Accretion of Chaplygin gas onto wormholes

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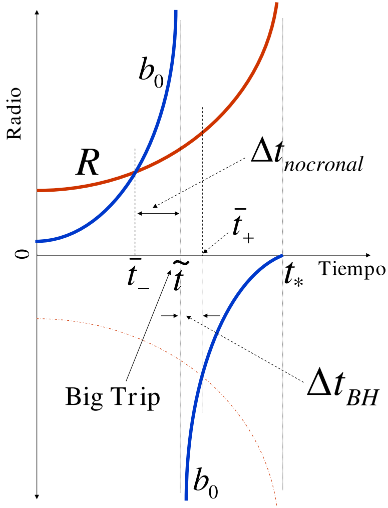
- This acceleration is due to dark energy (DE).
- Big Rip appears in models with phantom energy.
- Phantom energy lets existence of wormholes.
- If there is a wormhole in a universe with phantom energy, it could reach a big trip singularity.

Motivation (2)



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- This exotic substance may be used to construct wormholes.
- Does an universe filled with Chaplygin gas also escape from the big trip?

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Morris-Thorne metric

$$\mathrm{d}s^2 = e^{\Phi(r)}\mathrm{d}t^2 - \frac{\mathrm{d}r^2}{1 - \frac{K(r)}{r}} - r^2(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2),$$

where $\Phi(r)$ is the shift function and $K(r)$ is the shape function.

We model dark energy by the test perfect fluid with negative pressure and an arbitrary equation of state, with the energy-momentum tensor

$$T_{\mu\nu} = (p + \rho)\, u_\mu u_\nu - p g_{\mu\nu}, \quad (1)$$

where p is the pressure, ρ is the energy density, and $u^\mu = \mathrm{d}x^\mu/\mathrm{d}s$ is the 4-velocity.

Energy-momentum conservation law $T^{\mu\nu}{}_{;\nu} = 0$ can be written as

$$
\begin{aligned}
0 &= \frac{\mathrm{d}}{\mathrm{d}r}\left[e^{\Phi(r)}\left(P+\rho\right)\frac{\mathrm{d}t}{\mathrm{d}s}\frac{\mathrm{d}r}{\mathrm{d}s}\right] + \\
&+ e^{\Phi(r)}\left(P+\rho\right)\left[\Phi'(r) + \frac{K'(r)r - K(r)}{2r^2\left(1-\frac{K(r)}{r}\right)} + \frac{2}{r}\right]\frac{\mathrm{d}t}{\mathrm{d}s}\frac{\mathrm{d}r}{\mathrm{d}s}.
\end{aligned}
$$

Integration of conservation law

$$uM^{-2}r^2\left(1-\frac{K(r)}{r}\right)^{-1}(P+\rho)\sqrt{u^2+\frac{K(r)}{r}-1}=C,$$

where $u=\mathrm{d}r/\mathrm{d}s$,
Flow equation

$$M^{-2}r^2u\left(1-\frac{K(r)}{r}\right)^{-1/2}e^{\int_{\rho\infty}^{\rho}\frac{\mathrm{d}\rho}{P+\rho}}=-A,$$

where $u<0$ and A is a positive dimensionless constant.
One get

$$(P+\rho)\left(1-\frac{K(r)}{r}\right)^{-1/2}\sqrt{u^2+\frac{K(r)}{r}-1}\;e^{-\int_{\rho\infty}^{\rho}\frac{\mathrm{d}\rho}{P+\rho}}=C_2,$$

where $C_2=-C/A=\tilde{A}\left[P(\rho_\infty)+\rho_\infty\right]$, with $\tilde{A}$ a positive constant.

The rate of change of the exotic mass of wormhole

$$\dot{M} = -\int T_0{}^r \mathrm{d}S,$$

with $\mathrm{d}S = r^2 \sin\theta \mathrm{d}\theta \mathrm{d}\phi$.
This can rewritten as

$$\dot{M} = -4\pi D M^2 \sqrt{1 - \frac{K(r)}{r}} \left[P(\rho_\infty) + \rho_\infty \right],$$

with the constant $D = A\tilde{A} > 0$
Asymptotic regime

$$\dot{M} = -4\pi M^2 D \left(P + \rho \right).$$

Quintessence

$$M = \frac{M_0}{1 - DM_0\sqrt{\frac{8\pi\rho_0}{3}}\left[1+\sqrt{6\pi\rho_0}\,(1+w)\,(t-t_0)\right]^{-1}} \quad ,$$

where M_0 is the initial (exotic) mass of wormhole.
M has a singular behaviour $(w < -1)$

$$t_{\rm trip} = t_0 + \frac{M_0 D\sqrt{\frac{8\pi\rho_0}{3}} - 1}{(1+w)\sqrt{6\pi\rho_0}} < t_{\rm rip} \quad ,$$

where $t_{\rm rip}$ is the time where big rip occurs, is to say $t_{\rm rip} = t_0 - \frac{1}{\sqrt{6\pi\rho_0}(1+w)}$. Thus $t_{\rm trip}$ occurs before big rip.

Generalized Chaplygin gas

This can be described as a perfect fluid with the equation of state

$$P = -A_{\rm ch}/\rho^{\alpha},$$

Conservation of energy-momentum tensor:

$$\rho = \left[A_{\rm ch} + \frac{B}{R^{3(\alpha+1)}}\right]^{\frac{1}{1+\alpha}}$$

with $B \equiv (\rho_0^{\alpha+1} - A_{\rm ch})R_0^{3(\alpha+1)}$.
$B > 0 \Longrightarrow P + \rho > 0$
$B < 0 \Longrightarrow P + \rho < 0$.

Friedmann equation:

$$H^2 = \frac{8\pi G}{3}\left[A_{\rm ch} + \frac{B}{R^{3(\alpha+1)}}\right]^{\frac{1}{1+\alpha}}.$$

From this equation, one can get that phantom generalized Chaplygin gas avoids the big rip singularity.

Generalized Chaplygin Gas

$$M = \frac{M_0}{1 - DM_0\sqrt{\frac{8\pi}{3}}\left[\rho^{\frac{1}{2}} - \rho_0^{1/2}\right]}.$$

When time goes to infinity, then the exotic mass of wormhole approaches to

$$M = \frac{M_0}{1 - DM_0\sqrt{\frac{8\pi}{3}}\left(A_{\rm ch}^{\frac{1}{2(1+\alpha)}} - \rho_0^{1/2}\right)},$$

Precludes this the occurrence of the big trip ?

In order for avoiding a big trip, the following two conditions are also required

(I) $R \neq M$, or

(II) $N(R) = \dot{R}/\dot{M}$ be always an increasing function.

(I) implies that

$$f(M) = M - M M_0 D \sqrt{\frac{8\pi}{3}} \left[\left(A_{\rm ch} + \frac{B}{M^{3(1+\alpha)}} \right)^{\frac{1}{2(1+\alpha)}} - \rho_0^{1/2} \right] - M_0 \neq 0$$

We analyze this question by taking the zeros of the second derivative

$$f''(M) \equiv \frac{{\rm d}^2 f(M)}{{\rm d}M^2} = \sqrt{6\pi} D B \frac{M_0}{M^{3(1+\alpha)+1}} \left(A_{\rm ch} + \frac{B}{M^{3(1+\alpha)}} \right)^{\frac{-2\alpha-1}{2(1+\alpha)}}$$
$$\times \left[1 - 3(1+\alpha) + \frac{3B(2\alpha+1)}{2M^{3(1+\alpha)}} \left(A_{\rm ch} + \frac{B}{M^{3(1+\alpha)}} \right)^{-1} \right].$$

Taking into account that,

$$M > M_0 > \left(-\frac{B}{A_{\rm ch}}\right)^{\frac{1}{3(1+\alpha)}},$$

The question can be reduced to study the zeros

$$1 - 3\,(1+\alpha) + \frac{3B(2\alpha+1)}{2M^{3(1+\alpha)}}\left(A_{\rm ch} + \frac{B}{M^{3(1+\alpha)}}\right)^{-1} = 0,$$

whose solution would read $M^{3(1+\alpha)} = -B/[2(2+3\alpha)A_{\rm ch}]$.
If $-\frac{2}{3} < \alpha < -\frac{1}{2}$ one might expect just three points at most. And two outside this interval.

As to condition (II), we obtain

$$N(R) \equiv \frac{\dot{R}}{\dot{M}} = -\frac{1}{BM_0^2 D\sqrt{6\pi}} R^{3(1+\alpha)+1} \rho^{\frac{2\alpha+1}{2}} \left\{ 1 - M_o D \sqrt{\frac{8\pi}{3}} \left[\rho^{\frac{1}{2}} - \rho_0^{1/2} \right] \right\}^2 .$$

Where $N(R_0) \geq 1$, If $N(R)$ is a increasing function, then big trip don't occur. With $B < 0$ and α no close to -1, big trip is avoided. But inside the interval

$$-1 < \alpha < \frac{\ln A_{\mathrm{ch}}}{\ln \left(\sqrt{\frac{3}{8\pi}} \frac{1}{M_0 D} + \rho_0^{1/2} \right)^2} - 1,$$

a big trip would still take place.

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CONCLUSIONS

- We have reviewed the accretion formalism originally considered by Babichev, Dockuchaev and Eroshenko for the case of a wormhole.
- We have applied such formalism to the generalized Chaplygin gas model.
- The evolution of exotic mass with the accretion of Chaplygin dark energy has been first considered for the case that the dominant energy condition is satisfied. It has been seen that in that case the mass decreases with cosmic time.
- If accretion involves Chaplygin phantom energy, then M increases from its initial value, tending to reach a plateau as cosmic time goes to infinity.

CONCLUSIONS (2)

- It is obtained that for a wide region of the Chaplygin parameters no big trip is predicted, contrary to what happens in quintessence and K-essence dark energy models. However, as far as the Chaplygin regime tends to match the quintessence regime, but still within the Chaplygin region, the possibility for a big trip at a finite time in the future is not excluded.
- Finally, we also argued that the fate of the final wormhole is to be destabilized by quantum vacuum processes.

CONCLUSIONS (3)

- The generalized Chaplygin gas has several very interesting features and may circumvent several singularities that appear in the usual quintessence models. Whether or not the above features can be taken to imply that Chaplygin gas is a more consistent component than usual quintessential or K-essence dark energy component with $w < -1$ is a matter that will depend on both the intrinsic consistency of the models and the current observational data and those that can be expected in the future.