Quantitative theory of weights for rough homogeneous singular integrals

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The study of weighted inequalities in the Classical Harmonic Analysis started in the 1970's, with B. Muckenhoupt. At that time the question about how the operator norm depended on the constant associated with w, which we denote by $[w]_{A_p}$, was not considered (i.e., *quantitative estimates* were not investigated).

Since the beginning of 2000's, a great activity has been carried out in order to establish the sharp dependence for singular integral operators, reaching the solution of the so-called A_2 conjecture by T. H. Hytönen. So far, one of the most recent extensions of this result is contained in a work by M. T. Lacey.

In this talk we consider operators with homogeneous singular kernels, on which we assume smoothness conditions that are weaker than the standard ones (this is why they are called rough). Qualitative estimates are due to J. Duoandikoetxea and J. L. Rubio de Francia. For the norm of these operators in the space $L^2(w)$ we obtain a quantitative estimate which is quadratic in the constant $[w]_{A_2}$.

Our results are based on a classical decomposition of the rough operators as a sum of other operators with a smoother kernel, for which a quantitative reelaboration of a dyadic decomposition proposed by Lacey is applied.

We will overview as well the most recent advances by A. K. Lerner after our current research and some other possible investigation lines.

The present work has been accomplished in colaboration with Hytönen and O. Tapiola.