# The boundedness of operators in (Muckenhoupt weighted) Morrey spaces via duality and extrapolation techniques due to Rubio de Francia

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**Abstract**

We showed that the bidual of the closure of smooth functions with respect to the Morrey norm coincides with the Morrey space. This assertion is generalized to some Muckenhoupt weighted Morrey spaces. We combine this fact with basic extrapolation techniques due to Rubio de Francia adapted to weighted Morrey spaces. This leads to new results on the boundedness of operators even for the unweighted case. Let us mention that in many related papers dealing with various generalizations of the Morrey spaces the non-density of the smooth functions in the Morrey spaces is not taken into account. Therefore to estimate e.g. singular integrals on the smooth functions is not sufficient. Moreover, one has also to clarify in which way one extends the considered operator which is a priori given on the smooth functions (e.g. singular integrals) to the Morrey spaces. But one must be well aware that the extension is by no means unique.

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