Mean-field Limit and Semiclassical Expansion of a Quantum Particle System

Federica Pezzotti,

Dipartimento di Matematica Pura ed Applicata, Università degli Studi de l'Aquila, Italy

joint work with Mario Pulvirenti

Dipartimento di Matematica "Guido Castelnuovo", Università degli Studi di Roma "La Sapienza", Italy

Abstract

We consider a quantum system constituted by N identical particles interacting by means of a mean-field Hamiltonian. It is well known that, in the limit $N \to \infty$, the one-particle state obeys to the Hartree equation. Moreover, propagation of chaos holds. In this paper, we take care of the $\hbar$ dependence by considering the semiclassical expansion of the N-particle system. We prove that each term of the expansion agrees, in the limit $N \to \infty$, with the corresponding one associated with the Hartree equation. We work in the classical phase space by using the Wigner formalism, which seems to be the most appropriate for the present problem.