Stochastic background of gravitational waves “tuned” by f(R) gravity

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Gravitational Waves in f(R)-theories

Gravitational Waves Equation

$$\Box h_i^j = 0$$

Action in Generic f(R)-Theory of Gravity

$$\mathcal{A} = \frac{1}{2k}\int d^4x\sqrt{-g}f(R)$$

Conformal Transformation

$$\widetilde{g}_{\mu\nu} = e^{2\Phi}g_{\mu\nu} \qquad \text{with} \qquad e^{2\Phi} = f'(R)$$

Conformal scalar field

Conformally Equivalent Hilbert-Einstein action

$$\mathcal{A} = \frac{1}{2k}\int \sqrt{-\widetilde{g}}d^4x\left[\widetilde{R} + \mathcal{L}\left(\Phi,\Phi_{;\mu}\right)\right]$$

where

$$\widetilde{R}_{\mu\nu} = R_{\mu\nu} + 2\left(\Phi_{;\mu}\Phi_{;\nu} - g_{\mu\nu}\Phi_{;\delta}\Phi^{;\delta} - \Phi_{;\mu\nu} - \frac{1}{2}g_{\mu\nu}\Phi^{;\delta}{}_{;\delta}\right)$$

$$\widetilde{R} = e^{-2\Phi}\left(R - 6\Box\Phi - 6\Phi_{;\delta}\Phi^{;\delta}\right)$$

Gravitational Waves in f(R)-theories

GW Equations in conformal metric

$$\widetilde{\Box}\widetilde{h}_i^j = 0$$

is a conformal invariant

$$\widetilde{h}_i^j = \widetilde{g}^{lj}\delta\widetilde{g}_{il} = e^{-2\Phi}g^{lj}e^{2\Phi}\delta g_{il} = h_i^j$$

Amplitude of plane-wave

$$h(t) = e_i^j \exp(ik_i x^i)$$

Polarization tensor

d'Alembert operator trasforms as

$$\widetilde{\Box} = e^{-2\Phi}\left(\Box + 2\Phi^{;\lambda}\partial_{;\lambda}\right)$$

Cosmological stochastic background

GW equations in Friedmann-Robertson-Walker reduce to

$$\ddot{h} + \left(3H + 2\dot{\Phi}\right)\dot{h} + k^2 a^{-2} h = 0$$

being $\Box = \frac{\partial}{\partial t^2} + 3H\frac{\partial}{\partial t}$

scalar factor $a(t)$

wave number k

GW'solutions (in particular, the amplitudes) depend on:

- Specific cosmological background $a(t)$
- Specific theory of gravity (i.e. conformal field) $\Phi(t)$

Cosmological stochastic background

Power law behaviors for $a(t)$ and $\Phi(t) = \frac{1}{2}\ln f'(R(t))$

$$\Phi(t) = f'(R) = f'_0\,(t/t_0)^m\,, \quad a(t) = a_0\,(t/t_0)^n$$

General relativity is recovered for $m = 0$

The relation $n = \frac{m^2+m-2}{m+2}$ between the parameters of a generic $f(R) = f_0 R^s$

where $s = 1 - \frac{m}{2}$ with $s \neq 1$

See *Capozziello et al. 2005,2006*

The GW equation for the amplitude becomes

$$\ddot{h} + (3n+m)\,t^{-1}\dot{h} + k^2 a_0\,(t_0/t))^{2n}\,h = 0$$

with the general solution

$$h(t) = \left(\frac{t}{t_0}\right)^{\beta}\left[C_1 J_\alpha(x) + C_2 J_{-\alpha}(x)\right]$$

$$x = \frac{kt^{1-n}}{1-n}$$

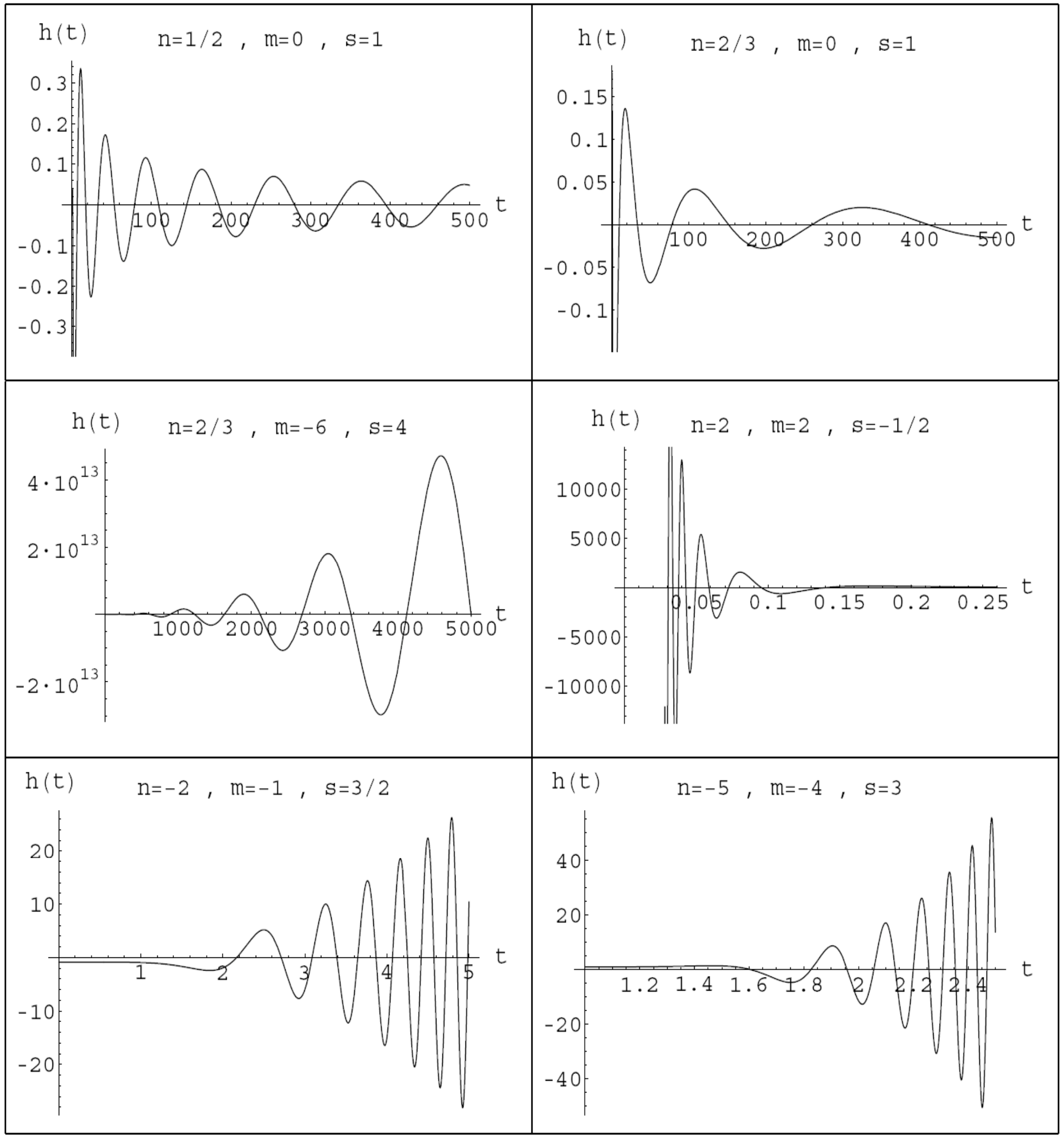
Bessel function

$$\beta = \frac{1-3n-m}{2}$$

$$\alpha = \frac{1-3n-m}{2(n-1)}$$

$t_0,\, C_1,\, C_2$ are integration constants related to the specific values of $m.$ and n

Waveforms



The plots are labelled by the set of parameters which assign the time evolution of $\Phi(t)$ and $a(t)$ with respect to a given power-low theory

$$f(R) = f_0 R^s$$

The time units are in terms of the Hubble radius H^{-1}

$n = 1/2$ **Radiation like-evolution**

$n = 2/3$ **dust like-evolution**

$n = 2$ **Power-low inflationary phases**

$n = -5$ **Pole-like inflation**

Singular case for $m = -2$ $s = 2$

The amplitude evolution of GW strictly depends on the background !!!

Production mechanisms contributing to the stochastic background

Several mechanisms can be considered :

- Cosmological populations of astrophysical sources
- Vacuum fluctuations
- Primordial phase transitions….

We could seek, in principle, for contributions due to every high-energy process in the early Universe evolution

Cosmological processes, in strict sense, are phase transitions related to inflation, where Hubble flow emerges in the radiation dominated era. Astrophysical processes are early star formation rates, where the production takes place during the dust dominated era.

Key Issue:

Stochastic GWs background is strictly related to the cosmological model and connected to the specific theory of gravity!

Production of GWs contributing to the stochastic background

Assume that the main contribution comes from the amplification of vacuum fluctuations at the transition between the inflationary phase and radiation dominated area.

In any inflationary model, we can assume that the GWs generated as zero-point fluctuation during the inflation undergo adiabatically damped oscillations $(\sim 1/a)$ until they reach the Hubble radius H^{-1}.

This is a particle horizon for the growth of perturbations.

Any other previus fluctuation is smoothed away by the inflationary expansion.

GWs freeze out for $a/k \gg H^{-1}$ and reenter the H^{-1} radius after reheating in the Friedmann era.

The reenter in radiation-dominated or in the dust-dominated era depends on the scale of GW.

GWs can be detected by their Sachs-Wolfe effect on the temperature anisotropy $\triangle T/T$ at the decoupling.

When Φ acts as the inflaton, we have $\dot{\Phi} \ll H$ during the inflation

Considering the conformal time $d\eta = dt/a$, the GWs equation reads

$$h'' + 2\frac{\chi'}{\chi}h' + k^2 h = 0$$

Where $\chi = ae^{\Phi}$

Inflation means that $a(t) = a_0 \exp(Ht)$ and $\eta = \int dt/a = (aH)^{-1}$

and $\chi'/\chi = -\eta^{-1}$

The exact solution is

$$h(\eta) = k^{-3/2}\sqrt{2/k}\,[C_1\,(\sin k\eta - \cos k\eta) + C_2\,(\sin k\eta + \cos k\eta)]$$

Inside the Hubble radius H^{-1}, we have $k\eta \gg 1$

Considering the absence of gravitons in the initial vacuum state, we have only negative modes and the adiabatic behavior is

$$h = k^{1/2}\sqrt{2/\pi}\frac{1}{aH}C\exp(-ik\eta)$$

At first horizon crossing $(aH = k)$, the average amplitude of perturbations $A_h = (k/2\pi)^{3/2}\,|h|$ becomes

$$A_h = \frac{1}{2\pi^2}C$$

when the scale a/k grows than H^{-1}, the growing mode of evolution is constant, that is it is frozen.

The GW amplitude A_h is preserved until the second horizon crossing. After, it can be observed, as an anisotropy perturbation on the CMBR.

Considering a single graviton in the form af a monochromatic wave, we have

$$[h(t,x),\, \pi_h(t,y)] = i\delta^3(x-y)$$

The Lagrangian is

$$\widetilde{\mathcal{L}} = \frac{1}{2}\sqrt{-\widetilde{g}}\widetilde{g}^{\mu\nu} h_{;\mu} h_{;\nu}$$

In FRW conformal metric $\widetilde{g}_{\mu\nu}$, h is conformally invariant. We obtain

$$\pi_h = \frac{\partial \widetilde{\mathcal{L}}}{\partial \dot{h}} = e^{2\Phi} a^3 \dot{h}$$

Where the commutation relation becomes

$$\left[h(t,x),\, \dot{h}(y,y)\right] = i\frac{\delta^3(x-y)}{a^3 e^{2\Phi}}$$

The fields h and $\dot{h}$ can be expanded in terms of creation and annihilation operators

$$h(t,x) = \frac{1}{(2\pi)^{3/2}} \int d^3k \left[h(t)e^{-ikx} + h^*(t)e^{+ikx}\right]$$

$$\dot{h}(t,x) = \frac{1}{(2\pi)^{3/2}} \int d^3k \left[\dot{h}(t)e^{-ikx} + \dot{h}^*(t)e^{+ikx}\right]$$

Commutation relations in conformal time are

$$[hh'^* - h^*h'] = \frac{i(2\pi)^3}{a^3 e^{2\Phi}}$$

We obtain $C = \sqrt{2}\pi^2 H e^{-\Phi}$

and then $A_h = \frac{\sqrt{2}}{2} H e^{-\Phi}$

The amplitude of GWs produced during inflation directly depends on the given $f(R)$ theory being $\Phi = \frac{1}{2}\ln f'(R)$

Explicitly

$$A_h = \frac{H}{\sqrt{2f'(R)}}$$

- The amplitude of GWs produced during inflation depends on the given theory of gravity that, if different from GR, gives extra degrees of freedom which assume the role of inflaton field in the cosmological dynamics.
- The Sachs-Wolfe effect related to the CMBR temperature anisotropy could constitute a powerful tool to test the *true* theory of gravity at early epochs, i.e, at very high redshifts.

This probe, related with data at medium and low redshift, could strongly contribute:

- to reconstruct cosmological dynamics at every scale,
- to further test GR or to rule out it against alternative theories
- To give constraints on the GW-stocastic background, if $f(R)$ theories are independently probed at other scales

Summary

- Amplitudes of tensor GWs are conformally invariant and their evolution depends on the cosmological background
- Background is tuned by conformal scalar field which is not present in the standard GR. Assuming that primordial vacuum fluctuations produce stochastic GWS, beside scalar perturbations, kinematical distorsions and so on, the initial amplitude of these ones is a function of the f(R) theory of gravity and then the stochastic background can be, in certain sense “tuned” by theory.
- Data coming from Sachs-Wolfe effect could contribute to select a suitable f(R) theory which can be consistently matched with other observations.
- Further and accurate studies are needed in order to test the relation between Sach-Wolfe effect and f(R) gravity.
- This goal could be achieved very soon through the forthcoming space (LISA) and ground-based (VIRGO, LIGO) interferometers.