Seminario de análisis matemático y aplicaciones
Analisi matematikoa eta aplikazioak mintegia

A variational formula for the free energy of a many-Boson system

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ABSTRACT: We consider N bosons in a box in the d-dimensional space with volume N/ρ under the influence of a mutually repellent pair potential. The particle density $\rho \in (0,\infty)$ is kept fixed. Our main result is the identification of the limiting free energy, $f(\beta,\rho)$, in the thermodynamic limit at positive temperature $1/\beta$, in terms of an explicit variational formula, for any fixed ρ if β is sufficiently small, and for any fixed β if ρ is sufficiently small.

The thermodynamic equilibrium is described by the symmetrised trace of the operator $\mathrm{e}^{-\beta\mathcal{N}}$, where $\mathcal{N}$ denotes the corresponding Hamilton operator. The well-known Feynman-Kac formula reformulates this trace in terms of N interacting Brownian bridges. Due to the symmetrisation, the bridges are organised in an ensemble of cycles of various lengths. The novelty of our approach is a description in terms of a marked Poisson point process whose marks are the cycles. This allows for an asymptotic analysis of the system via a large-deviations analysis of the stationary empirical field. The resulting variational formula ranges over random shift-invariant marked point fields and optimizes the sum of the interaction and the relative entropy with respect to the reference process.

In our proof of the lower bound for the free energy, we drop all interaction involving 'infinitely long' cycles, and their possible presence is signalled by a loss of mass of the 'finitely long' cycles in the variational formula. In the proof of the upper bound, we only keep the mass on the 'finitely long' cycles. We expect that the precise relationship between these two bounds lies at the heart of Bose-Einstein condensation and intend to analyse it further in future.

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