

Seminario de análisis matemático y aplicaciones

Analisi matematikoa eta aplikazioak mintegia

Existence of solutions to systems of coupled NLS equations and NLS-KdV equations

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Along this talk, will be shown some existence of solutions for systems of coupled NLS equations,

$$\begin{cases} -i \frac{\partial}{\partial t} E_j - \Delta E_j = \mu_j |E_j|^2 E_j + \sum_{k \neq j; k=1}^N \beta_{jk} |E_k|^2 E_j, & x \in \mathbb{R}^n, \quad t > 0 \\ E_j = E_j(x, t) \in \mathbb{C}, \quad E_j(x, t) \rightarrow 0 \text{ as } |x| \rightarrow \infty \end{cases} \quad (1)$$

for $j = 1, \dots, N$; $n = 1, 2, 3$; the coupled parameters $\beta_{jk} = \beta_{kj} \in \mathbb{R}$ for $j, k = 1, \dots, N$; $j \neq k$; $\mu_j > 0$ and $\mu_j > 0$ is for self-focusing in the j -th component of the beam. The solution E_j represents the j -th component of the beam. The coupling constant β_{jk} means the interaction between the j -th and k -th component of the beam. If they are positive, the interaction is attractive while if they are negative we have a repulsive interaction. The mixed case involves different sign on the coupling parameters β_{jk} . Here, we study the attractive and mixed interactions.

And also we will show existence of solutions for systems of coupled NLS-KdV equations,

$$\begin{cases} if_t + f_{xx} - \beta fg + |f|^2 f = 0, & x \in \mathbb{R}, \quad t > 0 \\ gt + g_{xxx} + gg_x - \frac{1}{2} \beta (|f|^2)_x = 0, & x \in \mathbb{R}, \quad t > 0, \end{cases} \quad (2)$$

where $f(x, t) \in \mathbb{C}$, $g(x, t) \in \mathbb{R}$, $\beta \in \mathbb{R}$ and $|f|, |g| \rightarrow 0$ as $|x| \rightarrow \infty$.

Precisely, we will show existence of positive bound and ground states for the corresponding stationary systems when one looks for solitary wave solutions of (1) and solitary-traveling wave solutions of (2).

The results presented in this talk are contained in the following papers.

- [1] A. Ambrosetti & E. Colorado, *Bound and ground states of coupled nonlinear Schrödinger equations*. C. R. Math. Acad. Sci. Paris **342** (2006), no. 2, 453-458.
- [2] A. Ambrosetti & E. Colorado, *Standing waves of some coupled nonlinear Schrödinger equations*. J. Lond. Math. Soc. (2) **75** (2007), no.1, 67-82.
- [3] E. Colorado, *Positive solutions to some systems of coupled nonlinear Schrödinger equations*. Nonlinear Analysis **110** (2014) 104-112.
- [4] E. Colorado, *Existence of bound and ground states for a system of coupled nonlinear Schrödinger-KdV equations*. Preprint ArXiv:1410.7638v2 .
- [5] E. Colorado, *On the existence of bound and ground states for a system of coupled nonlinear Schrödinger-KdV equations*. Preprint ArXiv:1411.7283v2 .

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