HARMONIC ANALYSIS MORNING
30 January, 2015

Extending sets by means of the Maximal function: Continuity estimates

(10:00) IOANNIS PARISSIS

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ABSTRACT: Let $\mathfrak{B}$ be a collection of bounded open sets in $\mathbb{R}^n$ such as balls, cubes or $n-$dimensional rectangles with sides parallel to the coordinate axes. We let $M_{\mathfrak{B}}f(x)$ denote the maximal operator associated with the collection $\mathfrak{B}$:

$$M_{\mathfrak{B}}f(x) := \sup_{x\in B\in\mathfrak{B}} \frac{1}{|B|}\int_B |f(y)|dy.$$

We are interested in the so-called *Halo* function of $\mathfrak{B}$:

$$C_{\mathfrak{B}}(\alpha) := \sup_{\substack{E\subset\mathbb{R}^n\\ 0<|E|<\infty}} \frac{1}{|E|}|\{x\in\mathbb{R}^n : M(\mathbf{1}_E)(x)>\alpha\}|,$$

where $\alpha\in(0,1)$ and the supremum is taken over all measurable sets E. For α very close to 1 we can think of the set $\{M_{\mathfrak{B}}(\mathbf{1}_E)>\alpha\}\supset E$ as an enlargement of E, defined in accordance with the geometry of $\mathfrak{B}$. It is natural to expect that this enlargement should converge to the set E itself, in some appropriate sense, as $\alpha\to 1^-$. We prove this heuristic statement by showing that

$$\lim_{\alpha\to 1^-} C_{\mathfrak{B}}(\alpha) = 1$$

for different choices of $\mathfrak{B}$. For more general collections $\mathfrak{B}$ (such as homothecy invariant collections of convex sets) we state a corresponding conjecture. This talk reports on joint work with Paul A. Hagelstein (Baylor)

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Sala de seminarios de la sección de matemáticas
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