Measure-valued solutions for hysteresis models with a nonlinear hardening term.

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Abstract.

We consider the following problem with a doubly-nonlinear equation of the form

$$z_t \in \partial G\big(f - \partial F(z)\big), \tag{1}$$
$$z(0) = z^0. \tag{2}$$

Here f and z^0 are given functions, ∂G and ∂F are subdifferentials of convex functionals G and F, respectively. Such equations have to model processes, which can display hysteretic behavior and arise for instance in the plasticity theory and in the models, which describe ferroelectric material behavior. The nonlinear operator ∂F, which is associated with a convex, non-quadratic hardening term in the energy function, causes difficulties for the existence theory. Although the operators ∂G and ∂F are monotone, their composition is in general not monotone. Therefore we cannot apply the theory of monotone operators to the equation (1) directly. We construct the sequence of approximating solutions using a time discretization. From the a-priori estimates we conclude that the approximating sequence converges weakly. The requirements of the models for the nonlinear operator ∂F do not allow using the compactness arguments to obtain the strong convergence of the approximating sequence, which would be enough to conclude that the weak limit is the solution of the problem (1),(2) in a classical sense. We prove an existence of measure-valued solutions for the problem (1),(2) using the theory of Young measures.