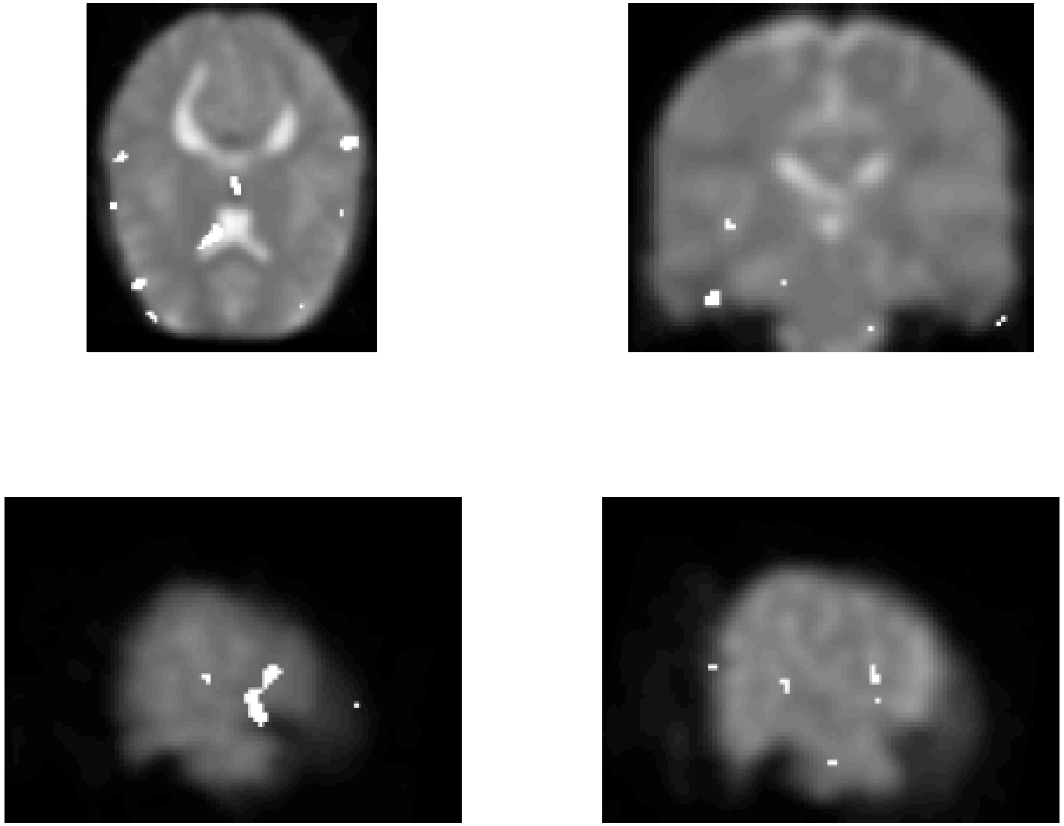


Fig. 3. Eleven time series sources detected by fastICA over the lattice normalized time series of the whole 3D volume.



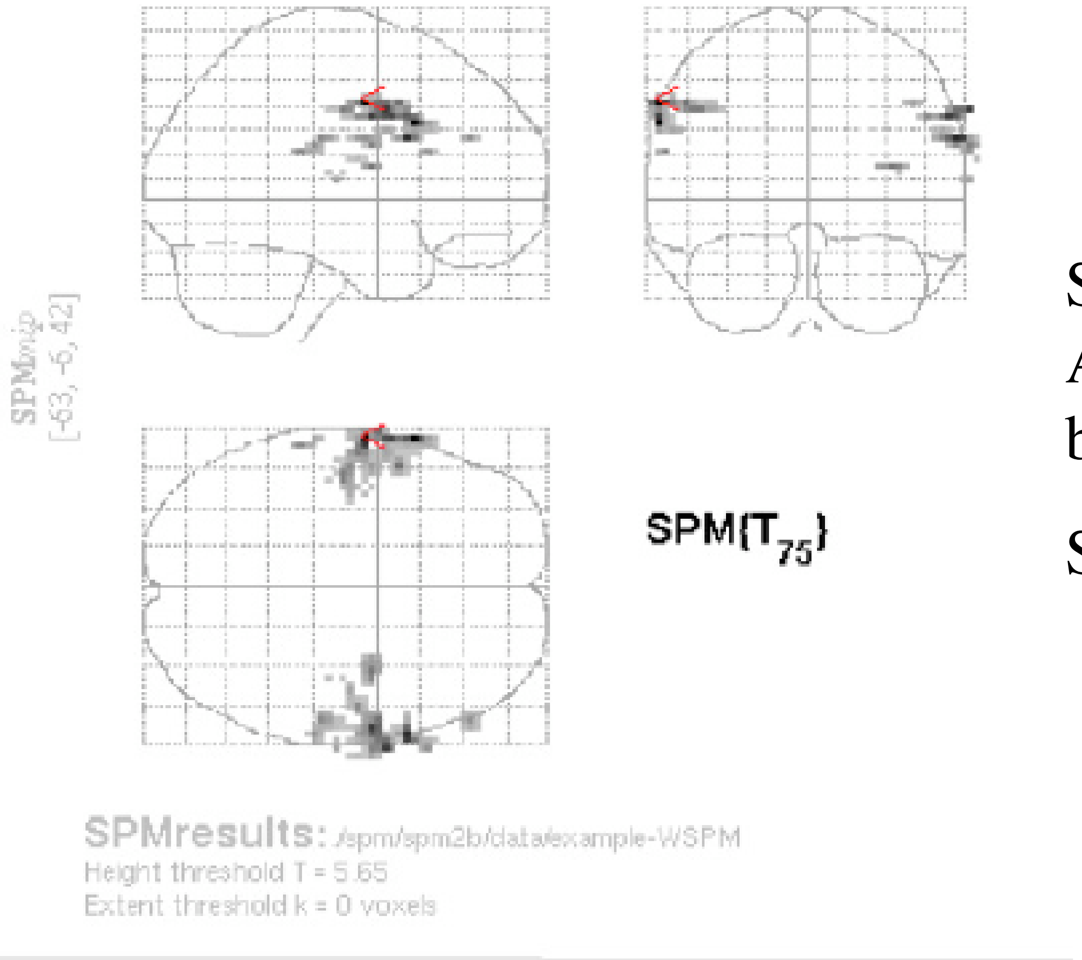
Lattice Independent Component Analysis

Fig. 2. Detected task related activations for endmember #9 from figure 1. White voxels correspond to abundance values above the 99% percentile of the distribution of the abundances for this endmember on the whole volume.



fastICA
activation

Fig. 4. Detected task related activations for source #6 from figure 3. White voxels correspond to mixing values above the 99% percentile of the distribution of the mixing coefficients for this source on the whole volume.



Standard Analysis by

SPM

Conclusions

- Lattice Component Analysis (LICA) finds activations in good agreement with SPM
- LICA has good agreement with results given by fastICA
- Further work:
  - Comparisons with other ICA approaches using quantitative performance measures
  - Application to event experimental designs