An approach from Lattice Computing to fMRI analysis

Manuel Graña, Maite García-Sebastián, Ivan Villaverde, Elsa Fernandez

www.ehu.es/ccwintco

LBM 2008 Workshop, CLA 2008, October 24-26 Olomouc

Outline

1. Introduction
   - fMRI
   - Our proposal
2. The Linear Mixing Model
3. Lattice Independence and Lattice Autoassociative Memories
4. Endmember Induction Heuristic Algorithm (EIHA)
5. A case study
6. Conclusions and discussion

Outline

1. Introduction
   - fMRI
   - Our proposal
2. The Linear Mixing Model
3. Lattice Independence and Lattice Autoassociative Memories
4. Endmember Induction Heuristic Algorithm (EIHA)
5. A case study
6. Conclusions and discussion

fMRI

- Noninvasive techniques can measure cerebral physiologic responses during neural activation
- fMRI uses the blood oxygenation level dependent (BOLD) contrast.
- signal changes are related to changes in the concentration of deoxyhemoglobin
- T2 weighted spin echo pulse sequences or T2* weighted gradient echo pulse sequences.
- good spatial and temporal resolution,
- Allows repeated single-subject studies

fMRI

- Appropriate postprocessing procedures for fMRI
- no consensus has been reached
  - many research groups
  - lack of a complete underlying theory of the BOLD effect
- The fMRI experiment consists of a functional template or protocol that induces a functional response in the brain.
  - aim: to detect this stimulus response
  - functional information of a voxel: extracted from its functional time course

fMRI

- for each functional time point one fMRI volume is recorded
- for each voxel of a volume a functional time course exists.
- The acquisition of these functional volumes runs over periods lasting up to several minutes.
- sources of noise in the fMRI signal
  - pulse sequence and the magnetic field stregth
  - head motions
  - Experiment designs

fMRI

- fMRI analysis approaches
  - Statistical Parametric Mapping (SPM)
    - free open source software
    - Independent Generalized Linear Model at each voxel
    - segmentation of the spatial distribution of the individual voxel t-test values as a parametric map.
  - Independent Component Analysis (ICA)
    - independent sources and the linear unmixing matrix.

Outline

1. Introduction
   - fMRI
   - Our proposal
2. The Linear Mixing Model
3. Lattice Independence and Lattice Autoassociative Memories
4. Endmember Induction Heuristic Algorithm (EIHA)
5. A case study
6. Conclusions and discussion

Our proposal

- The data is generated from a set of endmembers which are the vertices of a convex polytope
- Relation between the Lattice Independence and Affine Independence
- Lattice Associative Memories to serve as detectors of Lattice Independent sets of vectors
- Endmember Induction Heuristic Algorithm (EIHA)
- Lattice Normalization for the detection of meaningful Lattice Independence

Linear Mixing Model

Definition

linear mixing model

$$\mathbf{x} = \sum_{i=1}^{M} a_i \mathbf{s}_i + \mathbf{w} = \mathbf{S}\mathbf{a} + \mathbf{w}, \quad (1)$$

- $\mathbf{x}$ is the d-dimension pattern vector corresponding to the fMRI voxel time series
- $\mathbf{S}$ is the $d \times M$ matrix whose columns are the d-dimension vertices of the convex region
- $\mathbf{a}$ is the M-dimension fractional abundance vector
- $\mathbf{w}$ is the d-dimension additive observation noise vector.

Linear Mixing Model

Definition

Linear Unmixing: computation of the matrix inversion that gives the coordinates of the point relative to the convex region vertices.

- Unconstrained Least Squared Error (LSE) estimation

$$\widehat{\mathbf{a}} = \left(\mathbf{S}^T\mathbf{S}\right)^{-1}\mathbf{S}^T\mathbf{x}. \quad (2)$$

- Negative values are considered as zero values
- High positive values are interpreted as high voxel activation

LAM

Lattice Associative Memories (LAM) stems from $(\mathbb{R}, \vee, \wedge, +)$ as the alternative to $(\mathbb{R}, +, \cdot)$

Definition

Given a set of input/output pairs of pattern $(X, Y) = \left\{ \left(\mathbf{x}^{\xi}, \mathbf{y}^{\xi} \right); \xi = 1, .., k \right\}$,Lattice Memories (LM):

$$W_{XY} = \bigwedge_{\xi=1}^{k} \left[\mathbf{y}^{\xi} \times \left(-\mathbf{x}^{\xi} \right)' \right] \text{ and } M_{XY} = \bigvee_{\xi=1}^{k} \left[\mathbf{y}^{\xi} \times \left(-\mathbf{x}^{\xi} \right)' \right], \quad (3)$$

where $\times$ is any of the $\boxed{\vee}$ or $\boxed{\wedge}$ operators.

LAM

Definition

Here $\boxed{\vee}$ and $\boxed{\wedge}$ denote the max and min matrix product, respectively defined as follows:

$$C = A \boxed{\vee} B = [c_{ij}] \Leftrightarrow c_{ij} = \bigvee_{k=1,\dots,n} \left\{ a_{ik} + b_{kj} \right\}, \tag{4}$$

$$C = A \boxed{\wedge} B = [c_{ij}] \Leftrightarrow c_{ij} = \bigwedge_{k=1,\dots,n} \left\{ a_{ik} + b_{kj} \right\}. \tag{5}$$

Lattice Independence

Definition

Given a set of vectors $\left\{\mathbf{x}^1,\ldots,\mathbf{x}^k\right\} \subset \mathbb{R}^n$ a *linear minimax combination* of vectors from this set is any vector $\mathbf{x} \in \mathbb{R}^n_{\pm\infty}$ which is a *linear minimax sum* of these vectors:

$$x = \mathscr{L}\left(\mathbf{x}^1,\ldots,\mathbf{x}^k\right) = \bigvee_{j\in J}\bigwedge_{\xi=1}^{k}\left(a_{\xi j} + \mathbf{x}^{\xi}\right),$$

where J is a finite set of indices and $a_{\xi j} \in \mathbb{R}_{\pm\infty}$ $\forall j \in J$ and $\forall \xi = 1,\ldots,k$.

Lattice Independence

Definition

The *linear minimax span* of vectors $\left\{\mathbf{x}^1, \dots, \mathbf{x}^k\right\} = X \subset \mathbb{R}^n$ is the set of all linear minimax sums of subsets of X, denoted $LMS\left(\mathbf{x}^1, \dots, \mathbf{x}^k\right)$.

Definition

Given a set of vectors $X = \left\{\mathbf{x}^1, \dots, \mathbf{x}^k\right\} \subset \mathbb{R}^n$, a vector $\mathbf{x} \in \mathbb{R}^n_{\pm\infty}$ is *lattice dependent* if and only if $x \in LMS\left(\mathbf{x}^1, \dots, \mathbf{x}^k\right)$. The vector $\mathbf{x}$ is *lattice independent* if and only if it is not lattice dependent on X. The set X is said to be *lattice independent* if and only if $\forall \lambda \in \{1, \dots, k\}$, $\mathbf{x}^\lambda$ is lattice independent of $X \setminus \left\{\mathbf{x}^\lambda\right\} = \left\{\mathbf{x}^\xi \in X : \xi \neq \lambda\right\}$.

LAM and Lattice Independence

Conjecture

If $X = \{\mathbf{x}^1, ..., \mathbf{x}^k\} \subset \mathbb{R}^n$ is strongly lattice independent then X is affinely independent.

Theorem

Let $X = \{\mathbf{x}^1, ..., \mathbf{x}^k\} \subset \mathbb{R}^n$ and let W (M) be the set of vectors consisting of the columns of the matrix W_{XX} (M_{XX}.). Let $F(X)$ denote the set of fixed points of the LAM constructed from set X. There exist $V \subset W$ and $N \subset M$ such that V and N are strongly lattice independent and $F(X) = F(V) = F(N)$ or, equivalently, $W_{XX} = W_{VV}$ and $M_{XX} = M_{NN}$.

EIHA

Let us denote

- $\left\{\mathbf{f}(i) \in \mathbb{R}^d; i = 1,..,n\right\}$ the high dimensional data that may be the time series in a fMRI voxels,
- $\overrightarrow{\mu}$ and $\overrightarrow{\sigma}$ are, respectively, the mean vector and the vector of standard deviations computed over the data sample,
- α the noise correction factor
  - the patterns are corrected by the addition and subtraction of $\alpha\overrightarrow{\sigma}$, before being presented to the LAM's.
  - controls the amount of flexibility in the discovering of new endmembers
- E the set of already discovered vertices.
- $\mathbf{x} > \mathbf{0}$ the construction of the binary vector $\left(\{b_i = 1 \text{ if } x_i > 0; b_i = 0 \text{ if } x_i \leq 0\}; i = 1,..,n\right)$.

EIHA

Algorithm 1 Endmember Induction Heuristic Algorithm (EIHA)

1. Shift the data sample to zero mean $\{\mathbf{f}^c(i) = \mathbf{f}(i) - \overrightarrow{\mu}; i = 1, .., n\}$.
2. Initialize the set of vertices $E = \{\mathbf{e}_1\}$ with a randomly picked sample. Initialize the set of lattice independent binary signatures $X = \{\mathbf{x}_1\} = \{(e_k^1 > 0; k = 1, .., d)\}$
3. Construct the LAM's based on the lattice independent binary signatures: M_{XX} and W_{XX}.
4. For each pixel $\mathbf{f}^c(i)$
   - (a) compute the noise corrections sign vectors$\mathbf{f}^+(i) = (\mathbf{f}^c(i) + \alpha\overrightarrow{\sigma} > \mathbf{0})$ and $\mathbf{f}^-(i) = (\mathbf{f}^c(i) - \alpha\overrightarrow{\sigma} > \mathbf{0})$
   - (b) compute $y^+ = M_{XX} \boxvee \mathbf{f}^+(i)$
   - (c) compute $y^- = W_{XX} \boxwedge \mathbf{f}^-(i)$
   - (d) if $y^+ \notin X$ or $y^- \notin X$ then $\mathbf{f}^c(i)$ is a new vertex to be added to E, execute once 3 with the new E and resume the exploration of the data sample.
   - (e) if $y^+ \in X$ and $\mathbf{f}^c(i) > \mathbf{e}_{y^+}$ the pixel spectral signature is more extreme than the stored vertex, then substitute $\mathbf{e}_{y^+}$ with $\mathbf{f}^c(i)$.
   - (f) if $y^- \in X$ and $\mathbf{f}^c(i) < \mathbf{e}_{y^-}$ the new data point is more extreme than the stored vertex, then substitute $\mathbf{e}_{y^-}$ with $\mathbf{f}^c(i)$.
5. The final set of endmembers is the set of original data vectors $\mathbf{f}(i)$ corresponding to the sign vectors selected as members of E.

Experimental data

auditory stimulation test data of single person.

Preprocessed

acquired on a modified 2T Siemens MAGNETOM Vision system.

Each acquisition consisted of 64 contiguous slices. . There are 64x64x64 voxels of size 3mm x 3mm x 3mm.

The data acquisition took 6.05s, with the scan-to-scan repeat time (RT) set arbitrarily to 7s. 96 acquisitions were made (RT=7s) in blocks of 6, i.e., 16 42s blocks.

The condition for successive blocks alternated between rest and auditory stimulation, starting with rest.

Auditory stimulation was bi-syllabic words presented binaurally at a rate of 60 per minute.

The functional data starts at acquisition 4, image snrfMOO223-004.

We have discarded the first 10 scans.

Experimental data

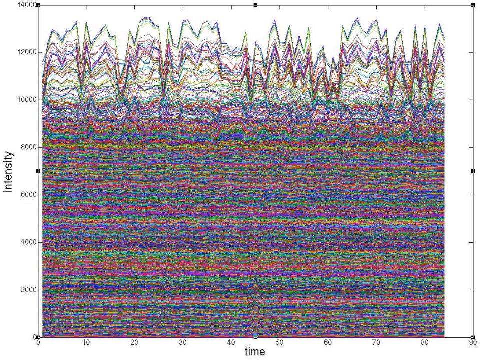

Figure: Plot of the time series for the voxels of axial slice #30.

Lattice Normalization

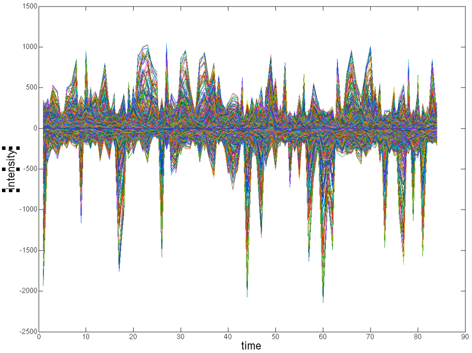

Figure: Plots of time series of voxels in axial slice #30 after susbtracting their mean values from them. The time series are collapsed in the neighborhood of zero.

Results

- The application of the EIHA algorithm with $\alpha = 20$ to the lattice normalized time series of the whole 3D volume produces the collection of eleven endmembers shown in figure 3.
- Attending to the intensity scale it can be assumed that the first endmember (top left plot) corresponds to the non activation pattern, while the remaining endmembers correspond to some kind of activation pattern.
- These patterns correspond to individual voxels and do not reflect aggregated spatial behaviors like in other approaches.

Results

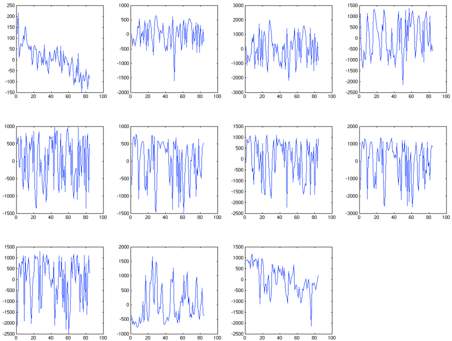

Figure: Eleven endmbers detected by EIHA over the lattice normalized time series of the whole 3D volume.

Results

- The unmixing process produces the abundance images that we interpret as the activation levels of each pattern.
- For interpretation, we refer to the standard results obtained with the SPM software.
- There it can be observed that the activation appears around the axial slice #30.

Results

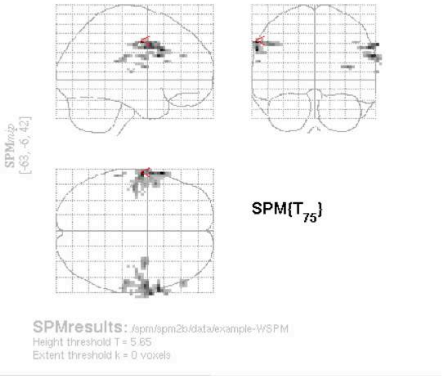

Figure: Activation maps from SPM results over the experimental data

Results

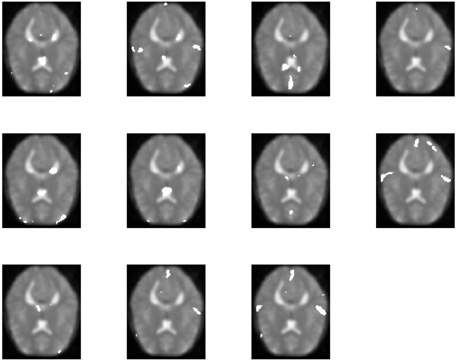

Figure: Abundances for axial slice #30 for all eleven endmembers. White voxels correspond to abundance values above the 99% percentile of the distribution of the abundances for each endmember at this slice.

Results

- It can be appreciated that the abundances for endmembers #8 and #11 have some activation regions overlapping the standard detections in figure 4,
    - some spurious activation regions.
- For a complete review of the activation detected by the endmember #11 abundances
    - we show the 99% percentile detection on all the slices in the axial direction in figure 6.

Results

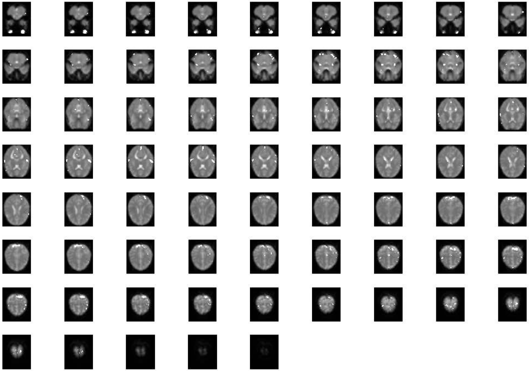

Figure: Activations detected by the 99% percentile of the abundance images of endmember #11 of figure 5 in the axial direction.

Conclusions and discussion

- We have proposed and applied the endmember induction algorithm EIHA to the task of brain region activation in fMRI.
- The idea is to try to mimic ICA application to fMRI activation detection
  - the sources correspond to endmembers detected by the EIHA algorithm and
  - the activation is computed as the abundance images obtained by unmixing the voxel time series on the basis of the found endmembers.

Conclusions and discussion

- The first obstacle that is that the distribution of the time series is not well aspected for the detection of Lattice Independence.
- To overcome this problem we propose a normalization which corresponds to a scale normalization in the sense of Lattice Computing.
    - We substract its mean to each voxel time series.

Conclusions and discussion

- Our computational experiment with a well known fMRI data set, provided with the distribution of the SPM software, show some promising results in the sense that we are able to partially detect activations as the standard analysis with the SPM software.
- There are however some false detections that show that our approach is not consistent with the SPM analysis.
- Finding ways to harmonize global random field analysis and our lattice computing approach may lead to such consistency.