Non-local SFT Tachyon and Cosmology

Alexey Koshelev

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Mainly based on JHEP **02** (2007) 041, `hep-th/0605085` by I.Ya. Aref'eva, A. K.
and JHEP **04** (2007) 029, `hep-th/0701103` by A. K.

Cosmological motivation

- Data on Ia supernovae
- Galaxy clusters measurements
- WMAP

} Universe exhibits an accelerated expansion

Equation of state: $p = w\rho, \qquad w < 0$ — Dark Energy

$$w = -1.06^{+0.13}_{-0.08}$$

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- $w > -1$ — Quintessence models
- $w = -1$ — Cosmological constant
- $w < -1$ — Phantom models

It is difficult to cook a Phantom divide crossing

$w = \text{const} < -1 \Rightarrow$ Big Rip singularity

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Consider spatially flat FRW universe

$$(1+3) \quad \text{dimensions,} \qquad ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$$

SFT (p-adic) Tachyon

Tachyon effective action ($\alpha' = 1$)

$$S = \frac{1}{g_4^2} \int dx \sqrt{-\eta} \left(\frac{1}{2} \Phi \mathcal{F}(\Box) \Phi - \frac{1}{p+1} \Phi^{p+1}(x) \right)$$

Cubic Super SFT: $\mathcal{F}(z) = \left(\xi^2 z + 1\right) e^{-\frac{1}{4} z}$, $\xi^2 \approx 0.9556$, $p = 3$

Aref'eva, Belov, A.K. Medvedev, NP**B638** (2002) 3

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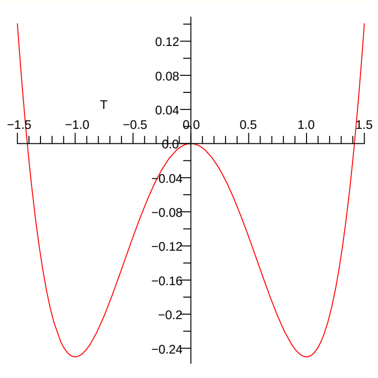

Tachyon potential (odd p)

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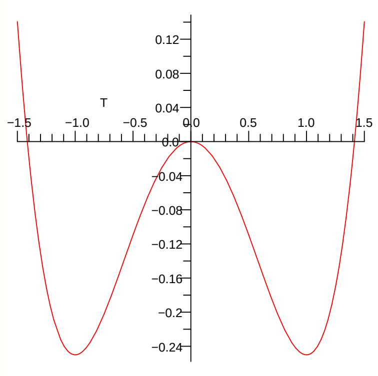

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We consider a generalization:

- $\mathcal{F}(z)$ is analytic in $\mathbb{C}$, i.e. $\mathcal{F}(z) = c_n z^n$
- $\mathcal{F}(0) = 1$, $c_n \in \mathbb{R}$
- Any $p > 1$
- $\eta \to g$, $\Box \to \Box_g = \frac{1}{\sqrt{-g}} \partial_\mu \sqrt{-g} g^{\mu\nu} \partial_\nu$

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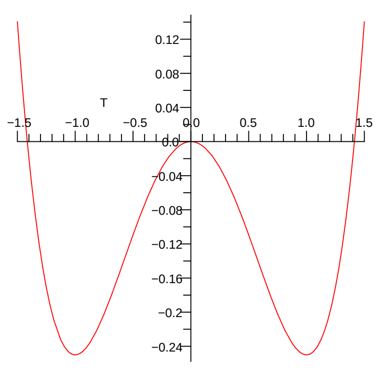

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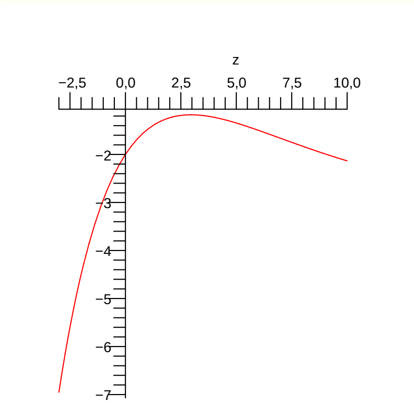
Our expectation:

Tachyon rolls down to the minimum and is expected to stop at the bottom in infinite time

Late time tachyon spectroscopy

$$\Phi = 1 - \psi \Rightarrow S = \frac{1}{g_4^2}\int dx\sqrt{-g}\left(\frac{1}{2}\psi\mathcal{F}(\Box_g)\psi - \frac{p}{2}\psi^2\right)$$

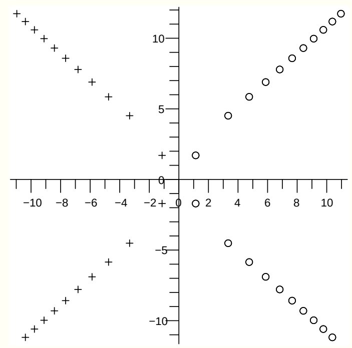


$\mathcal{F}(z) - p$ in CSSFT

Characteristic equation:

$$\mathcal{F}(\omega^2) = p$$

EOM: $(\mathcal{F} - p)\psi = 0$



Roots ω in CSSFT

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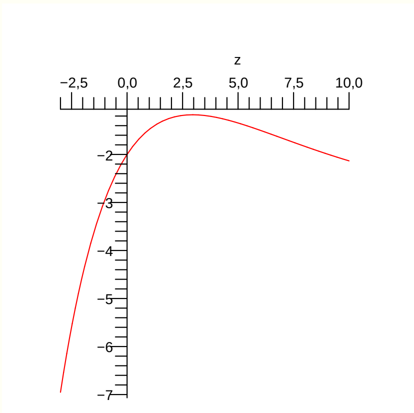

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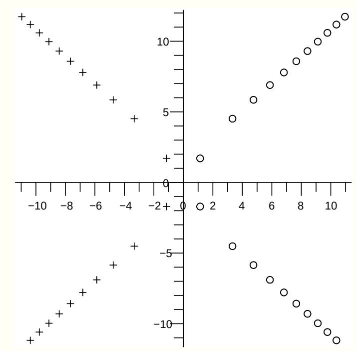

Roots ω in CSSFT

If $\Box_g f_\omega = \omega^2 f_\omega$ then $\mathcal{F}(\Box_g)f_\omega = \mathcal{F}(\omega^2)f_\omega$. Opposite is true for non-multiple roots only
We require no real roots and assume an absence of multiple roots.
For any root ω^2 its complex conjugate is the root too.

Infinitely many scalars vs. the non-locality

New action

$$S = \frac{1}{g_4^2}\int dx\sqrt{-g}\frac{1}{2}\sum_k \left(\varepsilon_k\psi_k(\Box_g - \omega_k^2)\psi_k + \bar{\varepsilon}_k\bar{\psi}_k(\Box_g - \omega_k^{2*})\bar{\psi}_k\right)$$

- EOMs are manifestly local and linear.
- Sum over k is indefinite until $\mathcal{F}$ is not specified explicitly
- $\varepsilon_k, \ \bar{\varepsilon}_k$ are constants to be determined through the energy-momentum tensor computation

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Since any ψ_k is an eigenfunction of operator $\Box_g$ with an eigenvalue which is a root of characteristic equation then it is an eigenfunction of operator $\mathcal{F}$ with an eigenvalue p

$\psi_k=\psi_{k+}+\psi_{k-}$ because $\Box_g$ is the second order differential operator

Equation for ψ is linear and, therefore, $\psi=\sum\limits_k\psi_k+\sum\limits_k\bar{\psi}_k$ is its solution.

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Thus, we have reproduced the spectrum by virtue of an (infinite) sum of non-interacting scalar fields.

Energy-momentum tensor

We consider spatially flat FRW universe as was mentioned above

$$\rho_\psi = \frac{K+P}{2}, \quad p_\psi = \frac{K-P}{2}$$

$$\text{where } K = \sum_{n=1}^{\infty} c_n \sum_{l=0}^{n-1} \partial_t \mathcal{D}^l \psi \partial_t \mathcal{D}^{n-1-l} \psi, \quad P = \sum_{n=1}^{\infty} c_n \sum_{l=0}^{n-1} \mathcal{D}^l \psi \mathcal{D}^{n-l} \psi$$

$$\text{with } \psi = \sum_k \left(\psi_{k+} + \psi_{k-} + \psi^*_{k+} + \psi^*_{k-}\right), \ \mathcal{D} = -\partial_t^2 - 3H\partial_t, \ H = \frac{\partial_t a}{a}$$

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Summation over n and l gives (dot denotes ∂_t)

$$K = \sum_k \left(\mathcal{F}'(\omega_k^2)\left(\dot\psi_{k+} + \dot\psi_{k-}\right)^2 + \mathcal{F}'(\omega_k^{2*})\left(\dot\psi_{k+}^* + \dot\psi_{k-}^*\right)^2\right),$$

$$P = \sum_k \left(\omega_k^2 \mathcal{F}'(\omega_k^2)\left(\psi_{k+} + \psi_{k-}\right)^2 + \omega_k^{2*}\mathcal{F}'(\omega_k^{2*})\left(\psi_{k+}^* + \psi_{k-}^*\right)^2\right).$$

Thus, fixing $\varepsilon_k = \mathcal{F}'(\omega_k^2)$, $\bar\varepsilon_k = \mathcal{F}'(\omega_k^{2*})$ and taking special solution $\bar\psi_k = \psi_k^*$ we reproduce with the local action the energy-momentum tensor too.

Remarkably, there is no mixing of modes with different k

(A generalization to a general (non-FRW) metric is straightforward)

Phantom emergence

Simplest consistent possibility: only single $\psi_{k+} \not\equiv 0$

We put $\psi = \alpha + i\beta$, $\omega^2 = M + iN$, $\mathcal{F}'(\omega^2) = x + iy$

Action for fields α and β becomes

$$S = \frac{1}{g_4^2}\int dx\sqrt{-g}\,(\alpha(x\mathcal{D} - xM + yN)\alpha - \beta(x\mathcal{D} - xM + yN)\beta - \\ -2\alpha(y\mathcal{D} - yM - xN)\beta)\,.$$

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- For any signs of parameters one normal and one phantom field present in the system
- Only field α is physical one since $\alpha = \frac{\psi+\psi^*}{2}$
- $N \neq 0$ because there are no real roots
- $M,\ x,\ y$ are not restricted but at least one of x or y is non-zero

This action may serve as a toy model for the tachyon around its vacuum.

Tachyon at large times must have phantom properties

Minimal coupling to gravity and Cosmology

$$S = \int dx \sqrt{-g} \left(\frac{R}{2\kappa^2} + \frac{1}{g_4^2} \left(\frac{1}{2} \Phi \mathcal{F}(\Box_g) \Phi - \frac{1}{p+1} \Phi^{p+1}(x) - \frac{p-1}{2(p+1)} - \tau \right) \right)$$

- $\kappa^2 = 8\pi G = \frac{1}{M_P^2}$
- τ is a correction to the brane tension dictated by an existence of the rolling solution
- We introduce I.Aref'eva, `astro-ph/0410443`

$$\Lambda = \frac{\tau}{g_4^2}$$

Independent equations

$$\frac{\ddot{a}}{a} = -\frac{1}{6 g_4^2 M_P^2} (\rho_\Phi + 3 p_\Phi) + \frac{\Lambda}{3 M_P^2}$$

$$\mathcal{F}(\mathcal{D}) \Phi = \Phi^p$$

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$$\mathcal{F}(\mathcal{D})\Phi = \Phi^p$$

We are about to expand $\Phi=1-\psi$

Approximate behavior near tachyon vacuum

$$S = \int dx\sqrt{-g}\left(\frac{R}{2\kappa^2} + \frac{1}{g_4^2}\left(\frac{1}{2}\psi\mathcal{F}(\Box_g)\psi - \frac{p}{2}\psi^2(x)\right) - \Lambda\right)$$

Using the developed machinery we pass to a local theory with many scalars and under an assumption that only one specific mode $\psi_{k+} \not\equiv 0$ one has

$$\psi = \alpha e^{-rt}\cos(\nu t + \varphi)$$

$$a = a_0 e^{H_0 t} +$$

$$+ \frac{e^{(H_0-2r)t}}{g_4^2 M_P^2}\left(s_K\sin(2\nu t + \varphi_K) + s_P\sin(2\nu t + \varphi_P) + c_K\cos(2\nu t + \varphi_K) + c_P\cos(2\nu t + \varphi_P)\right)$$

where r and ν are real and imaginary parts of $\frac{3}{2}H_0 \pm \sqrt{\frac{9}{4}H_0^2 - \omega_k^2}$ and $H_0 = \sqrt{\frac{\Lambda}{3M_P^2}}$

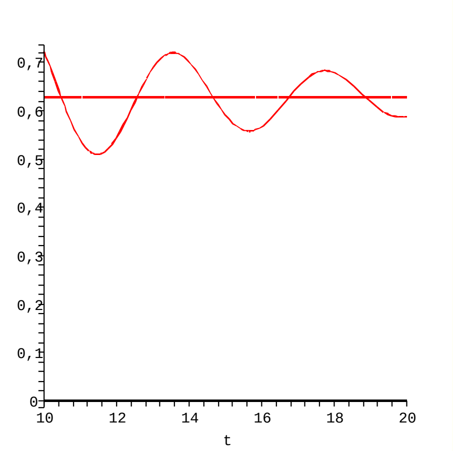
For $r = H_0/2$ oscillations in $a(t)$ will not die despite the fact that oscillations in Φ vanish.

Cosmological properties

Generic parameters, i.e. not necessarily $r = H_0/2$.

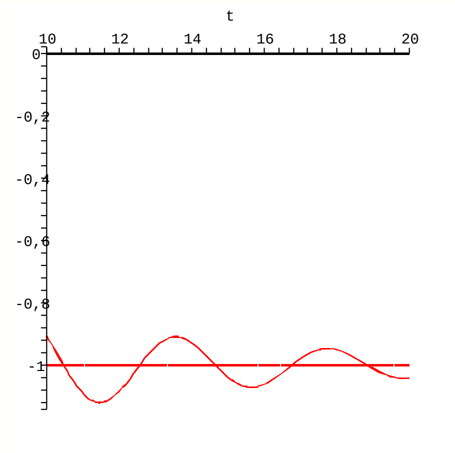
Hubble parameter

$$H = \frac{\dot{a}}{a}$$



Total effective state parameter

$$w = -1 - \frac{2}{3}\frac{\dot{H}}{H^2}$$



Quintessence and Phantom phases change one each other.

No Big Rip singularity

Crossing of the phantom divide

Summary

- Non-local action with a general operator $\mathcal{F}$ is analyzed and a local formulation for a linearization near a non-perturbative vacuum is given.
- The energy and pressure in FRW metric are formulated for a general function $\mathcal{F}$ without specifying its explicit form. Moreover, this analysis can be easily extended to a general background.
- It is shown that tachyon scalar field generates a crossing of the phantom divide in the cosmological constant background. This crossing is periodic one and a condition of non-vanishing oscillations is formulated.
- The Big Rip singularity problem is avoided.

Further directions

- Coupling to closed string scalars, vector field etc.
- Proof of stability of found solutions
- Cosmological perturbations of theories with infinitely many derivatives
- Numeric and may be analytic solution to full equations

- As well as many other questions