The primordial trispectrum from inflation

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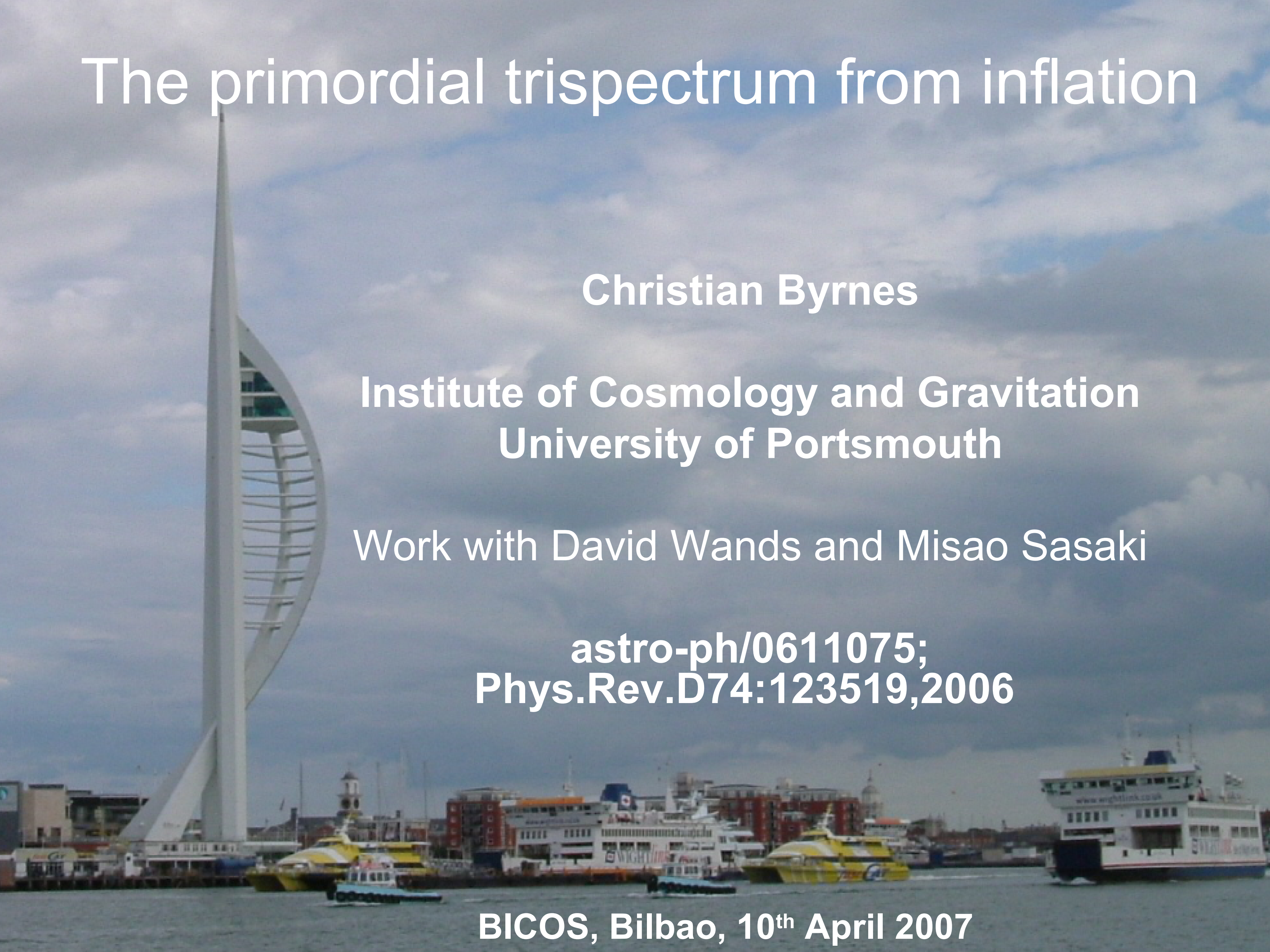


Inflation

- Inflation generates the primordial density perturbations from vacuum fluctuations in the scalar field

- The simplest models predict

  A nearly scale invariant spectrum of adiabatic (curvature) perturbations with a nearly Gaussian distribution

  There are LOTS of models of inflation:

  single field, multi field, new, chaotic, hybrid, power-law, natural, supernatural, assisted, Nflation, curvaton, eternal, F-term, D-term, brane, DBI, k- ....

  With so many models we need as many observables as possible to distinguish between them

  E.g. observational bound on bispectrum |f_NL|<100

Linear (Gaussian) field perturbations

$$\phi^A(t, x^i) = \phi_0^A(t) + \varphi_1^A(t, x^i)$$

2-pt function after Hubble exit, k<aH, for field perturbations on spatially-flat hypersurfaces

$$\langle \varphi_{\mathbf{k}}^A \varphi_{\mathbf{k}'}^B \rangle = C^{AB}(k)(2\pi)^3 \delta^3(\mathbf{k} + \mathbf{k}')$$

- to lowest order in slow-roll

$$C^{AB}(k) = \delta^{AB} P(k)$$

where

$$\mathcal{P}(k) = \frac{4\pi k^3}{(2\pi)^3} P(k) = \left(\frac{H_*}{2\pi} \right)^2$$

hence for scale-invariant spectrum $P(k) \propto 1/k^3$

- to next order in slow-roll $C^{AB} \neq \delta^{AB}$ van Tent (03); Byrnes & Wands (06)

2^{nd} and 3^{rd} order field perturbations

$$\delta\phi^A \equiv \varphi^A = \varphi_1^A + \frac{1}{2}\varphi_2^A + \frac{1}{6}\varphi_3^A + \cdots$$

3-pt function after Hubble exit, k<aH,
for field perturbations on spatially-flat hypersurfaces

$$\langle \varphi_{\mathbf{k}_1}^A \, \varphi_{\mathbf{k}_2}^B \, \varphi_{\mathbf{k}_3}^C \rangle \equiv B^{ABC}(k_1, k_2, k_3)(2\pi)^3 \delta^3(\mathbf{k_1} + \mathbf{k_2} + \mathbf{k_3})$$

Maldacena (2001); Seery & Lidsey (2005)

connected part of the 4-point function, after Hubble exit

$$\langle \varphi_{\mathbf{k}_1}^A \, \varphi_{\mathbf{k}_2}^B \, \varphi_{\mathbf{k}_3}^C \varphi_{\mathbf{k}_4}^D \rangle_c \; \equiv \; T^{ABCD}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4)(2\pi)^3 \delta^3(\mathbf{k_1} + \mathbf{k_2} + \mathbf{k_3} + \mathbf{k}_4)$$

Seery, Lidsey & Sloth (2006)

The primordial curvature perturbation ζ

Calculate using the δN formalism (valid on super horizon scales)

Starobinsky `85; Sasaki & Stewart `96
Lyth & Rodriguez '05 – works to any order

Efoldings $N = \int_{t_*}^{prim} H dt$

$$\zeta = \delta N = N_A \varphi^A + \frac{1}{2} N_{AB} \varphi^A \varphi^B + \cdots$$

Where $N_A \equiv \frac{\partial N}{\partial \phi^A}$ and φ^A is evaluated at Hubble-exit

Linear primordial perturbations

$$\langle \zeta_{\mathbf{k}} \zeta_{\mathbf{k}'} \rangle = P_\zeta(k)(2\pi)^3 \delta^3(\mathbf{k} + \mathbf{k}')$$

to lowest order in field perturbations

$$\zeta_1 = N_A \varphi_1^A \quad \Rightarrow \quad P_\zeta(k) = N_A N_B C^{AB}(k)$$

- to lowest order in slow-roll $C^{AB}(k) = \delta^{AB} P(k)$

$$P_\zeta(k) = N_A N^A P(k)$$

where

$$\mathcal{P}(k) = \frac{4\pi k^3}{(2\pi)^3} P(k) = \left(\frac{H_*}{2\pi}\right)^2$$

2nd order primordial perturbations

$$\zeta_2 = N_A \varphi_2^A + N_{AB} \varphi_1^A \varphi_1^B$$

Lyth & Rodriguez, astro-ph/0504045

to lowest (fourth) order in field perturbations

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle \equiv B_\zeta(k_1, k_2, k_3)(2\pi)^3 \delta^3(\mathbf{k_1} + \mathbf{k_2} + \mathbf{k_3})$$

where

$$\begin{aligned} B_\zeta(k_1, k_2, k_3) \; = \; & N_A N_B N_C B^{ABC}(k_1, k_2, k_3) \\ & + N_A N_{BC} N_D \left[C^{AC}(k_1) C^{BD}(k_2) + C^{AC}(k_2) C^{BD}(k_3) + C^{AC}(k_3) C^{BD}(k_1) \right] \end{aligned}$$

3rd order primordial perturbations

$$\zeta_3 = N_A \varphi_3^A + N_{AB}\left(\varphi_1^A \varphi_2^B + \varphi_2^A \varphi_1^B\right) + N_{ABC}\varphi_1^A \varphi_1^B \varphi_1^C$$

We define the trispectrum by

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \zeta_{\mathbf{k}_4} \rangle_c \equiv T_\zeta(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4)(2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 + \mathbf{k}_4)$$

$$\begin{aligned}
T_\zeta(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) \quad &= N_A N_B N_C N_D T^{ABCD}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) \\
&+ N_{A_1 A_2} N_B N_C N_D \left[C^{A_1 B}(k_1) B^{A_2 BC}(k_{12}, k_3, k_4) + (11 \text{ perms})\right] \\
&+ N_{A_1 A_2} N_{B_1 B_2} N_C N_D \left[C^{A_2 B_2}(k_{13}) C^{A_1 C}(k_3) C^{B_1 D}(k_4) + (11 \text{ perms})\right] \\
&+ N_{A_1 A_2 A_3} N_B N_C N_D \left[C^{A_1 B}(k_2) C^{A_2 C}(k_3) C^{A_3 D}(k_4) + (3 \text{ perms})\right] .
\end{aligned}$$

Seery & Lidsey, astro-ph/0611034;
Byrnes, Sasaki & Wands, astro-ph/0611075

Primordial Gaussianity from Gaussian fields

ζ is a function of multiple Gaussian fields

Bispectrum and trispectrum simplify, to leading order in slow roll

$$B_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) = \frac{6}{5} f_{NL}\left[P_\zeta(k_1)P_\zeta(k_2) + P_\zeta(k_2)P_\zeta(k_3) + P_\zeta(k_3)P_\zeta(k_1)\right]$$

$$T_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4) = \tau_{NL}\left[P_\zeta(k_{13})P_\zeta(k_3)P_\zeta(k_4) + (11\text{ perms})\right] + \frac{54}{25} g_{NL}\left[P_\zeta(k_2)P_\zeta(k_3)P_\zeta(k_4) + (3\text{ perms})\right]$$

There are 3 k independent parameters

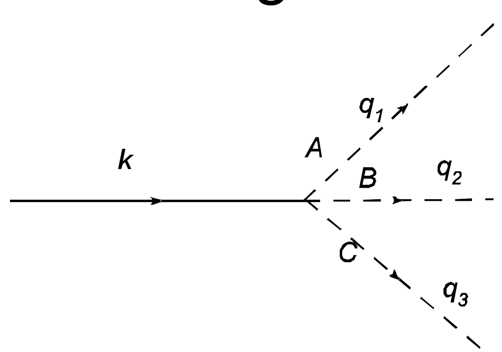
$$f_{NL} = \frac{5}{6}\frac{N_A N_B N^{AB}}{(N_C N^C)^2} \qquad g_{NL} = \frac{25}{54}\frac{N_{ABC}N^A N^B N^C}{(N_D N^D)^3} \qquad \tau_{NL} = \frac{N_{AB}N^{AC}N^B N_C}{(N_D N^D)^3}$$

If only 1 field generates the primordial curvature perturbation, 2 independent parameters remain

$$f_{NL} = \frac{5}{6}\frac{N''}{(N')^2} \qquad g_{NL} = \frac{25}{54}\frac{N'''}{(N')^3} \qquad \tau_{NL} = \frac{(N'')^2}{(N')^4} = \frac{36}{25}f_{NL}^2$$

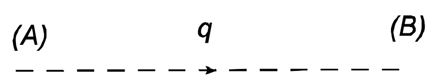
Diversion - Diagrammatic Approach

Vertex



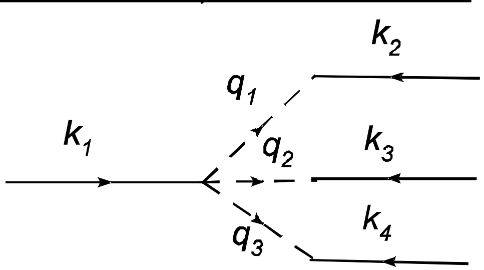
$$\delta^3(k - q_1 - q_2 - q_3)N_{ABC}$$

Propagator

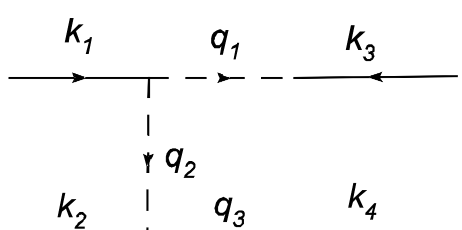


$$\delta^{AB}P(q)$$

<u>4-point function, at tree level</u> (not including permutations)



$$\delta^3(\sum k_i)N_A N_B N_C N^{ABC} P(k_1)P(k_2)P(k_3)$$



$$\delta^3(\sum k_i)N_A N_B N_C^A N^{BC} P(k_{13})P(k_3)P(k_4)$$

non-Gaussianity from slow-roll inflation?

single inflaton field

- can evaluate non-Gaussianity at Hubble exit (zeta is conserved)

$$f_{NL} = \frac{5}{6}\frac{N''}{(N')^2} = \frac{5}{6}(\eta - 2\epsilon) \quad g_{NL} = \frac{25}{54}\frac{N'''}{(N')^3} = \frac{25}{54}\left(2\epsilon\eta - 2\eta^2 + \xi^2\right)$$

- **undetectable** with WMAP or Planck data

multiple field inflation

- difficult to get large non-Gaussianity during inflation
  No explicit model has been constructed

Easier to generate non-Gaussianity after inflation
E.g. Curvaton, modulated (p)reheating, inhomogeneous end of inflation

Curvaton scenario

The primordial curvature perturbation is generated from a curvaton field that is subdominant during inflation
If the ratio of the curvaton's energy density to the total energy density is small $r = \left[\frac{3\rho_\chi}{3\rho_\chi + 4\rho_r}\right]_{\text{decay}} \ll 1$

Non-linearity parameters are large $f_{NL} = \mathcal{O}(1/r)\,, \qquad g_{NL} = \mathcal{O}(1/r^2)$

Sasaki, Valiviita and Wands 2006

Conclusions

- New data makes it worthwhile going beyond linear theory (WMAP, ACT, Planck)
- Need to compare models with data

- For Gaussian field fluctuations from slow roll inflation
  - Bispectrum has one new observable f_{NL}
  - Trispectrum has two new observables, τ_{NL} and g_{NL}

- Only in 'single field' models, $\tau_{NL} \propto f_{NL}^2$

- τ_{NL} and g_{NL} are small in the standard inflaton scenario
- Can be large in other models