New cosmological models for an accelerated universe

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Summary

1. ***Brief revision of quintessence models.***
2. ***Quintessence fluid + positive cosmological constant.***
3. ***Quintessence fluid + negative cosmological constant.***
4. ***RS1 + Quintessence fluid.***
5. ***Conclusions***

1. QUINTESSENCE

Friedmann equations

$$\ddot{a} > 0 \Rightarrow \rho + 3p < 0$$

$$p = w\rho$$

$$w = const$$

$$w < -1/3$$

$$a(t) = a_0\left(1 + \frac{3}{2}C(1+w)(t - t_0)\right)^{\frac{2}{3(1+w)}}$$

$$C = \frac{8\pi}{3}\rho_0$$

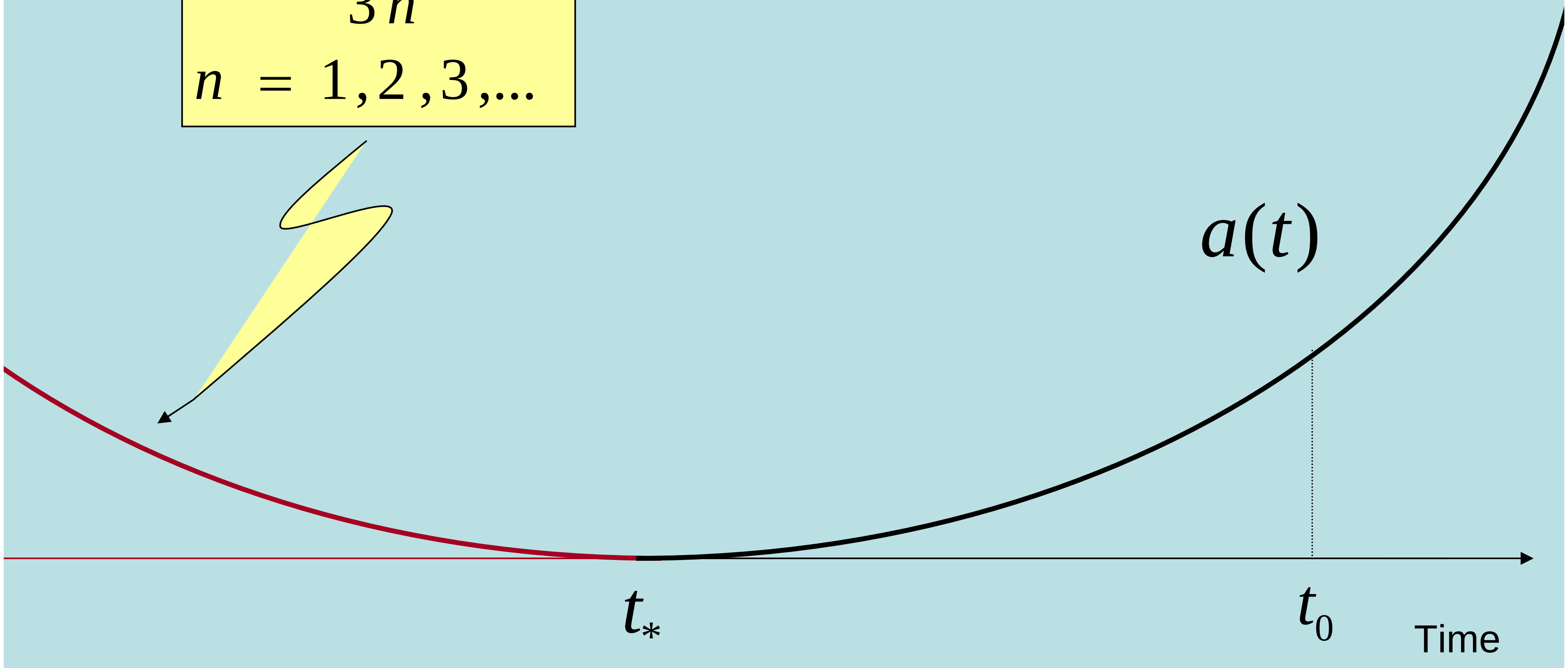
1.1. Dark energy

$$-1/3 > w > -1$$

$$t_* = t_0 - \frac{2}{3C(1+w)} \Rightarrow a(t_*) = 0, \rho \to \infty$$

$$w = \frac{1}{3n} - 1$$

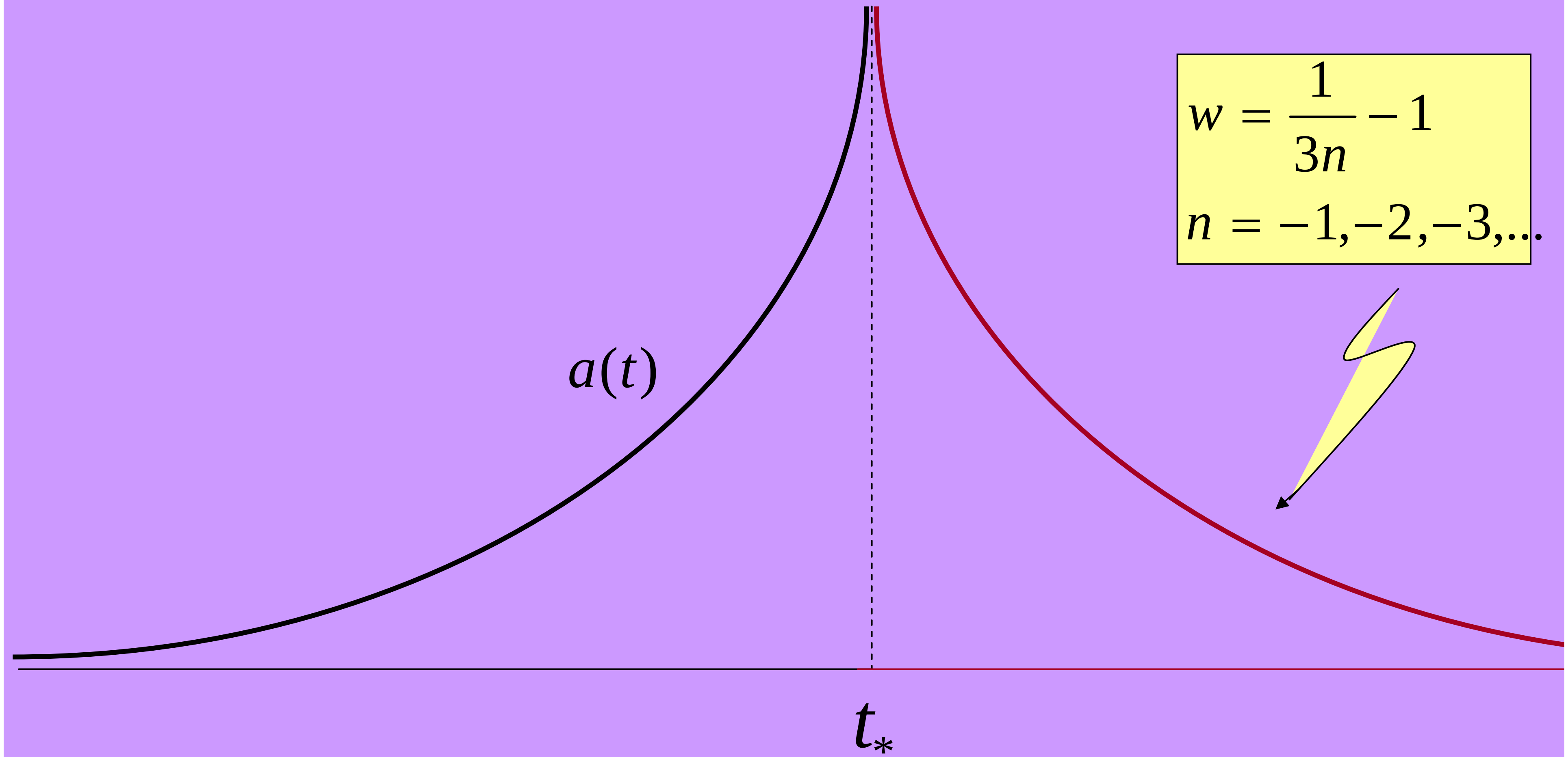
$$n = 1, 2, 3, \ldots$$



1.2. Phantom energy

$$w < -1$$

$$t_* = t_0 + \frac{2}{3C(|w|-1)} \quad \Rightarrow a(t_*) \to \infty, \rho(t_*) \to \infty$$



$$w = \frac{1}{3n} - 1$$

$$n = -1, -2, -3, \ldots$$

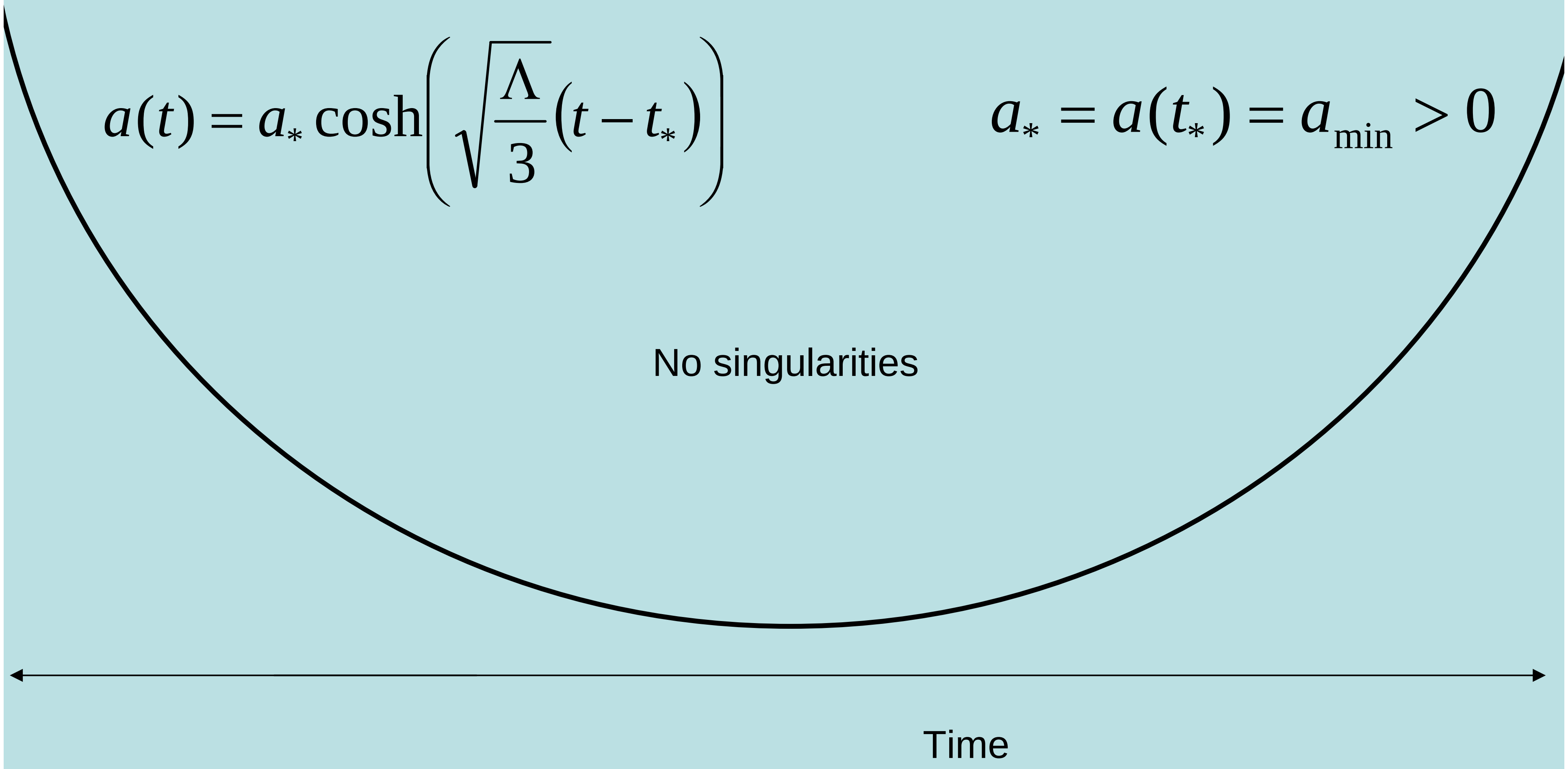
1.3. Limiting case: de Sitter

$$w = -1$$

This model is equivalent to a empty universe equipped with a cosmological constant.

$$a(t) = a_* \cosh\left(\sqrt{\frac{\Lambda}{3}}(t - t_*)\right)$$

$$a_* = a(t_*) = a_{\min} > 0$$



2. Vacuum energy = dynamical part + cosmological constant > 0

Friedmann equation:

$$\lambda = \frac{\Lambda}{3} > 0$$

$$p = w\rho, w = const$$

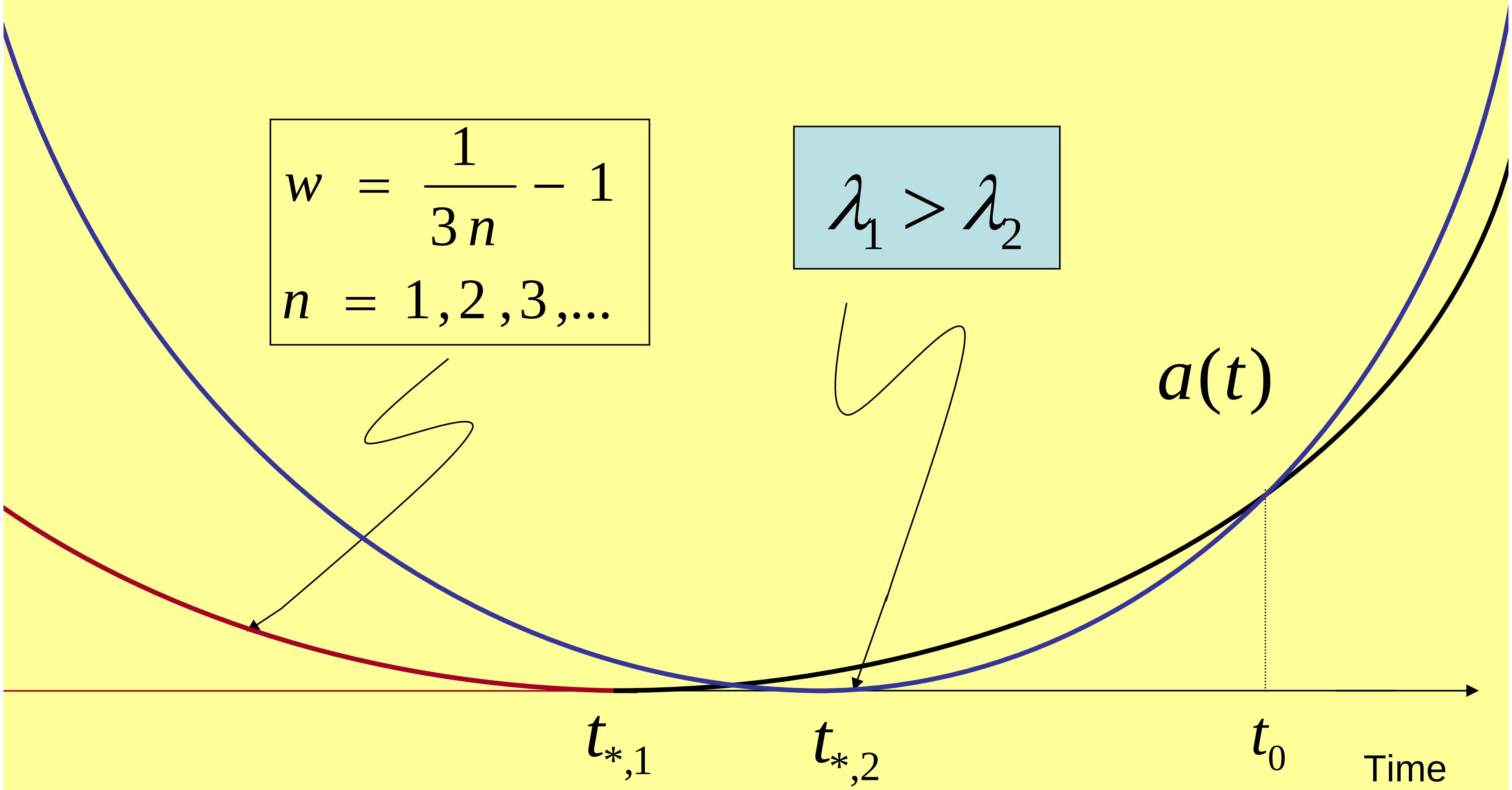
$$H^2 = \frac{8\pi}{3}\rho + \lambda$$

$$a(t) = a_0 \left(\frac{C}{4\lambda D} \right)^{\frac{1}{3(1+w)}} \left[e^{\frac{3}{2}(1+w)\sqrt{\lambda}(t-t_0)} - De^{-\frac{3}{2}(1+w)\sqrt{\lambda}(t-t_0)} \right]^{\frac{2}{3(1+w)}}$$

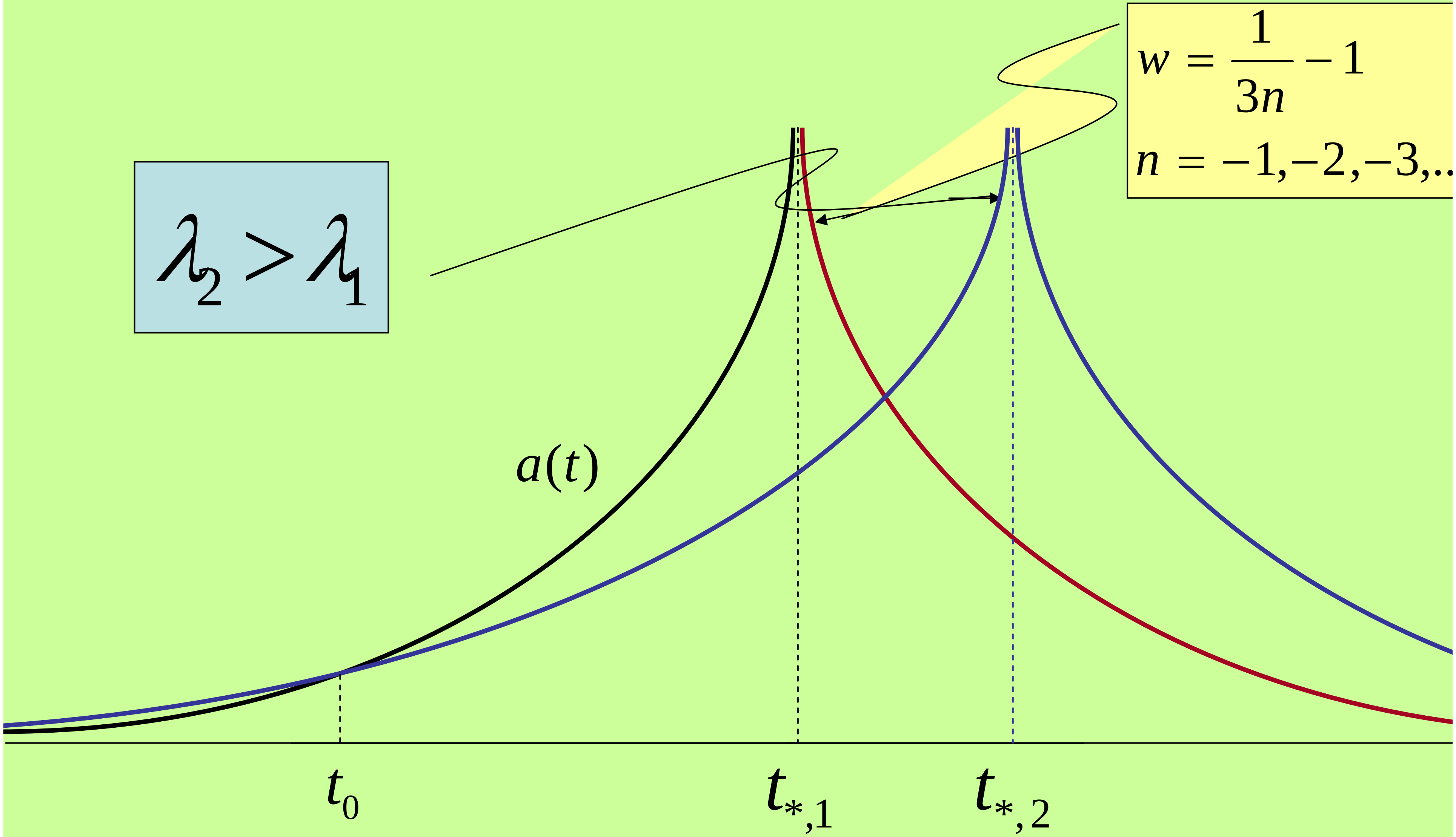
$$D = \frac{\sqrt{\lambda + C} - \sqrt{\lambda}}{\sqrt{\lambda + C} + \sqrt{\lambda}}, C = \frac{8\pi}{3}\rho_0$$

$$0 < D < 1$$

2.1. Universe equipped with a positive cosmological constant and filled with dark energy (w > -1)



2.2. Universe equipped with a positive cosmological constant and filled with phantom energy (w < -1)



- The presence of a positive cosmological constant consistently slows down the expansion of the quintessence models but still causes accelerating expansion.

- The equation of state parameter shows the same discretization as in the models without cosmological constant.

- The only way for not having a singularity is by making D=0, i. e., the dynamical part should vanish and one recovers the de Sitter space.

3. Vacuum energy = dynamical part + cosmological constant < 0

Friedmann equation:

$$H^2 = \frac{8\pi}{3}\rho - \lambda$$

$$\lambda = \frac{|\Lambda|}{3} > 0$$

$$p = w\rho, w = const$$

$$a(t) = a_0\left[\cos\left(c\left(t - t_0\right)\right) + b\sin\left(c\left(t - t_0\right)\right)\right]^{\frac{2}{3\beta}}$$

$$\beta = 1 + w, c = \frac{3\beta}{2}\sqrt{\lambda}, b = \sqrt{\frac{8\pi}{3\lambda} - 1}.$$

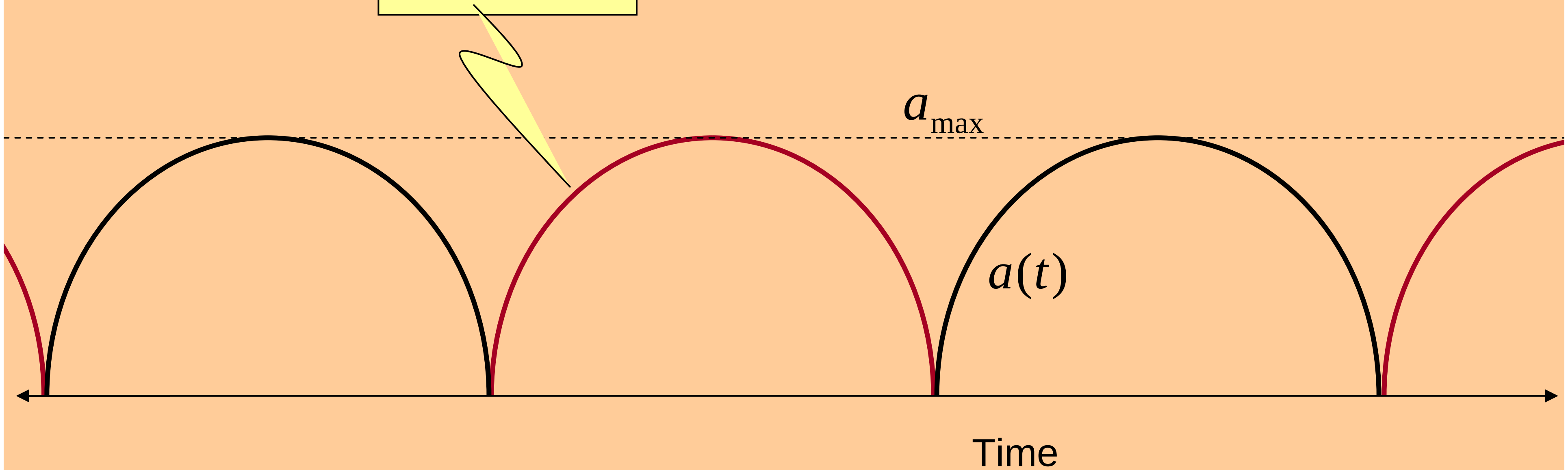
3.1. Universe equipped with a negative cosmological constant and filled with dark energy $\beta > 0$

$$t_{*m} = t_0 + \frac{1}{c}\arctan\left(-\frac{1}{b}\right) + \frac{m\pi}{c}, m = \ldots,-2,-1,0,1,2,\ldots \Rightarrow a\left(t_{*m}\right) = 0$$

$$w = \frac{1}{3n} - 1$$

$$n = 1,2,3,\ldots$$

It cannot describe our universe.

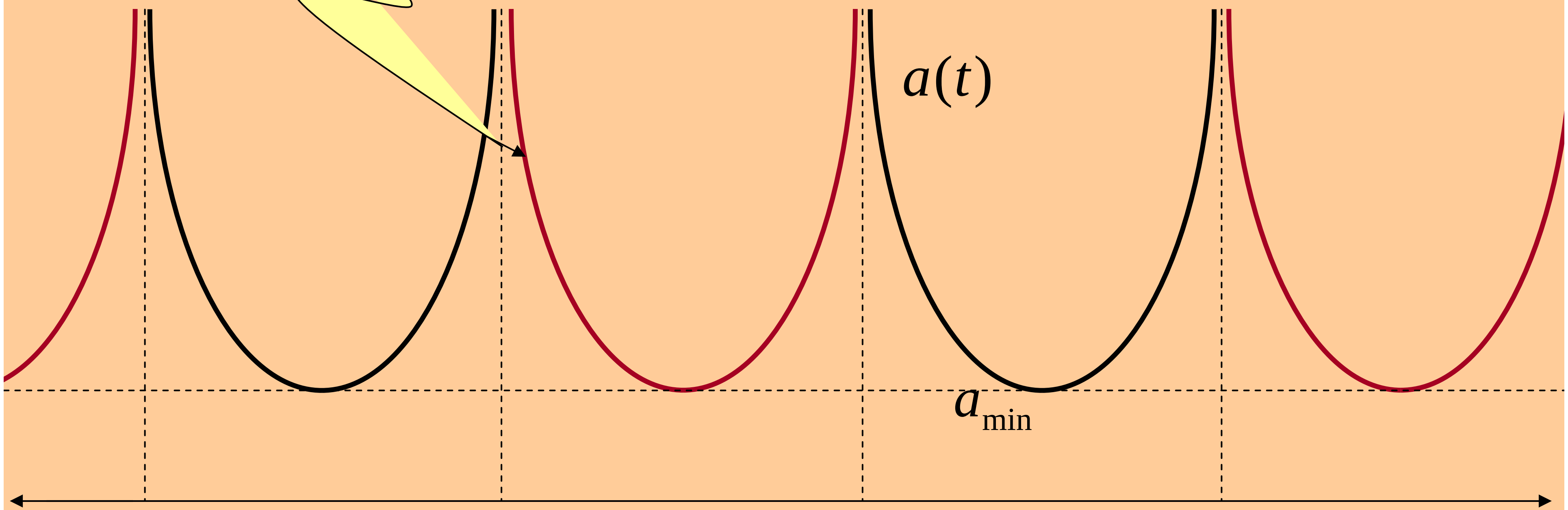


3.2. Universe equipped with a negative cosmological constant and filled with phantom energy $\beta < 0$

$$t_{*m} = t_0 + \frac{1}{c}\arctan\left(\frac{1}{b}\right) + \frac{m\pi}{c}, m = \ldots,-2,-1,0,1,2,\ldots \Rightarrow a\left(t_{*m}\right) \to \infty$$

$$w = \frac{1}{3n} - 1$$

$$n = -1,-2,-3,\ldots$$



- The same discretization as in the other cases is required in order to have a well defined scale factor in each region.

- There is a multiplicity of singularities.

- Although a negative cosmological constant induces an increase of acceleration, this occurs on regions without physical meaning in the model with dark energy; therefore, once one takes into account the discretizacion of w, the accelerating regions convert themselves into deccelerating. So this model cannot describe our universe.

- Classically, a singularity cuts off the space-time, so the different regions between singularities would be isolated. Each of them would correspond to a different universe, i. e., another space-time.

4. RS1 Scenario

RS1:

$$\left(\frac{\dot{a}}{a}\right)^2 = \rho\left(1+\frac{\rho}{2\sigma}\right), \sigma > 0$$

$$-2\frac{\ddot{a}}{a} = \rho + 3p + \frac{\rho}{\sigma}(2\rho + 3p)$$

$$\frac{p}{\rho} = w = -1 - 2\varepsilon$$

$$a^{6\varepsilon} = \frac{s}{1-18\sigma\varepsilon^2(t-t_1)^2}$$

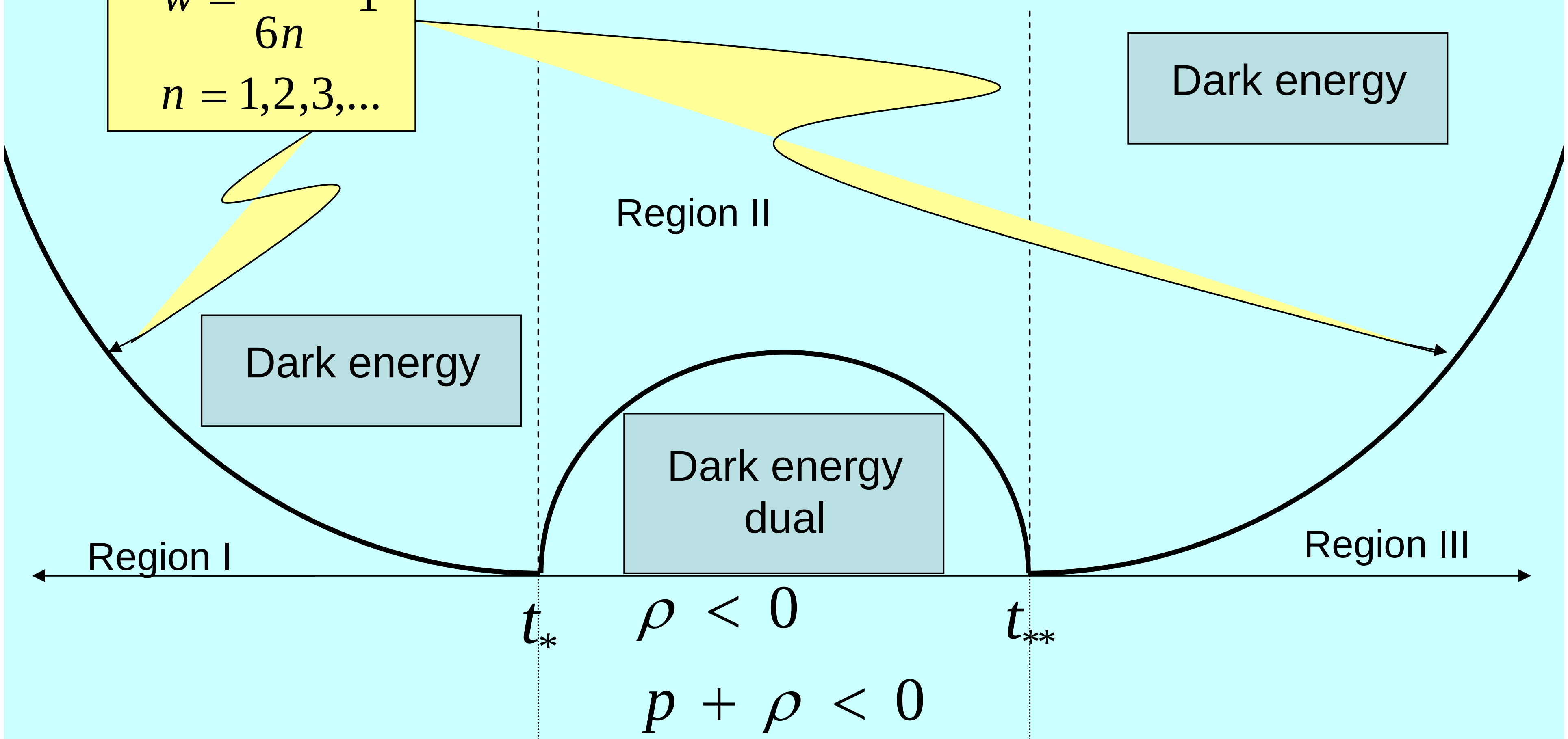
$$s = const > 0$$

4.1. RS1 with dark energy (w > -1) $\left(\varepsilon < 0\right)$

$$t_* = \frac{1}{3\varepsilon\sqrt{2\sigma}} = -t_{**} \Rightarrow a(t_*) = a(t_{**}) = 0, \rho \to \infty$$

$$w = \frac{1}{6n} - 1$$

$$n = 1,2,3,\dots$$

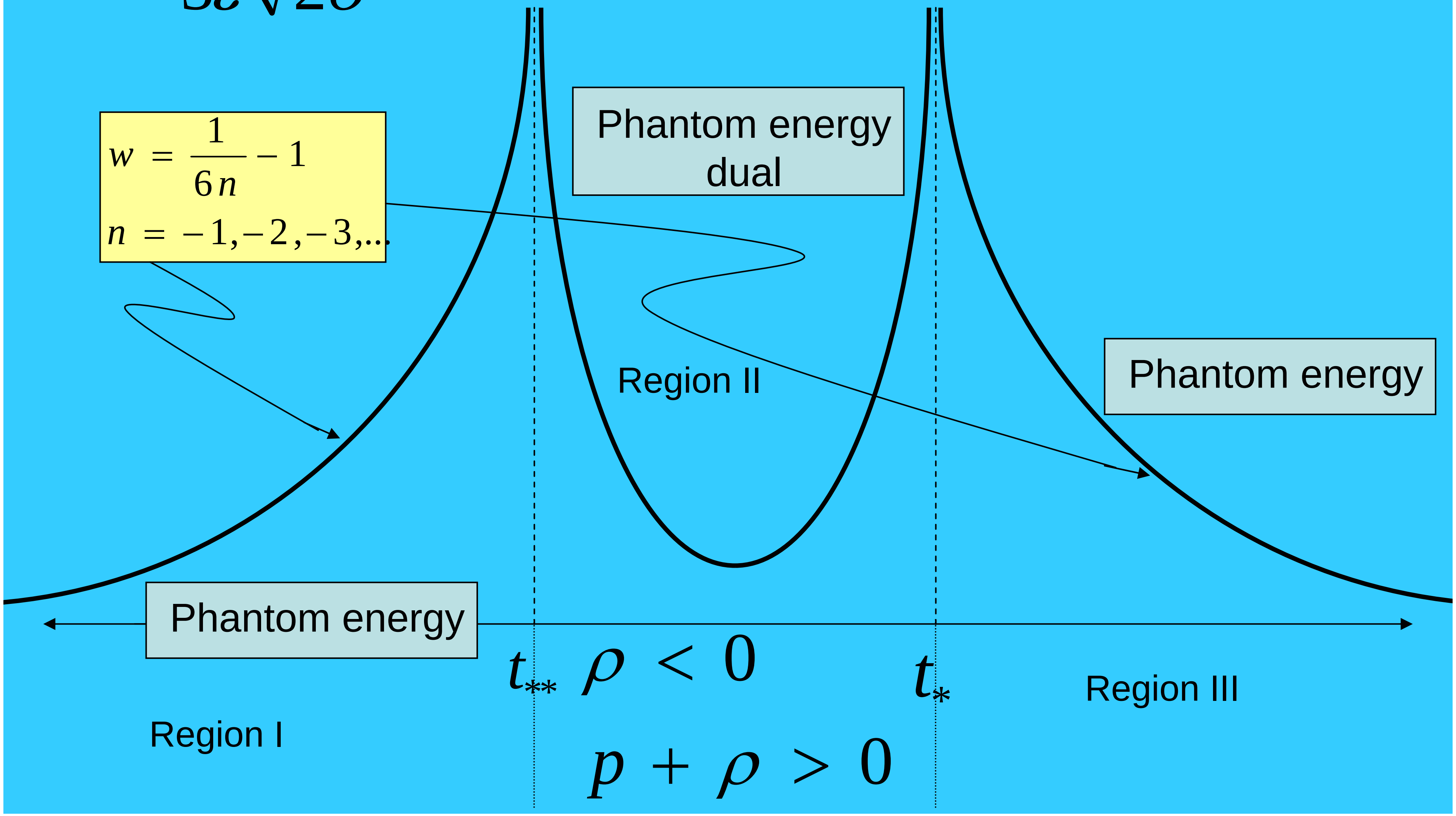


$$\rho < 0$$

$$p + \rho < 0$$

4.2. RS1 with phantom energy (w < -1) $(\varepsilon > 0)$

$$t_* = \frac{1}{3\varepsilon\sqrt{2\sigma}} = -t_{**} \Rightarrow a(t_*) = a(t_{**}) \to \infty, \rho \to \infty$$



- When we study a model RS1 (5D with a brane) with a dark/phantom energy fluid, the brane effect produces a dual dark/phantom energy fluid on a given region.

- The brane unfolds one single singularity into two singularities.

- On the regions with normal dark/phantom energy fluid it is necessary a discretization of the equation of state parameter.

- If the brane tension goes to infinity, the brane effects disappear: there is not a dual fluid and the two singularities become a single one at $t=0$.

5. Summary conclusions

- An accelerating universe requires a fluid with an equation of state parameter $-1 < w < -1/3$ (dark energy) or $w < -1$ (phantom energy). The latter produces superaccelerated expansion.
- To continue a quintessence model beyond a singularity one needs a discretization of w.
- If we supplement a quintessence model with a positive cosmological constant, the accelerated expansion is slowed down.
- A negative cosmological constant may give rise to surprising effects, like a slowing of acceleration in dark energy case due to the discretization of w or, more importantly, a multiverse generated from the existence of infinite singularities.
- In a RS1 model filled with those kinds of fluid the brane produces a dual fluid, splitting every singularity into two.