Optimal fiscal policy in a multisector model: The price consequences of government spending*

Steven P. Cassou[†]
Kansas State University

Arantza Gorostiaga[‡]
Universidad del País Vasco

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Abstract

This paper investigates optimal fiscal policy in a static multisector model. A Ramsey type planner chooses tax rates on each good type as well as spending levels on each good type subject to an exogenous total expenditure constraint. It is shown that, like taxes, government spending policy has price effects and that these price effects have significant implications for optimal policy. These price effects imply a U shape to the government's objective function and this U shape results in boundary values for the choice of the spending allocation. In particular, it is shown that the optimal allocation of government spending tends to be concentrated on one good rather than spread among many goods.

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[†] Department of Economics, 327 Waters Hall, Kansas State University, Manhattan, KS, 66506 (USA), (785) 532-6342, Fax:(785) 532-6919, email: scassou@ksu.edu.

[‡]Fundamentos del Análisis Económico II, Facultad de Ciencias Económicas y Empresariales, Universidad del País Vasco, Avd. Lehendakari Aguirre 83, 48015 Bilbao (SPAIN), Phone: +34 946013814, Fax: +34 946017123, email: arantza.gorostiaga@ehu.es.

1 Introduction

At least since Ramsey (1927), it is well known that the tax structure has important effects on welfare. It was here that economists first learned one of the principal findings of public finance, that efficient taxes will be highest on goods with the lowest elasticities. But Ramsey (1927) did more than contribute to our understanding of tax structure. Perhaps equally important, this paper provided what is now one of the standard approaches for formulating optimal tax problems.[1]

The original Ramsey formulation suggested that a policy planner would choose commodity taxes which would minimize welfare losses and provide sufficient revenue to the government to finance an exogenous spending program. As economists have become more interested in dynamic models, the Ramsey formulation has been adapted to dynamic settings and led to new dynamic results. Lucas and Stokey (1983) first generalized Ramsey's approach to present a dynamic stochastic model and study the structure and time consistency of optimal tax and monetary policy and a number of papers have further extended this theoretical framework. Chari and Kehoe (1999) review the most important contributions of this literature.[2] Other extensions of the Ramsey optimal tax formulation have investigated the spending side of the planners budget.[3]

[1] An important alternative approach was suggested by Mirrlees (1971) which has also generated a vigorous recent literature with work by Golosov, Kocherlakota and Tsyvinski (2003), Kocherlakota (2005) and Golosov and Tsyvinski (2006).

[2] A branch of special relevance within this literature has been that analyzing optimal taxation on capital. Judd (1985) and Chamley (1986) showed that long run capital tax rates should be zero. This issue has been further studied in more general theoretical setups. For instance, Chari, Christiano and Kehoe (1994) include aggregate shocks, Jones, Manuelli and Rossi (1997) consider human capital accumulation and endogenous growth and Erosa and Gervais (2002) use an overlapping generation model.

[3] Several papers have studied the properties and implications of the optimal policy when a public consumption good is assumed. Among them, Turnovsky (1996) studies the role of a tax on consumption in enhancing growth and welfare, Judd (1999) characterizes the optimal tax on capital in the long run, Gorostiaga (2003, 2005) describes the optimal policy under non-competitive labor markets, and Cassou and Lansing (2006) study the effect of tax reforms in an endogenous growth model. Other papers introduce government spending in the model as a productive input. For instance, Jones, Manuelli and Rossi (1993, 1997) study the size of optimal tax rates on capital, Fisher and Turnovsky (1998) analyze the impact of public investment on private capital accumulation, and Cassou and Lansing (1998, 1999) explore the relationship between nonoptimal fiscal policies and the observed productivity slowdown.

In this paper, we also extend the Ramsey formulation into the spending side, but in a way not yet investigated. Here we assume that government spending has no utility or production benefits as in Ramsey (1927), but in addition to choosing taxes, we allow the planner to choose the composition of its spending program. The presentation here is deliberately close to the original Ramsey structure so as to highlight an important effect that spending has on prices. Richer planning problems in which there are many goods, dynamics or government spending motivated by productive or utility benefits are certainly feasible, but such problems are likely to obscure or overwhelm the pricing effect described here with other economic effects. Furthermore, because this pricing effect will be present in any other setting, it is important to recognize not only its presence but also how it works. Thus, although this paper may appear modest in its modelling structure, this is by choice. It is done to describe in a clean and clear format this ever present price effect that may go unrecognized in richer settings. The fact that government spending has this price effect even in more sophisticated Ramsey planning settings, and, that these effects have not been described to the profession, makes the contribution here important to those investigating all types of Ramsey planning settings so that they do not mistake this effect for something else.[4]

That government spending policy creates pricing effects is well known, but is typically studied in contexts quite different than here. For instance, the idea that government spending can raise interest rates and crowd out private investment spending is a standard topic in the macroeconomic literature, while in microeconomic markets, Weidenbaum (1958, 1960) noted long ago that government policies can influence product prices. Some examples of product market effects that occasionally reach the public conscience include claims that the high rate of inflation for medical services

[4] In a related paper, Cassou, Gorostiaga, Gutiérrez and Hamilton (2006) we investigate a richer dynamic setting for a Ramsey planner to intervene in an economy with environmental issues. In that paper, the pricing trade off described here is hardly noticed in many settings as other economic effects overwhelm it. However, as these other economic effects diminish in size due to reasonable changes in parameters, the pricing trade off described here does become dominant and can present a puzzle to someone not expecting to see it.

and college tuition may be linked to the high rate of government spending in both of these markets.[5]

One might expect that a planner faced with a choice of government spending composition would choose some combination of the goods, but it is shown here that the planner finds it attractive to focus spending on only one good. Although this result is surprising, the intuition is straight forward. The standard approach to Ramsey planning exercises, including the one here, is to follow Ramsey (1927) who envisioned that total government spending is decided exogenously from the planner's problem in a political environment and that the government consumption does not enter consumer utility. What the planner then seeks to do is make policy choices which influence the competitive equilibrium taking as given this exogenous total government spending program. In Ramsey (1927), the objective of the planner is to minimize dead weight welfare losses in the market, but in more modern settings, including here, the planner seeks to maximize consumer utility. Now, because the planner uses the consumer utility function as its own objective function, goods consumed by the government yield no utility. This means that one can interpret the government consumption bundle as consisting of goods that are perfectly substitutable with each other since neither yield utility. On the other hand, consumers do get utility from consuming goods. So, given that the planner must spend a certain total amount on goods, the planner can maximize agent utility by purchasing as much of the most expensive good as possible where the term, most expensive good, is used loosely here so as to not complicate the intuition with qualifiers. As explained in the text, this most expensive good is not the good with the highest absolute price, but is the good whose price is higher than a baseline price which reflects agent utility. By purchasing the most expensive good, the planner thus leaves more of the other good for the consumer

[5] An introduction to these connections can be found in Pauly (2003) and Cunningham, Wellman, Clinedinst, Merisotis and Carroll, (2001). Pauly (2003) provides a thorough treatment of the origins and consequences of rising medical spending growth in the U.S. and Cunningham, Wellman, Clinedinst, Merisotis and Carroll, (2001) explore the sources for rising tuition costs in higher education. Both studies cite rising government spending in these areas as potential sources for inflation.

which maximizes their utility.

Even more interesting is that the planner's objective function is not monotonic in the government consumption goods. Instead, it is U shaped and has an interior minimum. Thus, in a two good setting, there are two maximum; one is a local maximum at one boundary where spending is concentrated on only one good and the other is a global maximum at the other boundary where spending is concentrated only on the other good. The reason for this U shaped objective is the effect that spending has on prices. To understand how government spending induces this U shape, first consider the interior minimum. As shown below, at the interior minimum, prices are equal to the zero total government spending prices. As government spending moves toward a higher concentration of one type of good, the price of that good goes up because of the higher demand. This higher price results in the gains noted above, where it becomes desirable to spend on the most costly good. Furthermore, this effect has an amplifying component in that as concentration rises more, the price is driven still higher and larger gains from the next increment can be had than arose from the last increment. This amplifying component leads to a U shaped objective function rather than a V shape where by the U shape we mean that each successive increment in government consumption produces a greater gain in utility than the previous increment and by V shape we mean that each successive increment in government consumption produces an equal gain in utility to the previous increment.

To present these results in a clear format, we have organized the paper as follows. Section 2 presents a simple two good logarithmic utility model in which government revenue is raised through consumption taxes. This simple formulation allows analytically tractability, and formal results for this model are presented in Section 3. Section 4 goes on to investigate several generalizations to the simple model. These generalizations include: 1) more general utility specifications; 2) a case in which government revenue is raised through lump sum taxes; and, 3) a case in which there are more than just two types of goods. It is shown that our main finding, that government spending has price effects, is robust to these other situations.

2 The Model

In the original formulation by Ramsey (1927), a planner who chose taxes so as to finance an exogenous total government spending program and minimize deadweight welfare losses associated with the competitive equilibrium was envisioned. In Ramsey's formulation, government consumption yielded no consumer utility or production benefits and was assumed to be determined in an exogenous political process. This meant the planner only focused on how to finance this spending. Since this pioneering work, economists have moved away from the deadweight welfare loss objective function toward one of maximizing consumer utility. Furthermore, these recent works have extended the Ramsey formulation to include dynamic aspects as well as integrate into the set of choice variables for the planner government spending which may impact productive activities such as highway infrastructure.

Here we also consider spending elements, but with a different twist than these other studies. As in Ramsey (1927) we assume government spending does not yield production or utility benefits. However, here we allow the planner to choose how it wishes to allocate its spending among the various types of goods. So in particular, we consider a planner who chooses taxes as well as spending allocations so as to maximize consumer utility, taking total government spending and the conditions of a competitive equilibrium as constraints.

In this section we formally describe this optimization problem by first describing the competitive economy in sections 2.1-2.4. Then we formally state the Ramsey problem in section 2.5.

2.1 The corporate sector

The corporate sector consists of two types of producers who manufacture different goods. We index the producers by j and distinguish the sectors by $j = a, b$ and assume there is an equal number of firms from each sector as there are consumers.[6]

[6] Such an assumption is common in models emphasizing competitive price taking agents and is in part justified when production exhibits constant returns to scale. This assumption allows us to focus on a representative agent running each type of firm and a reprentative consumer consuming

In these sectors, output is created through the employment of physical capital and labor according to

$$y_j = k_j^{\alpha_j} l_j^{(1-\alpha_j)} \quad \text{for} \quad j = a, b, \tag{1}$$

where $\alpha_j \in (0,1)$ and y_j, k_j and l_j denote output, capital input and labor input in sector $j = a, b$. In this formulation, when $\alpha_a \neq \alpha_b$, capital and labor inputs have different productive characteristics across sectors. This difference in the ability to produce goods is essential to our results.

We assume that capital and labor are free to move between sectors or between firms and are allocated according to whichever sector or firm pays the highest return. This free mobility has the effect of equating returns across production sectors so that

$$r = r_j = p_j \alpha_j \frac{y_j}{k_j} \quad \text{for} \quad j = a, b, \tag{2}$$

$$w = w_j = p_j (1-\alpha_j) \frac{y_j}{l_j} \quad \text{for } j = a, b, \tag{3}$$

where r and w denote the market capital rental rate and market wage rate and r_j, w_j and p_j denote the jth sector capital rental rate, wage rate and price of output. We will use the a good as the numeraire, so $p_a = 1$.

2.2 The consumer sector

The consumer sector consists of many identical agents who each own k units of capital and l units of labor which is provided to the corporate sector in exchange for capital and labor income. This income is then used to purchase two types of consumption goods, c_a and c_b which yield utility according to

$$U(c_a, c_b) = \ln c_a + \ln c_b. \tag{4}$$

Choices for consumption bundles are based upon a budget constraint given by

$$\sum_{j=a,b} (1+\tau_j) p_j c_j = \sum_{j=a,b} r_j k_j + \sum_{j=a,b} w_j l_j, \tag{5}$$

goods, all of whom are price takers.

where τ_j is a consumption tax chosen by the government and applied to consumption of good j. This constraint shows that income received on the right side of the budget constraint is used to purchase consumption goods on the left side of the budget constraint.

2.3 The government sector

The government engages in two types of activities. First, the government purchases goods from sector j at a level denoted by $g_j \geq 0$. These purchases are assumed to be nonproductive in utility and production. Second, the government chooses a tax policy which raises revenue to finance its expenditures. The tax instruments available for this purpose consist of the consumption taxes on each of the consumption goods which were introduced above. We interpret negative values for a tax as a subsidy. We assume the government runs a balanced budget given by,

$$\sum_{j=a,b} p_j g_j = \sum_{j=a,b} \tau_j p_j c_j, \tag{6}$$

where the left hand side of (6) represents the total government expenditures and the right hand side represents the government's tax receipts. We also assume that total government spending is a proportion of the total level of output according to

$$\sum_{j=a,b} p_j g_j = \phi \sum_{j=a,b} p_j y_j, \tag{7}$$

where $\phi \geq 0$ is the percentage of total output consumed by the government.

In the Ramsey problem described below, it will be shown that the Ramsey planner's objective function has a U shape which implies that the optimal outcome will occur at a boundary and thus the planner will find it optimal to concentrate its spending on only one type of good. However, it is possible to constrain the Ramsey planner's choice set so that a mix of spending arises as the optimal outcome. One way to do this is to assume that there are minimum expenditure constraints for each type of good. In particular, defining $\phi_j = \frac{g_j}{y_j}$ for $j = a, b$ one could require minimum expenditures for each good of ϕ_a^M and ϕ_b^M and formally write such a constraint as

$$\phi_j \geq \phi_j^M \quad \text{for} \quad j = a, b. \tag{8}$$

A natural minimum value is zero, as values less than zero imply the government is able to manufacture goods without a production function. With zero minimum values for the minimum expenditure constraints the optimal policy will be to concentrate spending on only one good. However, in some examples below we will assume minimum values larger than zero and motivate this as the outcome of some exogenous political process that not only chooses ϕ but also ensures that this spending is spread around on all types of goods. In other words, lobbyists for each industry ensure that their sector receives some minimum level of spending. In this set up, a situation in which lobbyists for each industry are equally effective could be imposed by assuming that $\phi_a = \phi_b = \phi$.

2.4 Competitive equilibrium

There are several types of market clearance conditions. First, input market clearance requires that capital across sectors adds up to the total capital stock,

$$k = \sum_{j=a,b} k_j \tag{9}$$

and that the total time allocation adds up to the total time available,

$$l = \sum_{j=a,b} l_j. \tag{10}$$

Second, goods market clearance requires that

$$c_j + g_j = y_j \quad \text{for} \quad j = a, b. \tag{11}$$

A competitive equilibrium is defined by the following. Given a capital stock k and a labor supply l, allocations $\{k_j, l_j, c_j, y_j : j = a, b\}$, prices $\{r_j, w_j, p_j : j = a, b\}$ and government policies $\{\tau_j, g_j, \phi_j, : j = a, b\}$ and ϕ, constitute a competitive equilibrium if the following conditions are satisfied:

(i) Given prices $\{r_j, w_j, p_j : j = a, b\}$ and taxes $\{\tau_j : j = a, b\}$ the allocation $\{k_j, l_j, c_j : j = a, b\}$ maximizes the consumer objective function (4) subject to the budget constraints (5), (9) and (10).

(ii) For each firm $j = a, b$, given prices r_j, w_j, p_j the allocation k_j, l_j, y_j maximizes its profits.

(iii) The government budget constraint (6), spending requirement (7) and minimum expenditure constraint (8) hold.

(iv) The capital, labor and goods markets clear as given in (9), (10) and (11).[7]

2.5 The Ramsey Problem

The Ramsey problem can now be formulated as one in which the planner chooses $\{\tau_j, g_j, \phi_j : j = a, b\}$ so as to maximize (4), taking total spending ϕ and the conditions for a competitive equilibrium as given.[8]

3 Optimal policy

First note that straightforward optimization of the consumer's problem implies

$$c_j = \frac{rk + wl}{2(1+\tau_j)p_j} \quad \text{for} \quad j = a, b. \tag{12}$$

Next note that equality of rental rates between the two production sectors implies the price of the b good is given by

$$p_b = \frac{\alpha_a}{\alpha_b}\frac{y_a}{y_b}\frac{k_b}{k_a}. \tag{13}$$

It is possible to characterize the equilibrium allocation of capital and labor in various ways, but for our purposes it will be useful to write them as functions of fiscal policy variables. Furthermore, as will be seen shortly, it is useful to write these expressions in terms of the particular fiscal variables τ_a and ϕ_a. Given τ_a, ϕ_a and ϕ, values of τ_b and ϕ_b are implied by (6) and (7). We summarize these expressions in the following Lemma.

[7] Equations (9) and (10) appear as both consumer budget constraints and as market clearance conditions because of the representative agent set up used here.

[8] As in Ramsey (1927), the assumption that ϕ is exogenous is motivated by assuming that total spending is decided in an exogenous political environment. Here, all that the planner is choosing is how to allocate and how to finance this spending.

Lemma 1: Given τ_a and ϕ_a, the equilibrium allocation of capital and labor is given by:

$$k_a = \frac{\alpha_a}{[2(1-\phi_a)(1+\tau_a)-1]\alpha_b + \alpha_a} k, \tag{14}$$

$$k_b = \frac{[2(1-\phi_a)(1+\tau_a)-1]\,\alpha_b}{[2(1-\phi_a)(1+\tau_a)-1]\alpha_b + \alpha_a} k, \tag{15}$$

$$l_a = \frac{(1-\alpha_a)}{[2(1-\phi_a)(1+\tau_a)-1]\,(1-\alpha_b) + (1-\alpha_a)} l, \tag{16}$$

and

$$l_b = \frac{[2(1-\phi_a)(1+\tau_a)-1]\,(1-\alpha_b)}{[2(1-\phi_a)(1+\tau_a)-1]\,(1-\alpha_b) + (1-\alpha_a)} l. \tag{17}$$

We can now use the expressions from Lemma 1 to obtain production levels and consumption expressions as functions of fiscal policy. Moreover, we can use these consumption expressions to define the indirect utility function $V(\tau_a, \phi_a) \equiv U\left[c_a(\tau_a, \phi_a), c_b(\tau_a, \phi_a)\right]$ as the maximum utility the consumer can achieve when the policy variables are τ_a and ϕ_a. This form of the indirect utility function is useful for our purposes because of its connection to fiscal policy parameters. The behavior of this function is summarized in the following proposition.

Proposition 1: For any value of τ_a, the derivative of the utility function with respect to ϕ_a is given by

$$\frac{\partial V(\tau_a, \phi_a)}{\partial \phi_a} = \frac{-2\left[(1-\phi_a)(1+\tau_a)-1\right](\alpha_b-\alpha_a)^2}{(1-\phi_a)\left[\left[2(1-\phi_a)(1+\tau_a)-1\right]\alpha_b+\alpha_a\right]\left[\left[2(1-\phi_a)(1+\tau_a)-1\right](1-\alpha_b)+(1-\alpha_a)\right]}.$$

Proposition 1 then implies the following corollary.

Corollary 1: For any value of τ_a:

1. If $\alpha_a = \alpha_b$, then $\frac{\partial V(\tau_a,\phi_a)}{\partial \phi_a} = 0$ for all ϕ_a.
2. If $\alpha_a \neq \alpha_b$, then (a) $\frac{\partial V(\tau_a,\phi_a)}{\partial \phi_a} = 0$ for $\phi_a = \frac{\tau_a}{1+\tau_a}$, (b) $\frac{\partial V(\tau_a,\phi_a)}{\partial \phi_a} > 0$ for $\phi_a > \frac{\tau_a}{1+\tau_a}$, and (c) $\frac{\partial V(\tau_a,\phi_a)}{\partial \phi_a} < 0$ for $\phi_a < \frac{\tau_a}{1+\tau_a}$.

Corollary 1, Case 2 presents the main results for understanding the boundary solution to the Ramsey problem. To understand this, first note that this result holds for any value of τ_a, including the Ramsey value of τ_a. For now let's simply focus on an arbitrary τ_a.

What Case 2 does is describe the shape of the indirect utility function over the range of ϕ_a values. Part (a) says that when $\phi_a = \frac{\tau_a}{1+\tau_a}$, the slope of the indirect utility function is zero. This zero derivative in combination with parts (b) and (c) then imply that when $\phi_a = \frac{\tau_a}{1+\tau_a}$ the indirect utility function is at a minimum. This follows because part (b) shows that for values of ϕ_a larger than $\frac{\tau_a}{1+\tau_a}$ the slope of the indirect utility function is positive and thus the indirect utility function is increasing in ϕ_a, while part (c) shows that for values of ϕ_a smaller than $\frac{\tau_a}{1+\tau_a}$ the slope of the indirect utility function is negative and thus the indirect utility function is decreasing in ϕ_a. Because the indirect utility function is minimized at $\phi_a = \frac{\tau_a}{1+\tau_a}$ and the derivatives are monotonic on either side of this, the maximum is obtained at one of the two boundaries. In general the value of the indirect utility function at the two boundaries will not be equal, so one will represent a local maximum and the other will represent a global maximum. For our purposes the global maximum is of main interest and what Case 2 implies is that for any τ_a the global maximum occurs at the boundary.

Next focus on the Ramsey solution. Because Corollary 1 holds for any value of τ_a, it will hold at the Ramsey tax rate. This means part 2 of Corollary 1 implies the following corollary.

Corollary 2: If $\alpha_a \neq \alpha_b$, then the solution to the Ramsey problem will always occur when one of the minimum expenditure constraints is binding.

To understand Corollary 2 intuitively, first note that one can substitute (14) and (15) into (13) and then make use of two production expressions from the appendix given by (24) and (25) to get

$$p_b = \frac{\alpha_a^{\alpha_a}(1-\alpha_a)^{(1-\alpha_a)}}{\alpha_b^{\alpha_b}(1-\alpha_b)^{(1-\alpha_b)}} \times \left(\frac{[2(1-\phi_a)(1+\tau_a)-1](1-\alpha_b)+(1-\alpha_a)}{[2(1-\phi_a)(1+\tau_a)-1]\alpha_b+\alpha_a}\frac{k}{l}\right)^{(\alpha_a-\alpha_b)}. \quad (18)$$

This expression shows that when $[2(1-\phi_a)(1+\tau_a)-1] = 1$ (i.e. $\phi_a = \frac{\tau_a}{1+\tau_a}$), the price equals the price that would be achieved in a no government situation (i.e. $\phi = \phi_a = \phi_b = \tau_a = \tau_b = 0$). Because of this, we will interpret $[2(1-\phi_a)(1+\tau_a)-1] = 1$ as a baseline for the following discussion. Note, that as ϕ_a rises (falls), $[2(1-\phi_a)(1+\tau_a)-1]$ falls (rises). So we can approach the comparative statics from either perspective and each will be used where it has an advantage.

Next note that

$$\frac{\partial p_b}{\partial \phi_a} = \frac{\alpha_a^{\alpha_a}(1-\alpha_a)^{(1-\alpha_a)}}{\alpha_b^{\alpha_b}(1-\alpha_b)^{(1-\alpha_b)}}\left(\frac{-2(1+\tau_a)(\alpha_b-\alpha_a)^2}{([2(1-\phi_a)(1+\tau_a)-1]\alpha_b+\alpha_a)^2}\frac{k}{l}\right)\times \left(\frac{[2(1-\phi_a)(1+\tau_a)-1](1-\alpha_b)+(1-\alpha_a)}{[2(1-\phi_a)(1+\tau_a)-1]\alpha_b+\alpha_a}\frac{k}{l}\right)^{(\alpha_a-\alpha_b-1)}$$

and that this derivative is always negative.[9] This implies that as ϕ_a rises from the baseline, the price of the b good falls, implying the b good is relatively cheaper than the baseline, while when ϕ_a falls from the baseline, the price of the b good rises, implying that the a good is relatively cheaper.

Recognizing this is really all that is needed for understanding the result. Essentially the government is like a consumer who has perfectly substitutable preferences between the a good and the b good. What they want to do is provide households with the most valued consumption bundle. They can do this by using their own spending budget up on the most costly good. Thus for ϕ_a larger than the baseline, the a good is relatively costly and spending on it leaves a more preferred consumption bundle for consumers. Similarly, when ϕ_a is smaller than the baseline, the b good is relatively more costly than its baseline price and the government can leave a more preferred bundle for consumers by purchasing more of the b good.

An alternative way to see this is to work directly with implications from the

[9] Alternatively note that

$$\frac{\partial p_b}{\partial[2(1-\phi_a)(1+\tau_a)-1]} = \frac{\alpha_a^{\alpha_a}(1-\alpha_a)^{(1-\alpha_a)}}{\alpha_b^{\alpha_b}(1-\alpha_b)^{(1-\alpha_b)}}\left(\frac{(\alpha_b-\alpha_a)^2}{([2(1-\phi_a)(1+\tau_a)-1]\alpha_b+\alpha_a)^2}\frac{k}{l}\right)\times \left(\frac{[2(1-\phi_a)(1+\tau_a)-1](1-\alpha_b)+(1-\alpha_a)}{[2(1-\phi_a)(1+\tau_a)-1]\alpha_b+\alpha_a}\frac{k}{l}\right)^{(\alpha_a-\alpha_b-1)}.$$

is always positive.

Ramsey planner. First note that consumer optimization implies

$$\frac{\partial U}{\partial c_b} = \frac{p_b(1+\tau_b)}{(1+\tau_a)}\frac{\partial U}{\partial c_a}. \tag{19}$$

Since the Ramsey planner takes consumer optimization as given, this expression also holds for the Ramsey planner. Next note that Corollary 1 says that for ϕ_a larger than the baseline (i.e. $\phi_a > \frac{\tau_a}{1+\tau_a}$)

$$\frac{\partial V(\tau_a,\phi_a)}{\partial \phi_a} = \frac{\partial U\left[c_a(\tau_a,\phi_a), c_b(\tau_a,\phi_a)\right]}{\partial \phi_a} = \frac{\partial U}{\partial c_a}\frac{\partial c_a}{\partial \phi_a} + \frac{\partial U}{\partial c_b}\frac{\partial c_b}{\partial \phi_a} > 0.^{10} \tag{20}$$

Plugging in (19) and cancelling terms implies

$$\frac{\partial c_a}{\partial \phi_a} > \frac{-p_b(1+\tau_b)}{(1+\tau_a)}\frac{\partial c_b}{\partial \phi_a}.$$

Notice that the right hand side can be interpreted as the number of a goods that can be obtained in the market for the b good reduction arising from the ϕ_a change. The equation implies that a small increase in ϕ_a implies a b good reduction and an a good increase which is such that the increase in a consumption is larger than would be obtained through simple market trading. Again what is happening is that as ϕ_a increases, the government gives up some b good which it trades back to the market to buy more of the a good. However, because the a good is relatively costly, it cannot buy very much, thus leaving more a good for consumers.[11]

Finally, an alternative way to visualize Corollary 2 is to consider Figures 1a and 1b. Figure 1a plots utility values implied by the competitive equilibrium for alternative values of τ_a and ϕ_a when $\alpha_a = 0.65$, $\alpha_b = 0.4$, $\phi = 0.2$, $k = 35$ and $l = 25$ while Figure 1b plots a cross section of the three dimensional graph for taxes held at $\tau_a = 0.26$.[12] As Figure 1b shows, the cross section is a U shaped curve with the

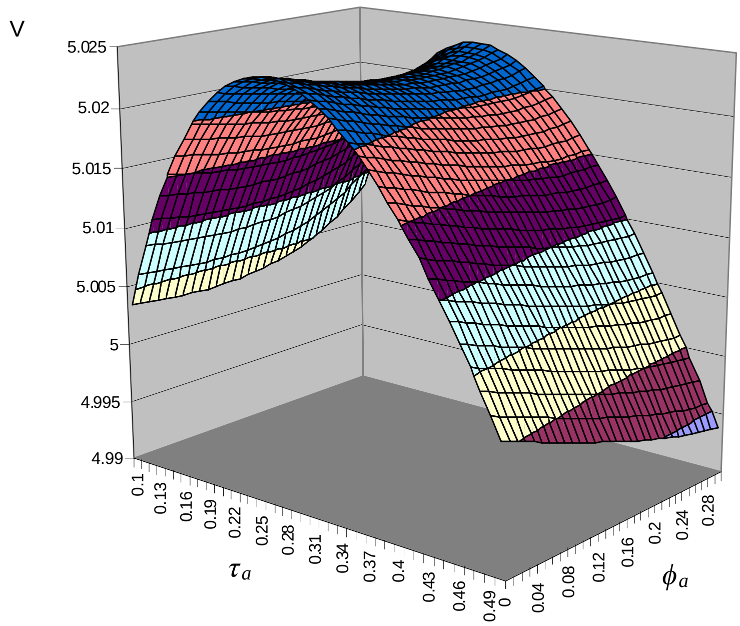
[10] An analogous argument holds for the $\phi_a < \frac{\tau_a}{1+\tau_a}$ case and is left to the reader.

[11] Other insights can be obtained by noting how the allocation of labor and capital given by (14), (15), (16) and (17) change as one moves away from the baseline. These exercises are left to the reader.

[12] The figures are plotted by computing values of $V(\tau_a, \phi_a)$ as follows. For a particular value of τ_a and ϕ_a, equations (14)-(17) yield the capital and labor allocations. These allocations can then be plugged into (1) to find output levels and then plugged into (2), (3) and (13) to get prices. Together, this information is then plugged into (12) to get consumption values and finally these consumption values are plugged into (4) to get the utility value which is plotted.

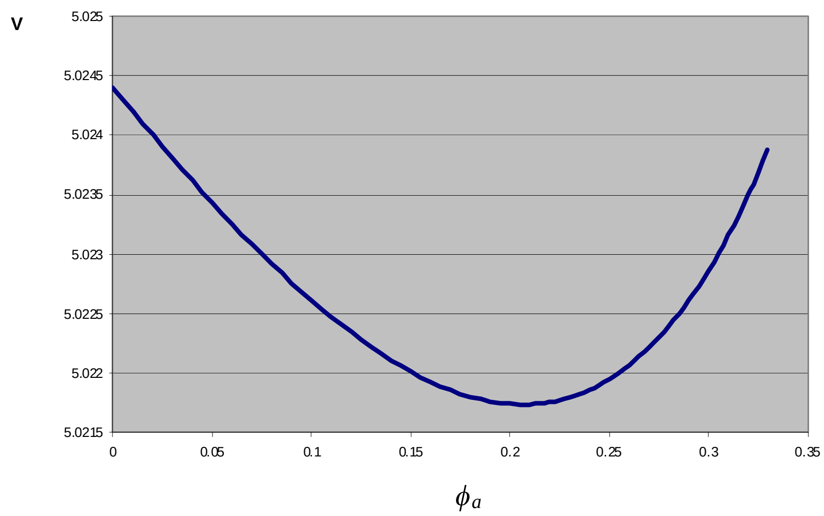
boundary values indicating the two maximum values for ϕ_a. Furthermore, one of these boundary values is a local maximum while the other is a global maximum. This is to be expected given the results of Corollary 1, part 2. What Corollary 2 concludes is that because the cross section for any τ_a is U shaped, the Ramsey solution will be a boundary value and this is clear from Figure 1a.[13]

Figure 1a: Log utility



[13]Note that if one uses minimum expenditure values of $\phi_a^M = \phi_b^M = 0$, then the optimal policy occurs at $\tau_a = .263$, $\phi_a = 0$ and $\phi_b = .331$ while other values for ϕ_a^M and ϕ_b^M would be located at other points in Figure 1.

Figure 1b: Log utility with $\tau_a = .26$



4 Extensions

In this section we consider several generalizations of the basic model structure to see how the results hold up. We begin by considering more general utility formulations. Then we discuss extensions to other tax structures and finally to a situation in which there are more than two goods.

4.1 Extensions to more general utility

When extending the model beyond the logarithmic utility, simple analytical results for the Ramsey problem become infeasible. This arises because the expressions characterizing elements of the competitive equilibrium are highly nonlinear and introduce marginal conditions into the Ramsey planner's first order conditions which do not readily simplify. However, despite this limitation, it is still possible to investigate extensions numerically by using the same routines that generated Figures1a and 1b.

We considered two extensions to the utility specification of the model. The first is for the consumption arguments in utility to exhibit elasticities with each other that differ from the unit elasticity implied by logarithmic preferences and the second is for labor to exhibit less than perfectly inelastic behavior. We formulated these extensions with a general CES utility function given by

$$U(c_a, c_b, l) = \left(\varepsilon_1 c^{\theta} + (1 - \varepsilon_1)(1 - l)^{\theta}\right)^{\frac{1}{\theta}}, \tag{21}$$

where

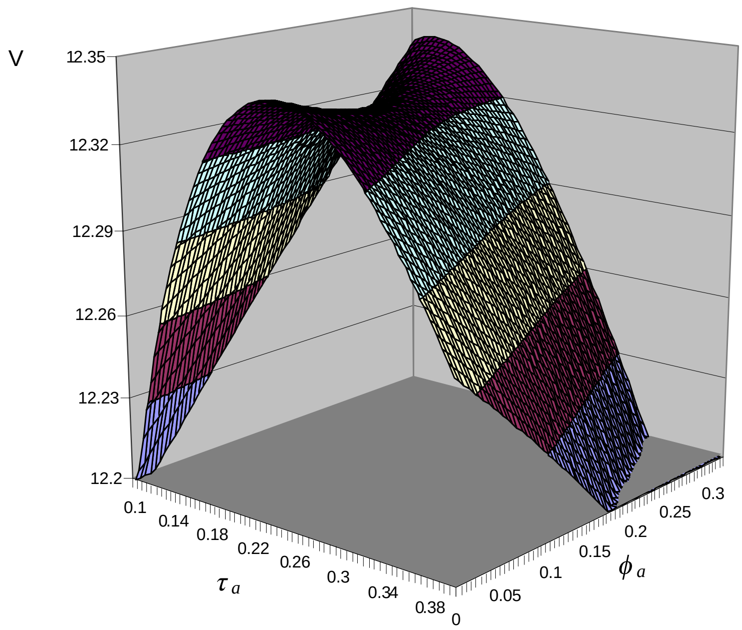
$$c = (\varepsilon_2 c_a^{\psi} + (1 - \varepsilon_2) c_b^{\psi})^{\frac{1}{\psi}}, \tag{22}$$

and parameters are restricted according to $0 < \varepsilon_1 < 1$, $0 < \varepsilon_2 < 1$, $\theta \leq 1$ and $\psi \leq 1$. The parameter θ is related to the elasticity of substitution between the consumption aggregate c and leisure and ψ is related to the elasticity of substitution between the consumption levels c_a and c_b. In this formulation, larger values of ϕ and ψ indicate greater rates of substitution while smaller values indicate greater levels of complementarity.

Our investigation carried out a grid search procedure to find the optimal policy values under the more general utility function. It was found that Corollary 1 did not generalize completely, but was true in a local region near the optimum, while Corollary 2, because it focused only on the optimal values, did generalize completely. In particular, the part of Corollary 1 which claims that a U shaped curve will hold for any value of τ_a was not always true for some elasticity values, but for τ_a near the optimum the U shaped behavior was always present. Since our interest is in finding results about Ramsey type decisions, the generalization of Corollary 2 is what is most important and leads us to conclude that the optimal policy will always occur at a boundary where one or the other minimum expenditure constraint is binding.

Figures 2 and 3 provide a sample of the types of diagrams we found. These are part of the investigation for the CES consumption good extension. Analogous diagrams for the investigation of the elastic labor extension are not included here because of their similarity to the inelastic labor case.[14] Figures 2 and 3 are drawn for $\psi = .5$ and $\psi = -1.5$ respectively while $\varepsilon_2 = 0.5$ and the other model parameters are the same as those used in Figures 1a and 1b. These curves are drawn so that attention can be focused on the optimal policy outcome.[15] The diagrams clearly show that for policy values near the optimum, holding τ_a constant, produces a U shaped curve along the ϕ_a dimension. This shows that Corollary 1 does generalize in a local region near the optimum. Furthermore, because of this U shaped behavior for local values of τ_a, we see that Corollary 2 generalizes completely since it focuses on the optimum.

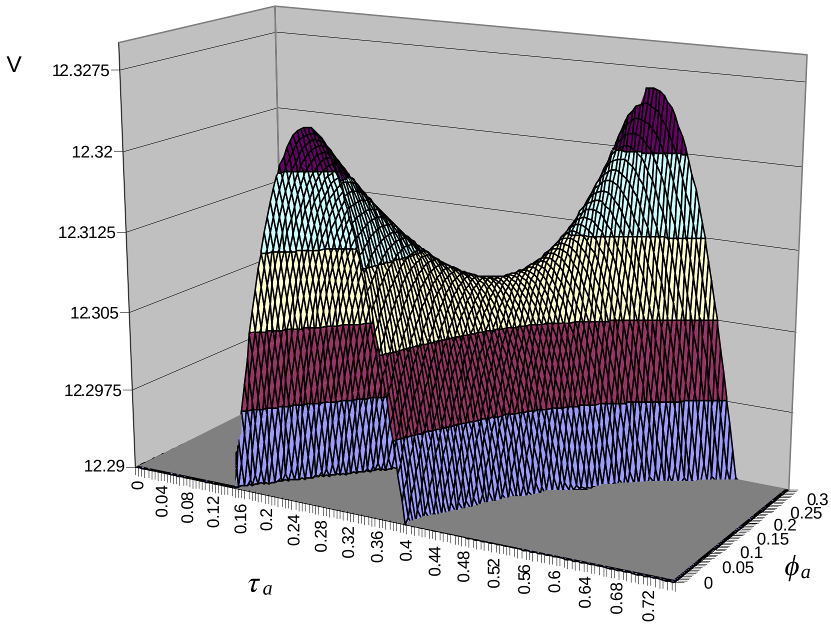
Figure 2: CES utility with ψ =.5



[14] These diagrams can be obtained from the authors upon request.

[15] Assuming minimum expenditure values of $\phi_a^M = \phi_b^M = 0$, the optimum values occur at $\tau_a = .245$, $\phi_a = 0$ and $\phi_b = .341$ for the model in Figure 2 and $\tau_a = .555$, $\phi_a = .335$ and $\phi_b = 0$ for the model in Figure 3.

Figure 3: CES utility with $\psi = -1.5$



4.2 Extensions to other taxes

The tax formulation used above was one in which the government raised revenues through consumption taxes. This formulation was chosen because it was the one that Ramsey (1927) used and because we wanted to show the impact of moving away from Ramsey's formulation by relaxing only the ability of the government to choose which goods it purchased. If we would have relaxed the spending decision and changed the tax structure, then one may question what is the source for our results. That said, however, it is natural to ask whether our results might be tied to the tax structure that was used. In this subsection we provide a further extension of the original Ramsey formulation by not only relaxing the spending decision, but also assuming that taxes are lump sum. It is shown that the results are qualitatively similar to those above and that the results are tied to the price effects of government spending. Since lump sum taxes are the most neutral type of tax, showing that the price effects arising from government spending still hold for this type of tax leads us to believe that these price effects will be present regardless of the tax structure.

Changing the tax system to a lump sum tax produces a few changes in the model foundations. Because there where no taxes in the original corporate sector specification, this sector is unchanged by the conversion. The only change to the consumer formulation requires that the budget constraint be modified to

$$\sum_{j=a,b} p_j c_j = \sum_{j=a,b} r_j k_j + \sum_{j=a,b} w_j l_j - T,$$

where T denotes the lump sum tax. Similarly the government budget constraint is modified to

$$\sum_{j=a,b} p_j g_j = T,$$

while (7) continues to hold. In addition, the definition for a competitive equilibrium and the formulation of the Ramsey problem are modified slightly to reflect these small changes in the model structure.

To analyze the Ramsey problem in this lump sum tax case, first note that optimization of the consumer's logarithmic utility problem now implies

$$c_j = \frac{rk + wl + T}{2p_j} \quad \text{for} \quad j = a, b,$$

where, as before, good a will be used as the numeraire and thus $p_a = 1$. Next note that equality of rental rates between the two production sectors implies that the same formula for the price of the b good, (13), holds. With these results, it is straightforward to show that given ϕ_a, the equilibrium allocation of capital and labor in Lemma 1 are now modified to:

$$k_a = \frac{\alpha_a}{\left(\frac{1+\phi-2\phi_a}{1-\phi}\right)\alpha_b + \alpha_a} k,$$

$$k_b = \frac{\left(\frac{1+\phi-2\phi_a}{1-\phi}\right)\alpha_b}{\left(\frac{1+\phi-2\phi_a}{1-\phi}\right)\alpha_b + \alpha_a} k,$$

$$l_a = \frac{(1-\alpha_a)}{\left(\frac{1+\phi-2\phi_a}{1-\phi}\right)(1-\alpha_b) + (1-\alpha_a)} l,$$

and

$$l_b = \frac{\left(\frac{1+\phi-2\phi_a}{1-\phi}\right)(1-\alpha_b)}{\left(\frac{1+\phi-2\phi_a}{1-\phi}\right)(1-\alpha_b)+(1-\alpha_a)} l.$$

From here one could go on to work through analogues to Proposition 1 and its corollaries. However, in the interest of brevity and to connect things to the price issues emphasized above, one can plug (1) into (13) and then make use of these allocation expressions to get

$$p_b = \frac{\alpha_a^{\alpha_a}(1-\alpha_a)^{(1-\alpha_a)}}{\alpha_b^{\alpha_b}(1-\alpha_b)^{(1-\alpha_b)}} \left(\frac{\left[\frac{1+\phi-2\phi_a}{1-\phi}\right](1-\alpha_b)+(1-\alpha_a)}{\left[\frac{1+\phi-2\phi_a}{1-\phi}\right]\alpha_b+\alpha_a} \frac{k}{l} \right)^{(\alpha_a-\alpha_b)}.$$

This expression shows that when $\left[\frac{1+\phi-2\phi_a}{1-\phi}\right]=1$ (i.e. $\phi_a=\phi$), the price equals the price that would be achieved in a no government situation (i.e. $\phi=\phi_a=\phi_b=0$). This value of $\phi_a=\phi$ is the minimum of the indirect utility function V for the analogue to Proposition 1. Furthermore, one can interpret this minimum by noting that

$$\begin{aligned} \frac{\partial p_b}{\partial \phi_a} &= \frac{\alpha_a^{\alpha_a}(1-\alpha_a)^{(1-\alpha_a)}}{\alpha_b^{\alpha_b}(1-\alpha_b)^{(1-\alpha_b)}} \left(\frac{-2\left(\frac{1}{1-\phi}\right)(\alpha_b-\alpha_a)^2}{\left(\left[\frac{1+\phi-2\phi_a}{1-\phi}\right]\alpha_b+\alpha_a\right)^2} \frac{k}{l} \right) \times \\ &\quad \left(\frac{\left[\frac{1+\phi-2\phi_a}{1-\phi}\right](1-\alpha_b)+(1-\alpha_a)}{\left[\frac{1+\phi-2\phi_a}{1-\phi}\right]\alpha_b+\alpha_a} \frac{k}{l} \right)^{(\alpha_a-\alpha_b-1)} \end{aligned}$$

and that this derivative is always negative. This means that as ϕ_a rises from the baseline value of $\phi_a=\phi$, the price of the b good falls, implying the b good is relatively cheaper than the baseline, while when ϕ_a falls from the baseline value of $\phi_a=\phi$, the price of the b good rises, implying that the a good is relatively cheaper. From here the intuition is the same as before and implies that a minimum occurs at the baseline value of $\phi_a=\phi$ and that a local maximum occurs at one boundary and a global maximum occurs at the other boundary.

4.3 Extensions to n goods

Thus far, the analysis has focused on economies in which there are two goods. Extending the logarithmic utility results to an n good model is straight forward once one recognizes that an n good model can be reduced to a two good model by simply assuming that the government holds constant its spending on the other $n-2$ goods. In particular, one can break the n good analysis into a sequence of two good comparisons. As we explain below, the result of this analysis will imply the same general statement as above, which is that the Ramsey planner should focus its spending entirely on the most expensive single good. Since building the model into an n good structure would require considerable space and yield little new insight into the policy implications, in what follows we will simply intuitively elaborate on the n good analysis.

One can begin by generalizing the consumer and production sectors to include more goods. In the logarithmic utility case, these changes would not break the tractability of the model and consumer demand functions could be found relatively easily. Next one can focus on the government's role in the competitive equilibrium and its implications for the Ramsey planner's problem. One strategy would be to simply make a sequence of two good comparisons between each pair of goods under the assumption that government consumption for the other $n-2$ goods is zero. Under a simple logarithmic utility function in which all goods are equally weighted, one would expect the two good comparisons to yield a series of figures analogous to Figure 1a which can be used to rank the goods. Then based on this ranking, the Ramsey planner would then find it optimal to allocate its entire budget to the most costly good. It is worth emphasizing that it would not want to allocate any spending to any other good because, based on the two good comparison between the most expensive and any other good, it would be best to allocate the budget solely to the most expensive good.

Extending the analysis to the CES utility case is more complicated since the model is already intractable in the two good case. Furthermore, a full generalization would allow the elasticities of substitution between the various goods as well as the weight in utility for each good to vary. However, despite these complications we expect the picture of the objective to be an n dimensional generalization of Figures

2 and 3 above and that again a boundary solution would prevail as the optimum. At this optimum we expect the same general statement to hold which is that the Ramsey planner would choose for its own personal consumption the good which is most costly.

5 Conclusion

This paper investigated an extension to the Ramsey planning structure in which the planner faces an exogenous total expenditure constraint, but is able to choose the composition of this spending. It is shown that, like taxes, expenditure policy also has price effects and that these price effects have a significant impact on optimal policy. In particular, these price effects result in a U shaped objective function and this implies that expenditures should be concentrated into one type of good. These results were demonstrated formally using a simple two good static model with logarithmic preferences and where shown to be robust to more general utility using graphical methods. It was also shown that this result held in a lump sum tax environment and when there are more than just two goods.

Although the economic structure used to demonstrate these price effects is simple, this was by choice as it allows a clean analytical proof for these results. Showing these price effects is important because there are many real world situations in which government spending does impact the price of goods. Furthermore, these effects are important to economic theorists because they will be present in any Ramsey problem in which there are multiple production sectors. So any investigation into more general Ramsey problems need to be aware of these effects so that they are not mistaken for something else. In a related dynamic model with environmental issues, Cassou, Gorostiaga, Gutiérrez and Hamilton (2006), these price effects are always present. Under some parameterizations, these price effects are dominated by other aspects of the economy while in other parameterizations these price effects dominate. If one were not aware of these price effects, it is possible that they could be mistaken for some other type of economic behavior.

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A Appendix

This appendix goes through the proofs of various propositions in the paper.

A.1 Proof of Lemma 1

Because the production technologies exhibit constant returns to scale technology, income equals GDP and can be written as

$$rk + wl = y_a + p_b y_b.$$

The market clearing condition for the A good market implies that: $c_a = y_a - g_a = (1-\phi_a)y_a$, which upon making use of (12) gives

$$c_a = \frac{y_a + p_b y_b}{2(1+\tau_a)} = (1-\phi_a)y_a.$$

Substituting in (13) gives

$$y_a + \frac{\alpha_a}{\alpha_b}\frac{y_a}{y_b}\frac{k_b}{k_a}y_b = 2(1-\phi_a)(1+\tau_a)y_a,$$

which can be written as

$$\alpha_b k_a + \alpha_a k_b = 2(1-\phi_a)(1+\tau_a)\alpha_b k_a. \tag{23}$$

Using the capital market clearance condition (9), the amount of capital in sectors a and b can be solved to get

$$k_a = \frac{\alpha_a}{[2(1-\phi_a)(1+\tau_a)-1]\alpha_b + \alpha_a}k,$$

and

$$k_b = \frac{[2(1-\phi_a)(1+\tau_a)-1]\,\alpha_b}{[2(1-\phi_a)(1+\tau_a)-1]\alpha_b + \alpha_a}k.$$

Next using (3), one gets

$$(1-\alpha_a)\,\frac{y_a}{l_a} = (1-\alpha_b)\,p_b\frac{y_b}{l_b}.$$

Using (13), one gets an expression for capital and labor ratios of

$$\frac{(1-\alpha_a)}{(1-\alpha_b)}\frac{k_a}{k_b} = \frac{\alpha_a}{\alpha_b}\frac{l_a}{l_b}.$$

Using this jointly with the equilibrium capital allocation, one can get

$$\frac{l_a}{l_b} = \frac{(1-\alpha_a)}{[2(1-\phi_a)(1+\tau_a)-1]\,(1-\alpha_b)}.$$

Finally, using (10) one can get

$$l_a = \frac{(1-\alpha_a)}{[2(1-\phi_a)(1+\tau_a)-1]\,(1-\alpha_b)+(1-\alpha_a)}l$$

and

$$l_b = \frac{[2(1-\phi_a)(1+\tau_a)-1]\,(1-\alpha_b)}{[2(1-\phi_a)(1+\tau_a)-1]\,(1-\alpha_b)+(1-\alpha_a)}l.$$

A.2 Proof of Proposition 1

Plugging (14), (15), (16) and (17) into (1) and rearranging gives

$$
\begin{aligned}
y_a &= \left(\frac{\alpha_a}{[2(1-\phi_a)(1+\tau_a)-1]\alpha_b+\alpha_a}\right)^{\alpha_a} \times \\
&\quad \left(\frac{(1-\alpha_a)}{[2(1-\phi_a)(1+\tau_a)-1](1-\alpha_b)+(1-\alpha_a)}\right)^{(1-\alpha_a)} \Omega_a \qquad (24)\\
y_b &= [2(1-\phi_a)(1+\tau_a)-1]\left(\frac{\alpha_b}{[2(1-\phi_a)(1+\tau_a)-1]\alpha_b+\alpha_a}\right)^{\alpha_b} \times \\
&\quad \left(\frac{(1-\alpha_b)}{[2(1-\phi_a)(1+\tau_a)-1](1-\alpha_b)+(1-\alpha_a)}\right)^{(1-\alpha_b)} \Omega_b \\
&= [2(1-\phi_a)(1+\tau_a)-1]\, y'_b. \qquad (25)
\end{aligned}
$$

where $\Omega_j = k^{\alpha_j} l^{(1-\alpha_j)}$ for $j = a, b$, and
$y'_b = \left(\frac{\alpha_b}{[2(1-\phi_a)(1+\tau_a)-1]\alpha_b+\alpha_a}\right)^{\alpha_b}\left(\frac{(1-\alpha_b)}{[2(1-\phi_a)(1+\tau_a)-1](1-\alpha_b)+(1-\alpha_a)}\right)^{(1-\alpha_b)}\Omega_b.$

Also, using (13) and $g_j = \phi_j y_j$ for $j = a, b$ in (7) we get

$$\phi_b = \phi - \frac{\alpha_b}{\alpha_a}\frac{k_a}{k_b}\left(\phi_a - \phi\right).$$

Next, note that this implies

$$
\begin{aligned}
1-\phi_b &= 1-\phi+\frac{1}{2(1-\phi_a)(1+\tau_a)-1}\phi_a - \frac{1}{2(1-\phi_a)(1+\tau_a)-1}\phi \qquad (26)\\
&= \frac{2(1-\phi)(1+\tau_a)-1}{2(1-\phi_a)(1+\tau_a)-1}(1-\phi_a).
\end{aligned}
$$

Also, using (14) and (15) we get $\frac{k_a}{k_b} = \frac{\frac{\alpha_a}{\alpha_b}}{2(1-\phi_a)(1+\tau_a)-1}$, which implies

$$\phi_b = \phi - \frac{1}{2(1-\phi_a)(1+\tau_a)-1}\left(\phi_a - \phi\right).$$

The market clearing condition for the j good implies that $c_j = y_j - g_j = (1-\phi_j)y_j$ for $j = a, b$. Using this along with (25) in (4), we define the indirect utility function $V(\tau_a, \phi_a)$ as:

$$V(\tau_a,\phi_a) \equiv U\left[c_a(\tau_a,\phi_a), c_b(\tau_a,\phi_a)\right] = \ln(1-\phi_a)+\ln(1-\phi_b)+\ln\left[2(1-\phi_a)(1+\tau_a)-1\right]+\ln y_a+\ln y'_b.$$

Differentiating the utility function with respect to ϕ_a gives

$$\frac{\partial V(\tau_a,\phi_a)}{\partial \phi_a} = -\frac{1}{(1-\phi_a)} - \frac{1}{(1-\phi_b)}\frac{\partial \phi_b}{\partial \phi_a} - \frac{2(1+\tau_a)}{2(1-\phi_a)(1+\tau_a)-1} + \frac{1}{y_a}\frac{\partial y_a}{\partial \phi_a} + \frac{1}{y'_b}\frac{\partial y'_b}{\partial \phi_a}. \qquad (27)$$

We now evaluate each of the terms in this derivative beginning with the second term. We compute the derivative of ϕ_b with respect to ϕ_a to get

$$\begin{aligned}\frac{\partial \phi_b}{\partial \phi_a} &= -\frac{2(1-\phi_a)(1+\tau_a)-1-(\phi_a-\phi)(-2)(1+\tau_a)}{[2(1-\phi_a)(1+\tau_a)-1]^2}\\ &= -\frac{2(1-\phi)(1+\tau_a)-1}{[2(1-\phi_a)(1+\tau_a)-1]^2},\end{aligned}$$

and using (26) we see

$$\frac{\partial \phi_b}{\partial \phi_a} = -\frac{(1-\phi_b)}{(1-\phi_a)}\frac{1}{[2(1-\phi_a)(1+\tau_a)-1]}.$$

Next use (24) to get

$$\begin{aligned}\frac{\partial y_a}{\partial \phi_a} &= \alpha_a\left(\frac{\alpha_a}{[2(1-\phi_a)(1+\tau_a)-1]\alpha_b+\alpha_a}\right)^{(\alpha_a-1)}\left(\frac{\alpha_a 2(1+\tau_a)\alpha_b}{([2(1-\phi_a)(1+\tau_a)-1]\alpha_b+\alpha_a)^2}\right)\times\\ &\left(\frac{(1-\alpha_a)}{[2(1-\phi_a)(1+\tau_a)-1](1-\alpha_b)+(1-\alpha_a)}\right)^{(1-\alpha_a)}\Omega_a\\ &+(1-\alpha_a)\left(\frac{\alpha_a}{[2(1-\phi_a)(1+\tau_a)-1]\alpha_b+\alpha_a}\right)^{\alpha_a}\times\\ &\left(\frac{(1-\alpha_a)2(1+\tau_a)(1-\alpha_b)}{([2(1-\phi_a)(1+\tau_a)-1](1-\alpha_b)+(1-\alpha_a))^2}\right)\times\\ &\left(\frac{(1-\alpha_a)}{[2(1-\phi_a)(1+\tau_a)-1](1-\alpha_b)+(1-\alpha_a)}\right)^{(-\alpha_a)}\Omega_a,\end{aligned}$$

which implies

$$\frac{1}{y_a}\frac{\partial y_a}{\partial \phi_a} = \left(\frac{2(1+\tau_a)\alpha_a\alpha_b}{[2(1-\phi_a)(1+\tau_a)-1]\alpha_b+\alpha_a}\right)+\left(\frac{2(1+\tau_a)(1-\alpha_a)(1-\alpha_b)}{[2(1-\phi_a)(1+\tau_a)-1](1-\alpha_b)+(1-\alpha_a)}\right).$$

Similarly, note that (25) gives

$$\begin{aligned}\frac{\partial y_b'}{\partial \phi_a} &= \alpha_b\left(\frac{\alpha_b}{[2(1-\phi_a)(1+\tau_a)-1]\alpha_b+\alpha_a}\right)^{(\alpha_b-1)}\left(\frac{\alpha_b 2(1+\tau_a)\alpha_b}{([2(1-\phi_a)(1+\tau_a)-1]\alpha_b+\alpha_a)^2}\right)\times\\ &\left(\frac{(1-\alpha_b)}{[2(1-\phi_a)(1+\tau_a)-1](1-\alpha_b)+(1-\alpha_a)}\right)^{(1-\alpha_b)}\Omega_b\\ &+(1-\alpha_b)\left(\frac{\alpha_b}{[2(1-\phi_a)(1+\tau_a)-1]\alpha_b+\alpha_a}\right)^{\alpha_b}\times\\ &\left(\frac{(1-\alpha_b)2(1+\tau_a)(1-\alpha_b)}{([2(1-\phi_a)(1+\tau_a)-1](1-\alpha_b)+(1-\alpha_a))^2}\right)\times\\ &\left(\frac{(1-\alpha_b)}{[2(1-\phi_a)(1+\tau_a)-1](1-\alpha_b)+(1-\alpha_a)}\right)^{(-\alpha_b)}\Omega_b\end{aligned}$$

which implies

$$\frac{1}{y'_b}\frac{\partial y'_b}{\partial \phi_a} = \left(\frac{2(1+\tau_a)\alpha_b^2}{[2(1-\phi_a)(1+\tau_a)-1]\alpha_b+\alpha_a}\right)+\left(\frac{2(1+\tau_a)(1-\alpha_b)^2}{[2(1-\phi_a)(1+\tau_a)-1](1-\alpha_b)+(1-\alpha_a)}\right)$$

Substituting out these derivatives in (27) gives

$$\begin{aligned}\frac{\partial V(\tau_a,\phi_a)}{\partial \phi_a} \quad &= \quad -\frac{1}{(1-\phi_a)}-\frac{1}{(1-\phi_b)}\left(-\frac{(1-\phi_b)}{(1-\phi_a)}\frac{1}{[2(1-\phi_a)(1+\tau_a)-1]}\right)-\frac{2(1+\tau_a)}{2(1-\phi_a)(1+\tau_a)-1}\\ &+\left(\frac{2(1+\tau_a)\alpha_a\alpha_b}{[2(1-\phi_a)(1+\tau_a)-1]\alpha_b+\alpha_a}\right)+\left(\frac{2(1+\tau_a)(1-\alpha_a)(1-\alpha_b)}{[2(1-\phi_a)(1+\tau_a)-1](1-\alpha_b)+(1-\alpha_a)}\right)\\ &+\left(\frac{2(1+\tau_a)\alpha_b^2}{[2(1-\phi_a)(1+\tau_a)-1]\alpha_b+\alpha_a}\right)+\left(\frac{2(1+\tau_a)(1-\alpha_b)^2}{[2(1-\phi_a)(1+\tau_a)-1](1-\alpha_b)+(1-\alpha_a)}\right).\end{aligned}$$

Putting things over a common denominator and rearranging gives

$$\frac{\partial V(\tau_a,\phi_a)}{\partial \phi_a} = \frac{-2\left[(1-\phi_a)(1+\tau_a)-1\right](\alpha_b-\alpha_a)^2}{(1-\phi_a)\left[\left[2(1-\phi_a)(1+\tau_a)-1\right]\alpha_b+\alpha_a\right]\left[\left[2(1-\phi_a)(1+\tau_a)-1\right](1-\alpha_b)+(1-\alpha_a)\right]}.$$