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THE RELATIONSHIP BETWEEN PATENT LICENSING AND COMPETITIVE BEHAVIOR*

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The analysis in this paper uncovers a heretofore unrecognized connection between the degree of competitive behavior in the product market and the preferred patent licensing mechanism. Using conjectural variations as a shorthand for several different supergame outcomes of a duopoly industry, we show that both the degree of competitive behavior and the extent of the asymmetries in the behavior of the firms in the market have a non-trivial effect on the patent licensing mechanism preferred by an external patentee. The relationship between the degree of competition in product markets and the preferred patent licensing mechanism, to the best of our knowledge, has not been addressed in the literature previously and represents a useful source of new empirical implications.

1 Introduction

In order to give private agents incentives to engage in costly research activities, patents and other forms of property rights are typically assigned to innovators. As a result, the analysis of patent licensing mechanisms occupies an important place in the literature on the creation and the diffusion of new knowledge. Empirically, the patent licensing mechanisms most often observed in practice are a royalty per unit of output produced with the new technology, a fee independent of the quantity produced with the new technology, or a combination of both.

An important theoretical literature has studied the implications of these and other mechanisms, as well as the preference for royalties often observed in practice. In particular, the literature has attempted to justify the presence of royalty payments in the licensing agreements by considering the role of, for example, product differentiation (Muto, 1993), the effects of risk sharing (Bousquet *et al.*, 1998), the patentee being one of the firms in the industry (Wang, 1998; Kamien and Tauman, 2002), the separation of ownership from management (Saracho, 2002) or moral hazard (Jensen and Thursby, 2001).

This paper offers an entirely new angle on this problem. We examine the relationship between the nature and degree of competition in the product

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market and the profitability of two one-stage patent licensing mechanisms (royalty and fixed-fee) from an external patentee's viewpoint. Specifically, we show how the superiority of the royalty over the fixed-fee mechanism typically observed empirically may be explained by simply taking into account the degree of competitive behavior of the market. To the best of our knowledge, despite the potential relevance of this connection, this natural relationship has not been addressed previously in the literature on patent licensing. Moreover, this new angle represents a complement to, rather than a substitute for, all previous attempts in the literature.

In order to implement the analysis and derive the results in a way that is as transparent and general as possible, we use the conjectural variation approach. As Dixit (1986) points out, there are two main advantages of using this approach. First, by specifying different conjectures appropriately, we can encompass many different classical models in the same formal framework. Second, the conjectural variations treated as parameters allow us to capture the idea of varying degrees of competition. Also, as Friedman and Mezzetti (2002) have recently shown, it is possible to provide a consistent reinterpretation of conjectural variations using a dynamic setting with boundedly rational firms.

The structure of the paper is as follows. In Section 2 we describe the model and then analyze and compare the fixed-fee mechanism and the royalty mechanism when firms are assumed to have identical conjectural variations. Section 3 extends the analysis to models where firms have different conjectural variations. This allows us evaluate a number of additional aspects with regard to the optimal licensing mechanism from the point of view of the patentee. Section 4 contains some concluding remarks.

2 The Framework and the Comparison between Fixed-fee and Royalty Licensing

Consider a quantity-setting, conjectural variation duopoly that produces a homogeneous good. The inverse demand function for the good is $P = a - bQ$, where Q is the total production of the industry. The marginal cost of production of each firm is constant and equal to c with $a > c$. An external research laboratory owns a patent on an innovation which reduces the marginal cost of production of the firms in the industry to c' where $c' = c - \varepsilon$. In choosing the production level that maximizes profits, each firm forms a conjectural variation γ about the response of the other firm to a change in its own output level. We first consider that conjectural variations are identical across firms: $\partial q_j/\partial q_i = \gamma$, $i \neq j$, where $-1 < \gamma \leq 0$. As indicated above, Section 3 extends the analysis to models in which firms have different conjectural variations. Notice that *ceteris paribus* a lower γ implies that firms will exhibit a greater degree of competitive behavior. It is not difficult to show that Cournot competition corresponds to the case where $\gamma = 0$ and the case of incentive delegation in Fershtman and Judd (1987) corresponds to the case where $\gamma = -\frac{1}{2}$.[1]

We are interested in evaluating and comparing the fixed-fee and royalty licensing mechanisms from the point of view of the patentee under different degrees of competition in the product market.[2] The analysis is modeled as a non-cooperative game that consists of three stages. In the first stage, the patentee sets either a royalty per unit of output produced with the new technology or a price independent of production. In the second stage, all firms in the industry are informed on the price set by the patentee and, simultaneously and independently, decide whether or not to buy a license at the announced fee or royalty. In the third stage firms engage in a competition game. We look for the subgame perfect Nash equilibrium in pure strategies. Therefore, we proceed by backward induction starting from the third stage.

In the third stage, each firm i, $i = 1, 2$, solves the following problem:

$$\max_{q_i}(a - bQ - c_i)q_i$$

subject to

$$q_i \geq 0$$

Without loss of generality, we assume that if there is only one adopter then that will be firm 1. It can be readily shown that, in equilibrium, the quantity produced by each firm i, q_i, and its profits, π_i, will be

$$q_i = \frac{(1+\gamma)a - (2+\gamma)c_i + c_j}{(\gamma+1)(\gamma+3)b} \qquad \pi_i = \frac{[(1+\gamma)a - (2+\gamma)c_i + c_j]^2}{(\gamma+1)(\gamma+3)^2 b}$$

$$\text{if } c_i < \frac{(1+\gamma)a + c_j}{2+\gamma}, i = 1, 2$$

$$q_1 = \frac{a-c}{b} \qquad q_2 = 0 \qquad \pi_1 = \frac{\varepsilon(a-c)}{b} \qquad \pi_2 = 0$$

$$\text{if } \frac{(1+\gamma)a + c_1}{2+\gamma} \leq c_2 < a - \varepsilon \tag{1}$$

[1] Strictly positive values of the conjectural variations, $0 < \gamma \leq 1$, imply that the behavior of firms is more collusive than that under Cournot competition. None of the results of the paper changes if we also consider these parameter values.

[2] As shown by Katz and Shapiro (1986), in the presence of interdependent demands, an auction mechanism is at least as good as the fixed-fee mechanism from the viewpoint of the patentee. However, as Kamien (1992) points out, the fixed-fee mechanism may be superior to the auction mechanism when we take into account the cost of organizing an auction. In the context analyzed in this section, if there are no costs of organizing an auction then the auction mechanism is at least as good as the fixed-fee mechanism, and superior to the royalty mechanism from the patentee's viewpoint. Jensen (1992a, 1992b) considers two different contexts in which the fixed-fee mechanism can dominate the auction mechanism.

$$q_1 = \frac{a-c+\varepsilon}{2b} \qquad q_2 = 0 \qquad \pi_1 = \frac{(a-c+\varepsilon)^2}{4b} \qquad \pi_2 = 0 \qquad \text{if } c_1 < a-\varepsilon \le c_2$$

Notice that when behavior in the product market is more competitive than under Cournot competition (i.e. $\gamma < 0$), a licensee may be the only one active firm in the industry even though the innovation is non-drastic in the sense of Arrow (1962), i.e., even though $c < a - \varepsilon$.

We analyze next the two licensing mechanisms and then we will compare them.

2.1 *Royalty Licensing*

The patentee sets a royalty r per unit of output produced with the new technology. Clearly, if $r > \varepsilon$, no firm will buy the license, and if $r \le \varepsilon$ both firms will buy the license. Thus, under this mechanism the marginal cost of production for both firms will be equal to $c - \varepsilon + r$. As a result, taking into account the expressions in (1), the optimal royalty per unit of output can be found by solving the following problem:

$$\max_r r\frac{2(a-c+\varepsilon-r)}{(\gamma+3)b}$$

subject to

$$r \le \varepsilon$$

Let $x \equiv (a - c)/\varepsilon$. The resolution of this problem implies that the royalty set by the patentee and his profits under the royalty mechanism are

$$r^* = \varepsilon \qquad \Pi = \frac{2\varepsilon^2 x}{(\gamma+3)b} \qquad \text{if } x > 1$$

$$r^* = \frac{a-c+\varepsilon}{2} \qquad \Pi = \frac{\varepsilon^2(x+1)^2}{2(\gamma+3)b} \qquad \text{if } x \le 1$$

It is important to note that the optimal royalty is a fee that is independent of the degree of competition in the market. However, given their marginal cost of production, firms behave more aggressively as γ decreases. As a result, total production in the industry and the total profits of the patentee are increasing in the degree of market competition.

2.2 *Fixed-fee Licensing*

Under this patent licensing mechanism every adopter pays for the license an amount that is independent of production. Given a number of licensees k, the maximum amount that a firm would be willing to pay for a license is equal to the difference between the profits obtained by an adopter and those that it would obtain as a non-adopter. This difference represents the inverse demand function for licenses. Clearly, given that profits as an adopter and as a non-adopter both decrease with the degree of market competition (i.e. they increase with γ) the willingness to pay for a license may or may not decrease when the degree of market competition increases. From equations (1), the inverse demand function is obtained as

$$\begin{array}{ll}
\dfrac{(2+\gamma)\varepsilon^2[\gamma+2(1+\gamma)x]}{(1+\gamma)(3+\gamma)^2 b} & \text{if } \dfrac{1}{1+\gamma} < x \text{ and } k = 2 \\
\dfrac{(2+\gamma)\varepsilon^2[2+\gamma+2(1+\gamma)x]}{(1+\gamma)(3+\gamma)^2 b} & \text{if } \dfrac{1}{1+\gamma} < x \text{ and } k = 1 \\
\dfrac{(1+\gamma)\varepsilon^2(x+1)^2}{(3+\gamma)^2 b} & \text{if } 1 < x \le \dfrac{1}{1+\gamma} \text{ and } k = 2 \\
\dfrac{x\varepsilon^2\left[(3+\gamma)^2-(1+\gamma)x\right]}{(3+\gamma)^2 b} & \text{if } 1 < x \le \dfrac{1}{1+\gamma} \text{ and } k = 1 \\
\dfrac{(1+\gamma)\varepsilon^2(x+1)^2}{(3+\gamma)^2 b} & \text{if } x \le 1 \text{ and } k = 2 \\
\dfrac{\varepsilon^2\left[(\gamma^2+2\gamma+5)x^2+2(\gamma+3)^2 x+(3+\gamma)^2\right]}{4(3+\gamma)^2 b} & \text{if } x \le 1 \text{ and } k = 1
\end{array}$$

In the first stage, the price set by the patentee will be the one that maximizes his revenues. Therefore, the number of licenses sold k^* and the patentee profits Π under the fixed-fee mechanism are

$$\begin{array}{lll}
k^* = 2 & \Pi = \dfrac{2(2+\gamma)\varepsilon^2[\gamma+2(1+\gamma)x]}{(1+\gamma)(3+\gamma)^2 b} & \text{if } \dfrac{2-\gamma}{2(1+\gamma)} \le x \\
k^* = 1 & \Pi = \dfrac{(2+\gamma)\varepsilon^2[2+\gamma+2(1+\gamma)x]}{(1+\gamma)(3+\gamma)^2 b} & \\
 & & \text{if } \dfrac{1}{1+\gamma} \le x < \dfrac{2-\gamma}{2(1+\gamma)} \\
k^* = 1 & \Pi = \dfrac{x\varepsilon^2\left[(3+\gamma)^2-(1+\gamma)x\right]}{(3+\gamma)^2 b} & \text{if } 1 < x < \dfrac{1}{1+\gamma} \\
k^* = 1 & \Pi = \dfrac{\varepsilon^2\left[(\gamma^2+2\gamma+5)x^2+2(\gamma+3)^2 x+(3+\gamma)^2\right]}{4(3+\gamma)^2 b} & \text{if } x \le 1
\end{array}$$

It is important to note that, for the patentee to prefer setting a price such that in equilibrium $k = 1$, that price must be at least equal to twice the fixed fee corresponding to $k = 2$. This in turn implies that, in general, the patentee will prefer to sell two licenses. This will not be the case when $\gamma \to -1$ or when the reduction in costs (ε) induced by the innovation is sufficiently high relative to $a - c$. If either $\gamma \to -1$ or ε is high, then a firm with a cost advantage will have high profits relative to those that it would obtain in the case in which both firms have the same marginal cost. Thus, the fixed fee corresponding to $k = 1$ will be large. Note that if ε is high relative to $a - c$ the licensee may become the only active firm in the industry. In addition, if the degree of competitive behavior is high and firms have identical marginal costs their profits will be low, and the fixed fee corresponding to $k = 2$ will also be low. As a result, the patentee will prefer to set the fixed fee corresponding to $k = 1$. In fact, if $\gamma \to -1$ then $(2 - \gamma)/2(1 + \gamma) \to \infty$, which implies that in this case $k^* = 1$ regardless of ε. If $\gamma = 0$ then $k^* = 1$ if and only if the innovation is drastic in the sense of Arrow (i.e. $x \leq 1$). The efficiency effect helps to explain why in this last case the patentee prefers to sell only one license.

2.3 The Comparison between Fixed-fee and Royalty Licensing

By comparing the patentee's profits under both licensing mechanisms we may establish the following result.

Proposition 1: The patentee will prefer the royalty mechanism to the fixed-fee mechanism if and only if either $-0.585 < \gamma < -0.438$ and $(2 + \gamma)^2/2(1 + \gamma) < x < -\gamma(2 + \gamma)/(1 + \gamma)^2$, or $-1 < \gamma \leq -0.585$ and $\gamma + 3 < x < -\gamma(2 + \gamma)/(1 + \gamma)^2$.

Proof: See the Appendix.

Figure 1 provides a graphical representation of the region of the parameter space where the royalty mechanism is preferred to the fixed-fee mechanism. Region A corresponds to the case of $-1 < \gamma \leq -0.585$ and $\gamma + 3 < x < -\gamma(2 + \gamma)/(1 + \gamma)^2$, while region B corresponds to the case $-0.585 < \gamma < -0.438$ and $(2 + \gamma)^2/2(1 + \gamma) < x < -\gamma(2 + \gamma)/(1 + \gamma)^2$.

The result in Proposition 1 has three important implications.

(i) Two necessary conditions for the patentee to prefer the royalty mechanism over the fixed-fee mechanism in a duopoly industry are that (1) the degree of competition in the market is *sufficiently high* (i.e. $\gamma < -0.438$) and (2) the reduction in marginal cost induced by the innovation is *sufficiently low* relative to $a - c$ (i.e. $\varepsilon < (a - c)/2$). This result encompasses previous results in the literature such as those in Kamien and Tauman (1986), who analyze Cournot competition (i.e. $\gamma = 0$, a case where the patentee prefers the fixed-fee mechanism over the royalty mechanism), or Saracho (2002) who considers the case of strategic delegation in Fershtman and Judd (1987), i.e. $\gamma = -0.5$, where the royalty mechanism may be superior to the fixed-fee mechanism.

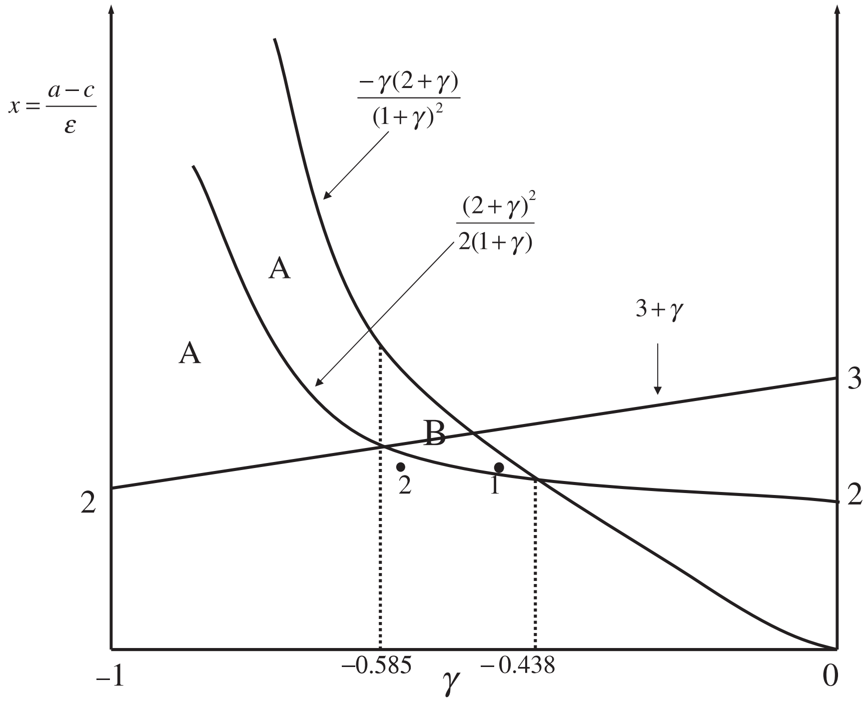

FIG. 1 The Superiority of the Royalty over the Fixed-fee Mechanism

The intuition for why the two conditions above are necessary for the patentee to prefer the royalty mechanism over the fixed-fee mechanism is the following. Let us consider the case of a drastic innovation (i.e. ε is very high relative to $a - c$). In this case, as explained above, the willingness to pay for being the only adopter is high and therefore the patentee will prefer the fixed-fee mechanism to the royalty mechanism.

On the other hand, if the degree of competitive behavior in the industry is high and the innovation is not important enough, then the profits of any one licensee will be low, regardless of whether or not its rival uses the innovation. As a result, under the fixed-fee mechanism the patentee's profits will be low. Moreover, given that the total production of the industry decreases with γ, the profits of the patentee under the royalty mechanism will be high, except if ε is too low.[3] In contrast, when the market is not very competitive, firms will have high profits even when both use the innovation. As a result, the profits of the patentee under the fixed-fee mechanism become relevant. In addition, the profits of the patentee under the royalty mechanism will be lower.

[3] Note that if, for example, $\gamma = -0.9$ then ε must be greater than $(a - c)/99$ and lower than $(a - c)/6.05$ for the patentee to prefer the royalty mechanism over the fixed-fee mechanism.

(ii) A greater degree of competitive behavior *does not* imply greater consumer surplus. Even though a more competitive behavior would appear, in principle, to be desirable from the consumers' viewpoint, it may actually turn out to have undesirable consequences for them. This is due to the relationship between the degree of competition in the product market and the patent licensing mechanism chosen by the patentee. Notice that if the innovation is not drastic in the sense of Arrow and the patentee chooses the royalty mechanism, then the actual marginal cost of production of the firms ($c - \varepsilon + r$) is identical to the one corresponding to the pre-innovation situation (c). As a result, neither total production of the industry nor consumer surplus change relative to the pre-innovation situation. However, under the fixed-fee mechanism the marginal cost of production for at least one of the firms in the industry is $c - \varepsilon$. As a consequence, both total production of the industry and consumer surplus always increase relative to the pre-innovation case. As we have shown, a greater degree of competition may imply that the patentee prefers the royalty mechanism over the fixed-fee mechanism. Hence, consumer surplus may decrease as the degree of market competition increases. For example, consider $x = 4.94$ and $\gamma = -0.6$. The preferred licensing method in this case is the royalty mechanism and consumer surplus is $8.472\varepsilon^2/b$. This amount is lower than $11.85\varepsilon^2/b$, which is the consumer surplus associated with the case $x = 4.94$ and $\gamma = -0.56$ where the optimal licensing method is the fixed-fee mechanism.

(iii) Lastly, it is also important to note that if the royalty mechanism is preferred to the fixed-fee mechanism for a given x and γ, then it *need not* be true that increasing the degree of market competition (decreasing the level of γ) for given x will imply that the royalty mechanism will continue being preferred to the fixed-fee mechanism. This is true in region A of Fig. 1 but it is not true in region B. For instance, compare points 1 and 2 in Fig. 1; the degree of market competition is greater at point 2 than at point 1, yet the royalty mechanism is preferred at point 1 while the fixed-fee mechanism is preferred at point 2.

The analysis thus far has identified some potentially important reasons based on the degree of competitive behavior of the firms for royalty licensing to be more profitable than fixed-fee licensing. At this point it is of interest to study whether the optimal two-part tariff contract that involves a combination of a fixed fee (ff) and a linear royalty on production (r) is either of them. It is shown in the Appendix that when we consider this mechanism, the patentee sells $k^* = 2$ licenses and the contract is such that

- if $x \geq (3-\gamma^2)/(1+\gamma)^2$

$$r = 0 \qquad \text{ff} = \frac{\varepsilon^2(2+\gamma)[\gamma + 2(1+\gamma)x]}{(1+\gamma)(3+\gamma)^2 b}$$

- if $(3+\gamma)/(5+3\gamma) \le x < (3-\gamma^2)/(1+\gamma)^2$

$$r = \frac{\varepsilon[3-\gamma^2-(1+\gamma)^2 x]}{2(3+2\gamma)}$$

$$\mathrm{ff}(r) = \frac{(1+\gamma)^2(a-c+\varepsilon-r)^2-[(1+\gamma)(a-c)-\varepsilon+r]^2}{(1+\gamma)(3+\gamma)^2 b}$$

- if $x < (3+\gamma)/(5+3\gamma)$

$$r = \frac{\varepsilon(1-\gamma)(x+1)}{4} \qquad \mathrm{ff} = \frac{\varepsilon^2(1+\gamma)(x+1)^2}{16b}$$

Given that $(3-\gamma^2)/(1+\gamma)^2$ decreases with γ, the range of values in which a combination of a royalty and a fixed fee is optimal increases with the degree of market competition γ. In consequence, not surprisingly, even when we consider two-part tariff contracts (royalty and fixed fee), royalty licensing plays a more important role when the degree of competitive behavior is high. Note also that while the fixed-fee mechanism alone may be the optimal contract, which occurs in the region $x \ge (3-\gamma^2)/(1+\gamma)^2$, this is never the case for the royalty mechanism.

The analysis is extended in the next section to models in which firms have different conjectural variations.

3 Patent Licensing when Firms Have Different Conjectural Variations

Allowing for conjectural variations to be different across firms, i.e. to be such that $\partial q_j/\partial q_i = \gamma_i$, $i \ne j$, where $\gamma_i \ne \gamma_j$ and $-1 < \gamma_i \le 0$ $\forall i = 1, 2$, allows us to consider new models. For example, the classical case of Stackelberg leadership by firm i corresponds to conjectural variations for firm i, $\gamma_i = -\frac{1}{2}$, and for firm j, $\gamma_j = 0$. Without loss of generality we assume that $\gamma_1 < \gamma_2$. Notice that this assumption implies that firm 1 will exhibit a greater degree of competitive behavior than firm 2.

In the third stage of the game each firm i chooses the level of production that maximizes profits. The equilibrium outputs and profits in this case are as follows.

(a) If $x \ge 1/(1+\gamma_1)$, if $1 \le x < 1/(1+\gamma_1)$ and $c_2 < c_1$ or if $c_2 = c_1$, we have

$$q_i = \frac{(1+\gamma_j)a+c_j-(2+\gamma_j)c_i}{(3+2\gamma_i+2\gamma_j+\gamma_i\gamma_j)b}$$

$$\pi_i = \frac{(1+\gamma_i)[(1+\gamma_j)a+c_j-(2+\gamma_j)c_i]^2}{(3+2\gamma_i+2\gamma_j+\gamma_i\gamma_j)^2 b}$$

(b) If $1 \leq x < 1/(1 + \gamma_1)$ and $c_1 < c_2$, then

$$q_1 = \varepsilon \frac{x}{b} \qquad q_2 = 0 \qquad \pi_1 = \varepsilon^2 \frac{x}{b} \qquad \pi_2 = 0$$

(c) If $x \leq 1$ and $c_i < c_j$, then

$$q_i = \varepsilon \frac{x+1}{2b} \qquad q_j = 0 \qquad \pi_i = \varepsilon^2 \frac{(x+1)^2}{4b} \qquad \pi_j = 0$$

The analysis corresponding to the royalty mechanism is equivalent to that in Section 2. Therefore, taking into account that $c_1 = c_2 = c - \varepsilon + r$, from the firms' equilibrium outputs obtained above we get that the output of the industry in equilibrium is $(2 + \gamma_1 + \gamma_2)(a - c + \varepsilon - r)/(3 + 2\gamma_i + 2\gamma_j + \gamma_i\gamma_j)b$. We may then conclude that the royalty set by the patentee and his profits under the royalty mechanism are

$$r^* = \varepsilon \qquad \Pi = \frac{(2+\gamma_1+\gamma_2)\varepsilon^2 x}{(3+2\gamma_i+2\gamma_j+\gamma_i\gamma_j)b} \qquad \text{if } x > 1$$

$$r^* = \frac{a-c+\varepsilon}{2} \qquad \Pi = \frac{(2+\gamma_1+\gamma_2)\varepsilon^2 (x+1)^2}{4(3+2\gamma_i+2\gamma_j+\gamma_i\gamma_j)b} \qquad \text{if } x \leq 1$$

The analysis of the fixed-fee mechanism when the degree of competitive behavior differs across firms has some special features. Under different conjectural variations, the general resolution of the problem means that the profits of each firm in equilibrium will be different even if both firms have identical marginal cost. In this last situation, $c_1 = c_2$, profits of firm 1 (the most aggressive one) will be greater than those corresponding to its rival. As a result, given a number of licensees k, the maximum amount that a firm will be willing to pay for a license will generally be different across firms. Obviously, in equilibrium, the fixed fee set by the patentee will be either the minimum of the amounts that each firm is willing to pay for the license if $k = 2$ or the maximum of the amounts that each firm is willing to pay for the license if $k = 1$. In the first case the patentee will sell two licenses but he will not be able to extract all the willingness to pay for the license from each of the two firms. In the second case he will sell the only license to the firm with a greater willingness to pay for the license.

As a result in order to calculate the inverse demand function for the licenses under the fixed-fee mechanism we must take into account that

(i) given a number of licensees k, the maximum amount that firm $i = 1, 2$ is willing to pay for a license, $P_i^{\max}(k)$, is equal to the difference between the profits with the license and the profits without it;

(ii) the inverse demand function for the licenses under the fixed-fee mechanism will be

$$\min\left[P_1^{\max}(2), P_2^{\max}(2)\right] \qquad \text{if } k = 2$$
$$\max\left[P_1^{\max}(1), P_2^{\max}(1)\right] \qquad \text{if } k = 1$$

It is straightforward to show that in all those cases in which both firms will remain active in the industry regardless of which one buys a license (i.e. $x \geq 1/(1 + \gamma_1)$), the greatest willingness to pay for a license for $k = 2$ corresponds to the most aggressive firm (i.e. $P_1^{\max}(2) > P_2^{\max}(2)$). In other situations we may have the opposite. Let us consider, for example, that the innovation is drastic in the sense of Arrow. In this case, if only one firm uses the innovation the industry becomes a monopoly. If $k = 1$, it is clear that, because of the replacement effect, firm 2 is the one that has a greater willingness to pay for the license. Notice that in this case $P_i^{\max}(1) = (a - c + \varepsilon)^2/4b - \pi_i(c, c)$, where $\pi_i(c, c)$ denotes the profits of firm i, $i = 1, 2$, when $c_i = c$. However, for $k = 2$ the greatest willingness to pay for a license corresponds to firm 1 given that it acts more aggressively and its profits will be greater than those corresponding to firm 2 if both use the innovation. In the rest of cases, which are intermediate cases between these two cases just considered, no definite results can be established.

Under the fixed-fee mechanism the price set by the patentee will be the one that maximizes his revenues. As shown in Section 2, when $\gamma_1 = \gamma_2$ under the fixed-fee mechanism the patentee, in general, will prefer to sell two licenses. However, when the degree of competitive behavior differs across firms the patentee must set a low price, lower than the greatest willingness to pay for the license, in order to have both firms buy the license. As a result, given γ_1 the patentee will be less interested in selling two licenses under the fixed-fee mechanism, and will also be less interested in using this mechanism instead of the royalty mechanism, than when $\gamma_1 = \gamma_2$.

An implication of the model is that, as expected, if the innovation is drastic then, regardless of the degree of competitive behavior of each of the firms, the patentee will prefer the fixed-fee mechanism to the royalty mechanism. In fact, he will sell only one license, which will be acquired by firm 2. However, given the characteristics of the general problem, each specific case must be solved separately.

As an illustration, let us consider the classic Stackelberg model often employed in the literature where firm 1 is the leader and firm 2 is the follower: $\gamma_1 = -\frac{1}{2}$ and $\gamma_2 = 0$. Other specific cases may of course be analyzed along the same lines.[4] It is straightforward to show that the patentee profits, Π, and the number of licenses sold, k^*, under the fixed-fee mechanism are

$$k^* = 2 \qquad \Pi = 2P_2^{\max}(2) = \frac{3\varepsilon^2(2x-1)}{8b} \qquad \text{if } 3.5 \leq x$$

[4] The derivation of the inverse demand function corresponding to this case is shown in the Appendix.

$$k^* = 1 \qquad \Pi = P_1^{\max}(1) = \frac{\varepsilon^2(x+1)}{2b} \qquad \text{if } 2 \le x < 3.5$$

$$k^* = 1 \qquad \Pi = P_1^{\max}(1) = \frac{x\varepsilon^2(8-x)}{8b} \qquad \text{if } 1.177 \le x < 2$$

$$k^* = 1 \qquad \Pi = P_2^{\max}(1) = \frac{3\varepsilon^2(2x+3)}{16b} \qquad \text{if } 1 \le x < 1.177$$

$$k^* = 1 \qquad \Pi = P_2^{\max}(1) = \frac{\varepsilon^2\left(3x^2+8x+4\right)}{16b} \qquad \text{if } x < 1$$

Taking into account the results corresponding to royalty licensing previously obtained in this section, we may conclude that in the Stackelberg model the patentee's profits under this mechanism are

$$\Pi = \frac{3\varepsilon^2 x}{4b} \qquad \text{if } x > 1$$

$$\Pi = \frac{3\varepsilon^2(x+1)^2}{16b} \qquad \text{if } x \le 1$$

By comparing the profits under both mechanisms it is straightforward to establish the following result.

Proposition 2: In the Stackelberg model the patentee will prefer the royalty mechanism to the fixed-fee mechanism if and only if $x > 2$.

If $x < 2$ then the patentee will prefer the fixed-fee mechanism and will sell only one license.[5] Notice that in this context the royalty mechanism is superior to the fixed-fee mechanism in most cases. More precisely, it is preferred in all those cases in which both firms will remain active in the industry independently of which one buys a license. Notice that when firms have identical conjectural variations the patentee will prefer the royalty mechanism $\forall x > 2$ if and only if $\gamma \to -1$ (see Fig. 1). As an empirical implication this result suggests that, relative to the case in which firms have identical conjectural variations, royalties should be observed more frequently in contexts in which identical firms have different conjectural variations, i.e. when there are asymmetries in the behavior of the firms.

Lastly, it is worth discussing the following three important aspects.

(i) The difference between the degree of competitive behavior of firms has an effect, in general, in the comparison between the profits of the pat-

[5] If $x = 2$ then the patentee is indifferent between both mechanisms.

entee under the two mechanisms. In the Stackelberg model just considered, this difference is high and, in consequence, the difference between the amount that firm 1 pays for the license for $k = 2$ and its willingness to pay for it is also high. In order to have both firms buying the license the patentee must set a 'low' price, which in turn induces the patentee to prefer the royalty mechanism. As an illustration, consider the example where $\gamma_1 = -\frac{1}{2}$ and, instead of $\gamma_2 = 0$, we have $\gamma_2 = -\frac{1}{4}$. It is not difficult to show that in this case the patentee will prefer the royalty mechanism to the fixed-fee mechanism if and only if $2.13 < x < 5.14$.

(ii) Consider the situations in which $\gamma_1 = -1$, independently of γ_2. If $c_i = c - \varepsilon + r$, $i = 1, 2$, the profits of each of the firms will be zero and the total production of the industry will be $\varepsilon x/b$ if the innovation is non-drastic and $\varepsilon(x + 1)/2b$ if it is drastic. As a result, the patentee's profits under the royalty mechanism will be

$$\Pi = \frac{\varepsilon^2 x}{b} \quad \text{if } x > 1 \qquad \text{and} \qquad \Pi = \frac{\varepsilon^2 (x+1)^2}{4b} \quad \text{if } x \leq 1$$

Consider now the fixed-fee mechanism. The patentee will sell only one license; otherwise, its profits will be zero. If the innovation is drastic the licensee will become a monopoly and if the innovation is non-drastic it will sell its product at a price equal to the marginal cost of its rival, c. Therefore, it is straightforward to conclude that the patentee will be indifferent between the two patent licensing mechanisms. Notice that the result applies for example (1) to the case of Bertrand competition, i.e. $\gamma_i = -1$, $i = 1, 2$, which is considered by Kamien and Tauman (1986), or (2) to the case where one of the firms is perfectly competitive.

(iii) Lastly, it may be important to note that given that the willingness to pay for the license differs across firms the patentee will be interested in charging a different price for the license to each firm for $k = 2$.[6] Obviously, this possibility does not affect the profits of the patentee under royalty licensing. It is straightforward to show that if price discrimination is allowed in the Stackelberg model then the patentee will not use the royalty mechanism and, as a consequence, for $x > 2$ consumer surplus will be greater than in the case of no price discrimination analyzed in the paper.

4 Concluding Remarks

This paper examines the effects of the nature and degree of competitive behavior in the product market on the optimal one-stage patent licensing mecha-

[6] In this case the patentee should offer simultaneously a different contract to each firm. He would not be able to announce two different prices because both firms would then buy at the lowest price. For this reason I do not refer to this mechanism as a fixed-fee mechanism.

nism from the patentee's viewpoint in a duopoly industry. Using conjectural variations as a shorthand for several different supergame outcomes of a duopoly industry, we find that the degree of competitive behavior in the market and the extent of the asymmetries in the behavior of the firms have a non-trivial effect on the patent licensing mechanism preferred by the patentee.

We consider that the main contribution of the analysis is that it highlights what may be an important connection between the degree of competitive behavior in the product market and the preferred patent licensing mechanism. This natural relationship, to the best of our knowledge, has not been addressed in the literature previously. The degree of competitive behavior in the market emerges as a natural determinant of the preferred patent licensing mechanism. In this sense the analysis contributes to an important literature on the determinants of the creation and the diffusion of new knowledge, and in particular to a literature that attempts to justify the empirically observed presence of royalty payments in patent licensing agreements. We also consider that the relationship between the degree of competitive behavior in the product market and the optimal patent licensing mechanism may be a valuable source of empirical predictions for future research.

Appendix

Proof of Proposition 1

The following proof shows that the patentee will prefer the royalty mechanism to the fixed-fee mechanism if and only if either $-0.585 < \gamma < -0.438$ and $(2 + \gamma)^2/2(1 + \gamma) < x < -\gamma(2 + \gamma)/(1 + \gamma)^2$, or $-1 < \gamma \le -0.585$ and $\gamma + 3 < x < -\gamma(2 + \gamma)/(1 + \gamma)^2$.

By comparing the patentee profits under both mechanisms, it is straightforward to conclude that the patentee prefers the royalty mechanism to the fixed-fee mechanism if and only if any of the following conditions is satisfied:

(a) $3+\gamma < x < \dfrac{1}{1+\gamma}$

(b) $\max\left[\dfrac{1}{1+\gamma}, \dfrac{(2+\gamma)^2}{2(1+\gamma)}\right] < x < \dfrac{2-\gamma}{2(1+\gamma)}$

(c) $\dfrac{(2+\gamma)^2}{2(1+\gamma)} < \dfrac{1}{1+\gamma} = x < \dfrac{2-\gamma}{2(1+\gamma)}$ or

(d) $\dfrac{2-\gamma}{2(1+\gamma)} \le x < \dfrac{-\gamma(2+\gamma)}{(1+\gamma)^2}$

Therefore, the result in the proposition follows given that

(i) $0 \ge \gamma > -0.484 \Rightarrow 3+\gamma > \dfrac{1}{1+\gamma}$ and $\dfrac{(2+\gamma)^2}{2(1+\gamma)} > \dfrac{2-\gamma}{2(1+\gamma)} > \max\left[\dfrac{1}{1+\gamma}, \dfrac{-\gamma(2+\gamma)}{(1+\gamma)^2}\right]$

(ii) $-0.484 > \gamma > -0.585 \Rightarrow 3+\gamma > \dfrac{1}{1+\gamma}$ and $\dfrac{-\gamma(2+\gamma)}{(1+\gamma)^2} > \dfrac{2-\gamma}{2(1+\gamma)} > \dfrac{(2+\gamma)^2}{2(1+\gamma)} > \dfrac{1}{1+\gamma}$

(iii) $-0.585>\gamma>-1 \Rightarrow 3+\gamma<\frac{1}{1+\gamma}$ and $\frac{-\gamma(2+\gamma)}{(1+\gamma)^2}>\frac{2-\gamma}{2(1+\gamma)}>\frac{1}{1+\gamma}>\frac{(2+\gamma)^2}{2(1+\gamma)}$

(iv) $\gamma=-0.585 \Rightarrow \frac{(2+\gamma)^2}{2(1+\gamma)}=\frac{1}{1+\gamma}=3+\gamma$

(v) $\gamma=-0.438 \Rightarrow \frac{-\gamma(2+\gamma)}{(1+\gamma)^2}=\frac{2-\gamma}{2(1+\gamma)}=\frac{(2+\gamma)^2}{2(1+\gamma)}$ and

(vi) $\gamma=-0.382 \Rightarrow \frac{-\gamma(2+\gamma)}{(1+\gamma)^2}=\frac{1}{1+\gamma}$

Derivation of the Inverse Demand Function under the Fixed-fee Mechanism

We must first take into account that the willingness to pay for a license, $P_i^{\max}(k)$ with $i = 1, 2$, differs across firms. Given a number of licensees k, the maximum amount that firm i is willing to pay for a license, $P_i^{\max}(k)$, is equal to the difference between the profits with the license and the profits without it. Replacing γ_1 by $-\frac{1}{2}$ and γ_2 by 0 in the profit functions obtained for each of the firms in Section 3, it is easy to show that

$$P_1^{\max}(2)=\frac{(a-c+\varepsilon)^2}{8b}-\frac{(a-c-\varepsilon)^2}{8b}=\frac{\varepsilon^2 x}{2b} \qquad \text{if } x\geq 1$$

$$P_2^{\max}(2)=\frac{(a-c+\varepsilon)^2}{16b}-\frac{(a-c-2\varepsilon)^2}{16b}=\frac{3\varepsilon^2(2x-1)}{16b} \qquad \text{if } x\geq 2$$

$$P_1^{\max}(1)=\frac{(a-c+2\varepsilon)^2}{8b}-\frac{(a-c)^2}{8b}=\frac{\varepsilon^2(x+1)}{2b} \qquad \text{if } x\geq 2$$

$$P_2^{\max}(1)=\frac{(a-c+3\varepsilon)^2}{16b}-\frac{(a-c)^2}{16b}=\frac{3\varepsilon^2(2x+3)}{16b} \qquad \text{if } x\geq 1$$

$$P_2^{\max}(2)=\frac{(a-c+\varepsilon)^2}{16b}-0=\frac{\varepsilon^2(x+1)^2}{16b} \qquad \text{if } x<2$$

$$P_1^{\max}(1)=\frac{\varepsilon(a-c)}{b}-\frac{(a-c)^2}{8b}=\frac{\varepsilon^2 x(8-x)}{8b} \qquad \text{if } 1\leq x<2$$

$$P_1^{\max}(2)=\frac{(a-c+\varepsilon)^2}{8b}-0=\frac{\varepsilon^2(x+1)}{8b} \qquad \text{if } x<1$$

$$P_1^{\max}(1)=\frac{(a-c+\varepsilon)^2}{4b}-\frac{(a-c)^2}{8b}=\frac{\varepsilon^2(x^2+4x+2)}{8b} \qquad \text{if } x<1$$

$$P_2^{\max}(1)=\frac{(a-c+\varepsilon)^2}{4b}-\frac{(a-c)^2}{16b}=\frac{\varepsilon^2(3x^2+8x+4)}{16b} \qquad \text{if } x<1$$

This means that the inverse demand function for the licenses under the fixed-fee mechanism

$$\min[P_1^{\max}(2), P_2^{\max}(2)] \qquad \text{if } k=2$$

$$\max[P_1^{\max}(1), P_2^{\max}(1)] \qquad \text{if } k=1$$

is

$$\frac{3\varepsilon^2(2x-1)}{16b} \quad \text{if } 2 \le x \text{ and } k=2$$

$$\frac{\varepsilon^2(x+1)}{2b} \quad \text{if } 2 \le x \text{ and } k=1$$

$$\frac{\varepsilon^2 x(8-x)}{8b} \quad \text{if } 1.18 \le x < 2 \text{ and } k=1$$

$$\frac{3\varepsilon^2(2x+3)}{16b} \quad \text{if } 1 \le x < 1.18 \text{ and } k=1$$

$$\frac{\varepsilon^2(x+1)^2}{16b} \quad \text{if } x < 2 \text{ and } k=2$$

$$\frac{\varepsilon^2(3x^2+8x+4)}{16b} \quad \text{if } x < 1 \text{ and } k=1$$

Derivation of the Optimal Fixed Fee Plus Royalty Contract

Next, we first compute the optimal contract for the case $k = 2$, then for $k = 1$ and, finally, we obtain the optimal contract by comparing the profits in these two cases.

1. Optimal contract when $k = 2$. Taking into account (1), we must solve the following two maximization problems:

(P1)

$$\max_r 2\left\{\frac{(1+\gamma)^2(a-c+\varepsilon-r)^2-[(1+\gamma)(a-c)-\varepsilon+r]^2}{(1+\gamma)(3+\gamma)^2 b}\right\}+2r\frac{a-c+\varepsilon-r}{(3+\gamma)b}$$

subject to $\max[0, \varepsilon-(1+\gamma)(a-c)] \le r$;

(P2)

$$\max_r 2\frac{(1+\gamma)(a-c+\varepsilon-r)^2}{(3+\gamma)^2 b}+2r\frac{a-c+\varepsilon-r}{(3+\gamma)b}$$

subject to $0 \le r \le \varepsilon-(1+\gamma)(a-c)$.

Assuming interior solutions we have that the first order condition of problem (P1) implies $\varepsilon[3-\gamma^2-(1+\gamma)^2x]-2(3+2\gamma)r=0$, while that of problem (P2) implies $(1-\gamma)(a-c+\varepsilon)-4r=0$. Therefore, we may conclude that

- for $(3-\gamma^2)/(1+\gamma)^2 < x$

$$r=0 \text{ and ff } = \frac{\varepsilon^2(2+\gamma)[\gamma+2(1+\gamma)x]}{(1+\gamma)(3+\gamma)^2 b}$$

- for $(3+\gamma)/(5+3\gamma) < x \le (3-\gamma^2)/(1+\gamma)^2$

$$r=\frac{\varepsilon[3-\gamma^2-(1+\gamma)^2 x]}{2(3+2\gamma)}$$

and

$$\text{ff}(r)=\frac{(1+\gamma)^2(a-c+\varepsilon-r)^2-[(1+\gamma)(a-c)-\varepsilon+r]^2}{(1+\gamma)(3+\gamma)^2 b}$$

- for $x \le (3+\gamma)/(5+3\gamma)$

$$r=\frac{(1-\gamma)(a-c+\varepsilon)}{4} \qquad \text{and} \qquad \text{ff}=\frac{\varepsilon^2(1+\gamma)(x+1)^2}{16b}$$

2. *Optimal contract when* k = 1. If a licensee is the only active firm in the industry when $x > 1$, then its profits, without subtracting the fixed fee, are $(\varepsilon - r)(a - c)/b$. We know from (1) that this will be the case if $0 \le r \le \varepsilon - (1 + \gamma)(a - c)$. On the other hand, if $0 \le r \le c + \varepsilon - a$ and a licensee is the only active firm in the industry, then it will behave as a monopolist. Therefore, taking into account (1), we must solve the following three maximization problems.

(T1) For any x:

$$\max_r \frac{(2+\gamma)^2(\varepsilon-r)^2+2(2+\gamma)(1+\gamma)(a-c)(\varepsilon-r)}{(1+\gamma)(3+\gamma)^2 b} + r\frac{(1+\gamma)(a-c)+(2+\gamma)(\varepsilon-r)}{(1+\gamma)(3+\gamma)b}$$

subject to $\max[0, \varepsilon - (1 + \gamma)(a - c)] \le r$. Assuming an interior solution, the first order condition implies $(1 + \gamma)^2(a - c) + (1 + \gamma)(2 + \gamma)\varepsilon + 2(2 + \gamma)r = 0$.

(T2) For $x > 1$:

$$\max_r (\varepsilon-r)\frac{a-c}{b}-\frac{(1+\gamma)(a-c)^2}{(3+\gamma)^2 b}+r\frac{a-c}{b}$$

subject to $0 \le r \le \varepsilon - (1 + \gamma)(a - c)$. Clearly, in this case $\Pi = \varepsilon^2 x[(3 + \gamma)^2 - (1 + \gamma)x]/(3 + \gamma)^2 b$ for all $r \in [0, \varepsilon - (1 + \gamma)(a - c)]$. Note that for $x \le 1$ and $c + \varepsilon - a \le r \le \varepsilon - (1 + \gamma)(a - c)$ the patentee obtains the same profits as in the $x > 1$ case with $r \in [0, \varepsilon - (1 + \gamma)(a - c)]$.

(T3) For $x \le 1$:

$$\max_r \frac{(a-c+\varepsilon-r)^2}{4b}-\frac{(1+\gamma)(a-c)^2}{(3+\gamma)^2 b}+r\frac{a-c+\varepsilon-r}{2b}$$

subject to $0 \le r \le c + \varepsilon - a$. Assuming an interior solution, the first order condition implies $-2r = 0$.

As a result, without loss of generality, for k = 1 the optimal contract is a fixed fee alone and the patentee profits are those obtained in Section 2.2.

- For $1/(1+\gamma) < x$:

$$r=0 \qquad \text{ff}=\frac{\varepsilon^2(2+\gamma)[2+\gamma+2(1+\gamma)x]}{(1+\gamma)(3+\gamma)^2 b}$$

- For $1 < x \le 1/(1+\gamma)$:

$$r=0 \qquad \text{ff}=\frac{\varepsilon^2 x\left[(3+\gamma)^2-(1+\gamma)x\right]}{(3+\gamma)^2 b}$$

- For $x \le 1$:

$$r = 0 \qquad \text{ff} = \frac{\varepsilon^2\left[(\gamma^2+2\gamma+5)x^2+2(\gamma+3)^2x+(3+\gamma)^2\right]}{4(3+\gamma)^2 b}$$

3. Optimal contract. We next compare the patentee profits for $k = 1$ and $k = 2$ to obtain the optimal contract. There are five possible cases to distinguish.

(A) Case $(2 - \gamma)/2(1 + \gamma) \le x$. Given that for $k = 1$ the optimal contract is a fixed fee alone, taking into account the results in Section 2.2 on the number of licenses sold under the fixed-fee mechanism, we may conclude that $k^* = 2$ and that the optimal contract is as follows.

- For $(3-\gamma^2)/(1+\gamma)^2 \le x$:

$$r = 0 \qquad \text{and} \qquad \text{ff} = \frac{\varepsilon^2(2+\gamma)[\gamma+2(1+\gamma)x]}{(1+\gamma)(3+\gamma)^2 b}$$

- For $(2-\gamma)/2(1+\gamma) \le x < (3-\gamma^2)/(1+\gamma)^2$:

$$r = \frac{\varepsilon\left[3-\gamma^2-(1+\gamma)^2x\right]}{2(3+2\gamma)}$$

and

$$\text{ff}(r) = \frac{(1+\gamma)^2(a-c+\varepsilon-r)^2-[(1+\gamma)(a-c)-\varepsilon+r]^2}{(1+\gamma)(3+\gamma)^2 b}$$

(B) Case $1/(1 + \gamma) \le x < (2 - \gamma)/2(1 + \gamma)$. We have that the difference in profits is

$$\Pi(k=2)-\Pi(k=1) = \frac{\varepsilon^2[(\gamma^4+4\gamma^3+6\gamma^2+4\gamma+1)x^2 + 2(\gamma^4+6\gamma^3+16\gamma^2+20\gamma+9)x+\gamma^4+4\gamma^3-16\gamma-15]}{2(1+\gamma)(3+\gamma)^2(3+2\gamma)b}$$

Note that since $-1 < \gamma$, both x^2 and x are multiplied by positive numbers. Therefore, since $1/(1 + \gamma) \le x$, we have that

$$\Pi(k=2)-\Pi(k=1) \ge \frac{\varepsilon^2(2+\gamma)(2+3\gamma+4\gamma^2+\gamma^3)}{2(1+\gamma)(3+\gamma)^2(3+2\gamma)b} > 0$$

Therefore, $k^* = 2$ and the optimal contract is $r = \varepsilon[3 - \gamma^2 - (1 + \gamma)^2x]/2(3 + 2\gamma)$ and $\text{ff}(r) = \{(1 + \gamma)^2(a - c + \varepsilon - r)^2 - [(1 + \gamma)(a - c) - \varepsilon + r]^2\}/(1 + \gamma)(3 + \gamma)^2b$.

(C) Case $1 < x \le 1/(1 + \gamma)$. We have that

$$\Pi(k=2)-\Pi(k=1) = \frac{\varepsilon^2(1+\gamma)\left[9+\gamma^2(-1+x)^2-12x+7x^2+2\gamma(3-5x+3x^2)\right]}{2(3+\gamma)^2(3+2\gamma)b}$$

Given that the term in brackets evaluated at $\gamma = -1$ is positive and that its partial derivative with respect to γ for $x > 1$ is also positive, the optimal contract is such that $k^* = 2$, $r = \varepsilon[3 - \gamma^2 - (1 + \gamma)^2x]/2(3 + 2\gamma)$ and $\text{ff}(r) = \{(1 + \gamma)^2(a - c + \varepsilon - r)^2 - [(1 + \gamma)(a - c) - \varepsilon + r]^2\}/(1 + \gamma)(3 + \gamma)^2b$.

(D) Case $(3 + \gamma)/(5 + 3\gamma) \le x \le 1$. We have that

$$\Pi(k=2)-\Pi(k=1)=\frac{-\varepsilon^2[9-30x+13x^2+2\gamma(3-14x+5x^2)+(1-6x+x^2)\gamma^2]}{4(3+2\gamma)(3+\gamma)^2 b}$$

Given γ, one of the roots of the polynomial in brackets is greater than 1 and the other is below $(3 + \gamma)/(5 + 3\gamma)$. Taking into account that for $x = 1$ we have $\Pi(k = 2) - \Pi(k = 1) = \varepsilon^2(2 + 3\gamma + \gamma^2)/(3 + 2\gamma)(3 + \gamma)^2 b > 0$, we obtain that the optimal contract is such that $k^* = 2$, $r = \varepsilon[3 - \gamma^2 - (1 + \gamma)^2 x]/2(3 + 2\gamma)$ and $\mathrm{ff}(r) = \{(1 + \gamma)^2(a - c + \varepsilon - r)^2 - [(1 + \gamma)(a - c) - \varepsilon + r]^2\}/(1 + \gamma)(3 + \gamma)^2 b$.

(E) Case $x \leq (3 + \gamma)/(5 + 3\gamma)$. It is straightforward to conclude that in this case the optimal contract is such that $k^* = 2$, $r = (1 - \gamma)(a - c + \varepsilon)/4$ and $\mathrm{ff}(r) = \varepsilon^2(1 + \gamma)(x + 1)^2/16b$. In this situation the patentee profits are equal to those that a monopolist with a marginal cost equal to $c - \varepsilon$ would obtain in the industry.

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