R&D Policy in Economies with Endogenous Growth and Non-Renewable Resources*

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October 2007

Abstract

The aim of this paper is to analyze how active R&D policies affect the growth rate of an economy with endogenous growth and non-renewable resources. We know from Scholz and Ziemens (1999) and Groth (2006) that in infinitely lived agents (ILA) economies, any active R&D policy increases the growth rate of the economy. To see if this result also appears in economies with finite lifetime agents, we developed an endogenous growth overlapping generations (OLG) economy à la Diamond which uses non-renewable resources as essential inputs in final good's production. We show analytically that any R&D policy that reduces the use of natural resources implies a raise in the growth rate of the economy. Numerically we show that in economies with low intertemporal elasticity of substitution (IES), active R&D policies lead the economy to increase the depletion of non-renewable resources. Nevertheless, we find that active R&D policies always imply increases in the endogenous growth rate, in both scenarios. Furthermore, when the IES coefficient is lower (greater) than one, active R&D policies affect the growth rate of the economy in the ILA more (less) than in OLG economies.

Key Words: endogenous growth, R&D, non-renewable resources, overlapping generations, infinitely lived agents, balanced growth path.
JEL *classification*: O13, O40, Q32

*Financial support from the Spanish Ministry of Education and Science, through Projects SEJ2006-12793/ECON, SEJ2007-62081/ECON and SEJ2006-10827/ECON, and Universidad del País Vasco through Grupo de Investigación Consolidado UPV GIU06/41.

1 Introduction

One of the central analytical findings of the literature on growth is that worldwide economic growth is possible in spite of the finite supply of exhaustible resources if there is sufficient technological progress. The feasibility of positive long-run growth, despise nonrenewable natural resources being an essential input in the production sector, has been extensively explored in the neoclassical exogenous growth framework, among others by Stiglitz (1974), Solow (1974), Dasgupta and Heal (1974) and Agnani *et al.* (2005). In all these models, the feasibility of positive long-run growth of per capita consumption depends on the scope and extent of technological progress relative to the endogenous depletion rate of non-renewable resources. Furthermore, even though technological progress is exogenously given, long-run growth is endogenously determined since it depends on the endogenous depletion rate.

The relationship between the use of exhaustible resources and technological progress has also been analyzed in the endogenous growth literature developed in the 1990s. Studies such as Aghion and Howitt (1998), Barbier (1999), Scholz and Ziemens (1999) analyze the sustainability of positive long-run growth paths in economies with exhaustible resources and infinitely-lived agents (ILA), where the engine of growth is the creation of new intermediate inputs that are used as imperfect substitutes in the final-good sector.[1] This present paper follows this research line. In particular, our aim is to investigate how active R&D policies may affect the growth rate in endogenous growth economies that use exhaustible resources, which are essential inputs in the production sector.

We already know from Scholz and Ziemens (1999) and Groth (2006) that in ILA economies, any active R&D policy increases the growth rate of the economy. In this paper, we analyze the equivalent economy studied in Scholz and Ziemens (1999) with finite lifetime agents instead of considering infinitely-lived individuals. In particular, we develop an overlapping generations (OLG) model à la Romer (1990) where each generation consists of finite households that live for two periods and are not altruistic as in Diamond (1965). Authors such as Solow (1986) point out that OLG models appear to be "the natural habitat" for discussing on the impact of current

[1] Aghion and Howitt (1998), Barbier (1999) and Nili (2001) solve the central planner's problem in this type of economy with constant elasticity of intertemporal substitution. Scholz and Ziemes (1999) extend this analysis by studying the market equilibrium in this kind of economy.

resource extraction decisions on future generations. Other research such as Agnani *et al.* (2005) justifies the use of an OLG framework vs. that of the ILA models to analyze long-run growth with exhaustible resources, relying on the existence of empirical evidence against the altruism assumed in ILA models. Therefore, a comparison between the results in an OLG framework with respect to the ILA setup appears to be necessary. One of our main findings is that the policy implication is different in both scenarios.

From the theoretical point of view, we find that any active R&D policy affects the growth rate of the economy through two channels. First, the direct channel, which shows that the more productive the R&D sector is, the higher the growth rate of the stock of knowledge, regardless of the use of the exhaustible resources. This ceteris paribus result is quite intuitive, since this is the standard result in Romer's model, without exhaustible resources. Second, the indirect channel which comes through the use of the exhaustible resources in the final output sector. The sign of this indirect effect is ambiguous. We prove analytically that for both frameworks, ILA and OLG, any active R&D policy that leads the economy to deplete less exhaustible resources will increase the growth rate of the economy (this is the case in which direct and indirect effects work in the same direction).

For the ILA economy, we show that the indirect effect is positive whenever the intertemporal elasticity of substitution is greater than one and not too high. Contrariwise, the indirect effect is negative for values of the intertemporal elasticity of substitution lower than one. However, even for cases where the indirect effect goes in the opposite direction to the direct effect, we prove that the final effect of an active R&D policy on the growth rate is unambiguously positive.

For the OLG economy the determination of the stationary depletion rate is even more complex than in the ILA set up, so we are not able to characterize analytically the cases in which the indirect effect is positive or negative. Because of this complexity we numerically simulate the effects of active R&D policies in the economy under the two scenarios, ILA and OLG. First of all, the parameters of the model are selected such that the benchmark case for both scenarios represents the same economy, and mimics some empirical facts of the economy. Secondly, we compare the results under both scenarios when productivity in the R&D sector increases. Our main numerical findings are as follows: First, OLG and ILA economies are similar in terms of growth rates; however they are very different in the composition of the growth process. Whereas under the ILA scenario, economic growth relies more on a lower

use of non-renewable resources, under the OLG economy the growth process depends on higher growth in the R&D sector. In this sense we could say that ILA economies are more exhaustible-conservationist. The intuition behind this result is clear. Since in ILA economies agents live up to infinity, they are able to wait until later to consume. Thus agents consume less today, depleting fewer resources, and devoting a high percentage of human capital to the R&D sector. Second, in both OLG and ILA economies, where agents are more willing to wait to consume in the future (i.e. with high IES coefficient), active R&D policies are more conservationist, depleting exhaustible resources less. And third, active R&D policies always increase the growth rate, under both scenarios. Furthermore, when current and future consumption are substitutes (complementaries), i.e. when the IES coefficient is lower (greater) than one, active R&D policies affect the growth rate more (less) in economies where agents live infinitely than those with finite lifetime agents.

The rest of the paper is organized as follows. Section 2 presents the OLG model. The market equilibrium in an OLG framework is defined in Subsection 2.1 and the balanced growth path is characterized in Subsection 2.2. In Section 3 we analyze the effect of an R&D policy on the two types of economies, ILA vs. OLG. Conclusions are presented in Section 4.

2 The Overlapping Generations (OLG) Model

We develop the basic two-period overlapping generations framework (Diamond (1965)) in an endogenous growth economy à la Romer (1990), with exhaustible resources which are essential inputs for production in the final good sector. From now on we refer to this set up as the OLG model. In order to analyze the role of the agents with finite lifetimes, we solve the equivalent model but with infinitely-lived agents. This is the model analyzed in Scholz and Ziemens (1999), but in continuous time rather than discrete time, and is developed in Appendix 2.

We assume that each generation consists of L new individual agents who live for two periods. There is no population growth. There are three production sectors: the final-good sector, the intermediate sector and the R&D sector.

Consumers/Households:

All individual agents have rational expectations and are identical except

for their age. As usual in growth literature, since we are interested in economies for which balanced growth paths exist, we consider consumer preferences with constant elasticity of intertemporal substitution (King and Rebelo (1993))[2]. In particular the preferences of a representative agent born at period t are represented by

$$u\left(c_{1t}, c_{2,t+1}\right) = \frac{c_{1,t}^{1-\epsilon} - 1}{1-\epsilon} + \frac{1}{1+\theta}\left(\frac{c_{2,t+1}^{1-\epsilon} - 1}{1-\epsilon}\right),$$

where $c_{1,t}$ and $c_{2,t+1}$ represent consumption for young and old age, respectively; $\theta \geq 0$ is the subjective discount rate of the agent and $1/\epsilon > 0$ is the intertemporal elasticity of substitution (IES). The closer to (farther from) zero the parameter ϵ is, the more substitute (complementary) current and future consumptions are. In particular $\epsilon = 1$ represents the logarithmic preferences case.

Each agent born at period t, is endowed when young with a fixed quantity of human capital, h. Since all agents are identical, except for their age, the individual human capital of a young individual and the average level of human capital in the young population (which is assumed to be fixed in the economy) coincide. He/she receives the wage, w_{Ht}, per unit of labor, which can be used either to consume the final good, $c_{1,t}$, to buy the ownership rights to the resource stock, m_{t+1}, or to save, s_{t+1} (physical capital or bonds issued by the intermediate sector)[3]. The final consumption good is taken as a numerary and p_t is the price of the exhaustible resource in terms of final consumption good.

When the agent is old, at period $t+1$, his/her income comes from different sources. The return of his/her savings is $(1 + r_{t+1})\, s_{t+1}$ which includes the rental from his/her physical capital stock and from the bonds issued by the intermediate firms. On the other hand, old agents receive income from selling resource property rights, m_{t+1}, to the young generation and to final-good

[2]Constant elasticity of intertemporal substitution is a sufficient but not a necessary condition to guarantee the existence of balanced growth. See Stokey and Lucas (1984) for more details.

[3]Consumers own the existing durable goods-producing firms, therefore the (net) intermediate sector's profits are paid to them. Alternatively, we could have assumed that consumers diversify their savings in the three forms, in physical capital, bonds (issued by intermediate firms) and exhaustible resource, but only in that amount of the exhaustible resource that is not used in the production process. (See Mourmouras (1993)).

firms.[4] The revenue from this sale is $p_{t+1}m_{t+1}$ (in per worker terms).

Therefore, the representative agent born at period t, maximizes his/her utility function with respect to young and old consumption taking prices as given. This problem can be set out in per worker terms as follows: $\forall t = 1, 2, ...$

$$\underset{\{c_{1,t}, c_{2,t+1}, s_{t+1}, m_{t+1}\}}{Max} \quad \frac{c_{1,t}^{1-\epsilon} - 1}{1-\epsilon} + \frac{1}{1+\theta}\left(\frac{c_{2,t+1}^{1-\epsilon} - 1}{1-\epsilon}\right),$$

$$s.t. \quad c_{1,t} + p_t m_{t+1} + s_{t+1} = w_{H,t}h, \tag{1}$$

$$c_{2,t+1} = (1 + r_{t+1})\, s_{t+1} + p_{t+1} m_{t+1}, \tag{2}$$

$$w_{H,t}, p_t \text{ and } r_{t+1} \text{ are given.}$$

The first order conditions for this maximization problem can be expressed as

$$\frac{c_{2,t+1}}{c_{1,t}} = \left(\frac{1 + r_{t+1}}{1+\theta}\right)^{1/\epsilon}, \tag{3}$$

$$1 + r_{t+1} = \frac{p_{t+1}}{p_t}. \tag{4}$$

Equation (3) indicates that each consumer equates the marginal rate of substitution between current and future consumption to their relative prices, or marginal rate of transformation given by $1 + r_{t+1}$. Equation (4) is the standard arbitrage condition that characterizes the optimal investment between the two forms of savings such that the marginal returns on both must be equal. In other words, the marginal rate of saving in the exhaustible resource, p_{t+1}/p_t, must be equal to the marginal rate of saving in physical capital or bonds issued by the intermediate firms, $1 + r_{t+1}$[5].

Combining first order conditions (3)-(4) and taking into account consumer budget constraints, the consumer saving function can be characterized as

$$s_{t+1} + p_t m_{t+1} = \frac{w_{H,t}h}{1 + (1+\theta)^{1/\epsilon}\,(1 + r_{t+1})^{-\left(\frac{1-\epsilon}{\epsilon}\right)}}.$$

[4] We are assuming that there exists a market for the exhaustible resource stock which is sold by older generations to younger generations, and a market for the exhaustible resource that is finally extracted and used in the production process (see, for example, Olson and Knapp (1997) or Agnani *et al.* (2005)).

[5] This arbitrage condition satisfies the well-known Hotelling rule of optimal resource extraction for exhaustible resource in partial equilibrium models, under assumption of costless extraction (Hotelling, 1931).

Notice that given the arbitrage condition, (4), the consumer's income in the second period depends on $s_{t+1}+p_t m_{t+1}$. Therefore, from the consumer's point of view, any combination of physical capital, bonds and exhaustible resources satisfying this saving function maximizes his/her utility.

Production Sectors:

There are three production sectors: the final-good sector, the intermediate-goods sector and the R&D sector.

a) The final-good sector:

This sector produces a homogeneous good, Y_t, that can be consumed or invested in the form of physical capital. All firms share the same production technology and use as inputs human capital, $H_{Y,t}$, a variety of intermediate goods, X_t^i with $i = 1, ...A_t$, and the exhaustible resource extracted, E_t. A_t indicates how large the variety of the intermediated goods is, and it also represents the stock of knowledge of the economy.

The aggregate production function for the final output is given by $Y_t = H_{Yt}^{\alpha_1}\left[\sum_{i=1}^{A_t}\left(X_t^i\right)^{\alpha_2}\right]E_t^{\alpha_3}$. Following Romer (1990), we assume the same technology for producing any intermediate good and, in consequence, their unit cost is the same. Therefore, since they enter in the final-good sector symmetrically, in equilibrium an identical amount of each intermediate good will be produced: $X_t^i = X_t, \quad \forall i = 1, ...A_t$. This implies that *in equilibrium* the total amount of intermediate-goods in the economy can be denoted by $\sum_{i=1}^{A_t}\left(X_t^i\right)^{\alpha_2} = A_t X_t^{\alpha_2}$ and the aggregate production function by

$$Y_t = A_t H_{Yt}^{\alpha_1} X_t^{\alpha_2} E_t^{\alpha_3}. \tag{5}$$

Constant returns to scale with respect to all private inputs are assumed, i.e., $\alpha_1 + \alpha_2 + \alpha_3 = 1$.

Final-good firms hire labor, intermediate goods and exhaustible resources to maximize profits taking prices and the stock of technology as given. Therefore, the representative firm's problem can be set down as follows in each period t,

$$\underset{\left\{H_{Yt},\{X_t^i\}_{i=1}^{A_t},E_t\right\}_{t=0}^{\infty}}{Max} \quad Y_t - w_{H,t}H_{Y,t} - q_t\sum_{i=1}^{A_t} X_t^i - p_tE_t,$$

$$s.t. \quad Y_t = H_{Yt}^{\alpha_1}\sum_{i=1}^{A_t}\left(X_t^i\right)^{\alpha_2}E_t^{\alpha_3},$$

$$w_{H,t}, q_t, p_t, \text{and } A_t \text{ are given.}$$

where q_t is the price of the intermediate goods.

In the case of an interior solution, the first-order conditions for the firm's maximization problem are given by the following equations

$$\alpha_1 H_{Yt}^{(\alpha_1-1)}\left(A_tX_t^{\alpha_2}\right)E_t^{\alpha_3} = w_{H,t}, \tag{6}$$

$$\alpha_2 H_{Yt}^{\alpha_1}X_t^{(\alpha_2-1)}E_t^{\alpha_3} = q_t, \tag{7}$$

$$\alpha_3 H_{Yt}^{\alpha_1}\left(A_tX_t^{\alpha_2}\right)E_t^{\alpha_3-1} = p_t, \tag{8}$$

which indicate that firms hire labor, intermediate goods and exhaustible resources until their marginal products equal their factor prices.

b) The monopolistic intermediate-goods sector / design market:

The intermediate-goods sector uses the designs innovated by the R&D sector to produce the intermediate goods that are available in each period. This sector is composed of A_t firms indexed by "i". The only input used in the production of the intermediate good i is physical (man-made) capital, K_t, [6] and the production function is given by

$$X_t^i = K_t^i/\eta, \ \forall i \in [0, A_t],$$

where K_t^i is physical capital used in the production of intermediate good i, and η denotes the units of physical capital required to produce one unit of intermediate good, X_t^i.

Each firm indexed by "i" owns an infinite life-time patent that allows it to produce monopolistically its corresponding intermediate good. This patent is bought in a competitive market for new designs (patents) and it financed through a bond issued with an interest rate of r_{t+1}.

[6] Capital goods are produced in a separate sector that has the same technology as the final-output sector, i.e. K_t can be accumulated as foregone output. We assume that physical capital stock, K_t, does not depreciate, $\delta = 0$.

Since the patent has an infinite life-time, the equilibrium price of the patent will be equal to the present discount value of the infinite stream of profits it will generate. So, the price for the patent, P_t^A, is given by the following expression in each period t

$$P_t^A = \sum_{\tau=t+1}^{\infty} \frac{\pi_\tau}{\prod_{s=t+1}^{\tau}(1+r_s)}$$

or equivalently solving the above equation we can write,

$$P_t^A = \left(\frac{1}{1+r_{t+1}}\right)[P_{t+1}^A + \pi_{t+1}], \tag{9}$$

where π_t is the profit of a representative monopolist producing intermediate good i at period t.

Once the patent has been paid, each intermediate-good i is produced monopolistically by a single firm, which sells it to the final good sector at a price, q_t. Taking into account the demand function for intermediate good, (7), the monopoly problem for the intermediate firm producing a good i is given by,

$$\begin{aligned} &\underset{\{X_t\}_{t=0}^{\infty}}{Max}\ \pi_t = q_t X_t - r_t K_t^i, \\ s.t. \quad &\begin{cases} q_t = \alpha_2 H_{Yt}^{\alpha_1} X_t^{(\alpha_2-1)} E_t^{\alpha_3}, \\ X_t = K_t^i/\eta, \\ H_{Y,t}, E_t \text{ and } r_t \text{ are given.} \end{cases} \end{aligned}$$

The first-order condition for this maximization problem is

$$q_t = \frac{r_t \eta}{\alpha_2}. \tag{10}$$

The resulting monopoly price given by equation (10) is a markup over the marginal cost, and this markup is determined by the elasticity of the demand curve, $1/(\alpha_2 - 1)$. The flow of monopoly profit is positive and works out at

$$\pi_t = (1-\alpha_2)\, q_t X_t > 0. \tag{11}$$

c) The R&D sector:

This sector uses human capital and the existing stock of knowledge to produce new knowledge, which consists in designs for new intermediate goods. There are j competitive firms producing designs and sharing the same technology, $\sigma H_{A,t}^{j} A_t$, which depends upon a productivity parameter, σ, the amount of human capital devoted to R&D activities by the research firm j, $H_{A,t}^{j}$, and the stock of knowledge available (number of designs) in the economy, A_t.

The aggregate stock of designs evolves according to the following law of motion,

$$A_{t+1} - A_t = \sigma H_{A,t} A_t, \tag{12}$$

where we $H_{A,t} = \sum_j H_{A,t}^{j}$ is the aggregated amount of human capital used by the R&D firms.

The technological productivity parameter, σ, is an intrinsic parameter that characterizes the R&D process of the economy. It captures all those factors that affect productivity in the R&D sector, apart from the amount of human capital devoted to the R&D activities. Such factors might include property rights, corruption, R&D infrastructure, even the ability to imitate from outside. In fact, in some articles, such as Benhabib and Spiegel (1994) or Córdoba and Ripoll (2005), the R&D sector productivity depends on the distance of the technology in the country relative to the technology frontier, which is assumed to be exogenous to the country. In general, the higher the value of σ, the higher the growth rate of new knowledge, for a given amount of human capital devoted to the R&D sector. Therefore, if an economy wishes to accelerate the creation of new knowledge it should develop technological policies that increase this productivity parameter.

A representative research sector firm hires the stock of human capital to maximize its profits given the dynamics of the stock of technology and taking wages, patent price and initial stock of technology as given,

$$\underset{\left\{H_{A,t}\right\}_{t=0}^{\infty}}{Max} \quad P_t^A \left(A_{t+1} - A_t\right) - w_{H,t} H_{A,t},$$

$$s.t. \quad \begin{cases} A_{t+1} - A_t = H_{A,t} \sigma A_t, \\ H_{A,t} \geq 0, \\ w_{H,t}, P_t^A \text{ and } A_0 \text{ are given.} \end{cases}$$

The first order condition for maximization of the R&D firm's problem is given by

$$P_t^A \sigma A_t \leq w_{H,t}, \tag{13}$$

with equality if $H_{A,t} > 0$. This condition indicates that R&D firms hire human capital until their marginal product equals its factor price. As in Romer (1990), if the stock of human capital in the economy is not high enough, the economy will allocate no resources to produce new designs.

Exhaustible Resources

The economy is initially endowed with a positive amount of exhaustible resources, M_0. The stock of exhaustible resources in the current period, M_t, is determined by the stock available in the previous period minus those resources extracted for the use of the final-good sector, i.e. $M_t = M_{t-1} - E_{t-1}$. If we define the depletion rate of exhaustible resources as

$$\tau_t = \frac{E_t}{M_t} \tag{14}$$

the equilibrium dynamics for exhaustible resources can be expressed as

$$M_{t+1} = (1 - \tau_t) M_t. \tag{15}$$

Human Capital

The human capital stock, $H = hL$, is fixed and is addressed either to the final good sector, H_Y, or to the R&D sector, H_A:

$$H = H_{Y,t} + H_{A,t}. \tag{16}$$

2.1 The Equilibrium Solution

In the endogenous growth model described above, a dynamic equilibrium is a sequence of quantities $\{c_{1,t}, c_{2,t}, s_{t+1}, K_{t+1}, Y_t, X_t, \tau_{t+1}, E_t, M_{t+1}, H_{Y,t}, H_{A,t}, A_{t+1}, \pi_t\}_{t=0}^{\infty}$ and prices $\left\{P_t^A, w_{H,t}, r_t, p_t, q_t\right\}_{t=0}^{\infty}$ such that: *i)* consumers maximize utility subject to their intertemporal budget constraint taking prices as given; *ii)* firms in the final-good sector maximize profits choosing labor and intermediate inputs taking their prices as given; *iii)* each design owner produces its corresponding intermediate good maximizing monopolistic profits, taking human capital and the demand they face as given; *iv)* producers of the new designs maximize profits choosing labor, taking wages, patent price and initial stock of technology as given; and *v)* all markets clear.

Market clearing

Market clearing conditions are given by the following:

i) Human capital allocates between the final-good sector and the R&D sector such that equations (6), (13) and (16) are satisfied.

ii) The resource market clears when non-renewable resources supplied by old agents are equal to the demand of firms and young agents. Therefore, the equilibrium evolution of the stock of exhaustible resources is given by equation (15)

iii) The designs market clears when the demand for each new design equals its supply, i.e. whenever equation (9) holds.

iv) The physical capital market clears when the stock of capital in the economy is equal to the demand for capital in the intermediate good sector. This means that

$$\eta A_t X_t = K_t. \tag{17}$$

v) The final good market clears when demand equals supply. The final good is devoted to consumption or to investment in physical capital, patents or non-renewable resources. Since we have used s_t to denote the savings per worker in physical capital and patents, the condition under which the final good market clears can be written in the standard way, i.e. the stock of physical capital per worker is given by

$$K_{t+1} + P_t^A A_{t+1} = s_{t+1} L. \tag{18}$$

The equilibrium characterization is summarized in the following definition.

Definition 1 *For any arbitrary initial value of τ_0, an equilibrium of this OLG economy is an infinite sequence of quantity allocations $\{c_{1,t}, c_{2,t+1}, s_{t+1}, K_{t+1}, Y_t, X_t, \tau_{t+1}, E_t, M_{t+1}, H_{Y,t}, H_{A,t}, A_{t+1}, \pi_t\}_{t=0}^{\infty}$ and prices $\left\{P_t^A, w_{H,t}, r_t, p_t, q_t\right\}_{t=0}^{\infty}$ such that consumers, final-goods producers and research firms maximize their objective functions taking prices as given, the intermediate firms maximize their monopolist profits and all markets clear, given the initial conditions $K_0, M_0, A_0 > 0$[7]. In other words, an equilibrium is a solution of the non-linear system (1)-(18).*

[7]Since physical capital, exhaustible resources and technology are essential for production, K_0, M_0 and A_0 must be positive. Otherwise young consumers of the initial generation would have no income and consumption would remain zero forever.

Note that the equilibrium is unable to determine the initial depletion rate. This problem has been solved in other related articles in different ways. Aghion and Howitt (1998) choose K_0 such that the economy starts on the balanced growth path and τ_0 is chosen under this assumption. Scholz and Ziemes (1999) choose τ_0 such that the steady state is a saddle path. Stiglitz (1974) takes the initial price for the exhaustible resource as given. Barbier (1999) takes the initial depletion rate of the exhaustible resource as given. Since we are interested only in the balanced growth equilibrium we do not address this issue.

2.2 The Balanced Growth Path

In general terms, balanced growth paths are those where all variables grow at a constant rate. As Groth (2006) notes, *compliance with Kaldor's styled facts is generally equivalent with the existence of balanced growth paths.* Furthermore, King *et. al.* (1988) point out that economies characterized by constant growth rates in the long-run provide clear evidence of industrialization. This is why in this section we focus on the equilibria paths where all variables grow at constant rates. In particular, we analyze balanced growth paths defined as follows :

Definition 2 *A balanced growth path is an equilibrium path where all variables grow at a constant rate and the depletion rate of exhaustible resources and stock of human capital allocations among the final-good and R&D sectors remain constant.*

Let us define γ_z as the ratio z_{t+1}/z_t, on the balanced growth path for all endogenous variables except for depletion rate and the stock of human capital in the final-good and R&D sectors. For the latter ones, we define $\tau = \tau_{t+1} = \tau_t, H_Y = H_{Y,t+1} = H_{Y,t}$ and $H_A = H_{A,t+1} = H_{A,t}$. With these definitions the balanced growth path will be determined by a zero growth rate for the depletion rate and human capital allocations and by constant growth rates $(\gamma_z - 1)$ for the rest of the endogenous variables. The following proposition states conditions that any balanced growth path of this OLG economy must satisfy.

Proposition 1 *Any balanced growth path of this OLG economy is given by a vector* $\left\{\gamma_Y, \gamma_K, \gamma_A, \gamma_M, \gamma_\pi, \gamma_s, \gamma_{c_1}, \gamma_{c_2}, \gamma_X, \gamma_E, \gamma_p, \gamma_q, \gamma_r, \gamma_{P^A}, \gamma_{w_H}, H_Y, H_A, \tau\right\}$ *satisfying the following system of equations*

$$\left.\begin{array}{c} \frac{\alpha_2^2\gamma}{\gamma-(1-\tau)} = \frac{(1-\alpha_2)\alpha_2}{\gamma_A-(1-\tau)}\left(\frac{\sigma H}{1+(1+\theta)^{\frac{1}{\epsilon}}\left(\frac{\gamma}{1-\tau}\right)^{-\left(\frac{1-\epsilon}{\epsilon}\right)}} - \gamma_A\right) - \frac{\alpha_3}{\tau}, \\ \gamma_A = \frac{(1-\tau)[(1+\sigma H)(1-\alpha_2)\alpha_2+\alpha_1]}{(1-\tau)(1-\alpha_2)\alpha_2+\alpha_1}, \\ H_Y = \frac{\alpha_1([\gamma_A-(1-\tau)]}{(1-\tau)\sigma(1-\alpha_2)\alpha_2}. \end{array}\right\},\ if\ H_A = H-H_Y > 0,$$

$$\left.\begin{array}{c} \frac{(1-\tau)\alpha_2^2\gamma}{\gamma-(1-\tau)} = \frac{\alpha_1}{1+(1+\theta)^{\frac{1}{\epsilon}}\left(\frac{\gamma}{1-\tau}\right)^{-\left(\frac{1-\epsilon}{\epsilon}\right)}} - \frac{\gamma_M(\alpha_3+(1-\alpha_2)\alpha_2)}{\tau}, \\ \gamma_A = 1, \\ H_Y = H. \end{array}\right\},\ if\ H_A = 0,$$

$$\begin{aligned} \gamma_M &= \gamma_E = 1-\tau, \\ \gamma_X &= \gamma_{P^A} = \gamma_\pi = (1-\tau)^{\frac{\alpha_3}{1-\alpha_2}}, \\ \gamma_p &= \frac{\gamma}{1-\tau}, \\ \gamma_r &= \gamma_q = 1, \\ \gamma_K &= \gamma_Y = \gamma_{c_1} = \gamma_{c_2} = \gamma_s = \gamma_{w_H} = \gamma. \end{aligned}$$

where $\gamma = \gamma_A(1-\tau)^{\frac{\alpha_3}{1-\alpha_2}}$.

Proof. See Appendix 1. ■

This proposition states that in this OLG framework the stationary depletion rate, τ, is obtained endogenously from a non-linear equation and is determined in the last instance for all the parameters of the economy.[8] This is not surprising since Agnani *et al.* (2005) obtain a similar result for OLG economies with exhaustible resources but with an exogenous engine of growth. Proposition 3, in Appendix 2, shows that in ILA economies, the stationary depletion rate also depends on all parameters of the model but in a different manner. But for the particular case of logarithmic consumer preferences ($\epsilon = 1$), the stationary depletion rate is given solely by the

[8]It is not possible to characterize the uniqueness of the equilibrium of this economy. We have found numerical parametrizations for which there are multiple equilibria.

consumers' subjective discount rate, θ, in ILA economy (see Corollary 1 in Appendix 2) while it depends on all the parameters in the OLG setup.

There are two types of balanced growth paths: those with interior solutions for the stock of human capital devoted to the R&D sector and those with corner solutions characterized by a null allocation of human capital in the R&D sector.

The growth rate of the economy, both in OLG and ILA economies, is given by $\gamma - 1 = \gamma_A (1-\tau)^{\alpha_3/(1-\alpha_2)} - 1$, where γ_A also depends on the depletion rate, τ, in the case of an interior solution (i.e. when $H_A > 0$). Therefore, any change in the exogenous variables that affects the endogenous growth rate solely through their effect on the endogenous stationary depletion rate (such as θ and ϵ), will depend only on how the endogenous depletion rate τ affects the endogenous growth rate.[9] There are two effects, both of the same sign, such that any exogenous change (in θ or ϵ) that negatively affects the endogenous depletion rate will have a positive effect on the endogenous growth rate[10]. The direct effect shows that the greater the endogenous depletion rate of the non-renewable resources, the lower the endogenous growth rate, regardless of the growth of the stock of knowledge, γ_A. That is $\partial\gamma/\partial\tau < 0$, taking γ_A as a constant. The indirect effect works through the allocation of the stock of human capital between R&D and the final output sector. Analyzing γ_A we can see that the higher the stationary depletion rate, the lower the growth of the stock of knowledge in the economy, and, consequently, the lower the growth rate.[11] Note that this indirect effect

[9] This is not the case for the R&D parameter, σ. In this case this parameter affects γ directly through γ_A. See Proposition 1.

[10] This result is very similar to the one obtained in Groth (2006) under an endogenous growth model, with non-renewable resources, without technical progress. In particular, he finds that *along a BGP, policies that decrease (increase) the depletion rate (and only such policies) will increase (decrease) the per capita growth rate.*

[11] Note that from the definition of γ we have

$$\frac{\partial\gamma}{\partial\tau} = \frac{\partial\gamma_A}{\partial\tau}(1-\tau)^{\frac{\alpha_3}{1-\alpha_2}} - \frac{\alpha_3}{1-\alpha_2}(1-\tau)^{\frac{-\alpha_1}{1-\alpha_2}}\gamma_A.$$

The second sum of the left hand side expresses the direct effect. The indirect effect comes through γ_A, that is

$$\frac{\partial\gamma_A}{\partial\tau} = -\frac{\alpha_1\left[(1+\sigma H)(1-\alpha_2)\alpha_2 + \alpha_1\right]}{\left[(1-\tau)(1-\alpha_2)\alpha_2 + \alpha_1\right]^2} < 0.$$

Therefore $\partial\gamma/\partial\tau < 0$.

appears because the engine of growth in the economy is endogenous.[12]

From Propositions 1 and 3 in Appendix 2, the following remark shows necessary conditions that guarantee positive growth in ILA and OLG economies.

Remark 1 *A necessary (but not sufficient) condition for an ILA and OLG economy to exhibit positive growth is that part of the human capital has to be allocated to the R & D sector,* $H_A > 0$.

Note that equilibria with $H_A = 0$ imply that $\gamma_A = 1$ and $\gamma = (1-\tau)^{\alpha_3/(1-\alpha_2)} < 1$ whenever the exhaustible resource is an essential input in the final-good production, $\alpha_3 > 0$.

Propositions 1 and 3 also state that on the balanced growth path, income, physical capital, consumption, savings and wage rates grow at the same rate, $(\gamma - 1)$, which depends on all parameters of the economy. The price of non-renewable resources grows at a higher rate than income, indicating that these resources are exhaustible and consequently the supply decreases over time. Since the gross interest rate is the growth rate of the price of exhaustible resources (arbitrage condition (4)), the interest rate must be constant on the balanced growth path. Observe that the stock of non-renewable resources and the use of such resources in the production process, M_t and E_t, decline over time. Moreover, if the growth rate of the economy is positive, i.e. if $\gamma > 1$, then the price of exhaustible resources must increase.

Another interesting result from Proposition 1 and 3 is stated in the following remark.

Remark 2 *The patent price decreases along the balanced growth path.*

This result contrasts with Romer's growth model (1990) solution, where the patent price remains constant, because the wage per unit of labor must grow at the same rate in both sectors (final-good and R&D sector) (equations (6) and (13)), and income and physical capital grow at a lower rate than the number of designs (stock of knowledge) due to the use of the exhaustible resources in the final-good sector. In consequence, the patent price decreases, capturing the fact that on the balanced growth path the productivity of the stock of knowledge decreases with the use of non-renewable resources.

[12] When the engine of growth is exogenous, the indirect effect does not appear (see Agnani *et al.* (2005)).

Although OLG and ILA economies show the same relationship between the endogenous growth rate and the endogenous stationary depletion rate, the relationships between the endogenous stationary depletion rate and the exogenous variables are different in both frameworks. In the following section we compare the balanced growth paths obtained in the ILA and OLG scenarios. In particular we focus on how the balanced growth varies when R&D policies are implemented in both scenarios.[13]

3 R&D Policy and the Finite Lifetimes

The aim of this paper is to analyze how an active R&D policy may affect the growth rate of the economy. In particular, we compare the results of an increase in the R&D productivity parameter, σ, in infinitely lived economies and in economies with finite lifetime agents.

From Proposition 1 and 3 (last in Appendix 2), we know that the relationship between the endogenous growth rate of the economy and the technological parameter of the R&D sector is not obvious. The next proposition shows that when increases in the R&D parameter lead an economy to reduce the extraction of resources, this implies an increase in the growth rate of the economy in question. However, the opposite may not occur.

Proposition 2 *In ILA and OLG economies, if* $\partial\tau/\partial\sigma < 0$ *then* $\partial\gamma/\partial\sigma > 0$.

Proof. Since the growth rate of the economy is given by $\gamma = \gamma_A(1-\tau)^{\frac{\alpha_3}{1-\alpha_2}}$, then

$$\frac{\partial\gamma}{\partial\sigma} = \frac{\partial\gamma_A}{\partial\sigma}(1-\tau)^{\frac{\alpha_3}{1-\alpha_2}} - \frac{\alpha_3}{1-\alpha_2}(1-\tau)^{\frac{-\alpha_1}{1-\alpha_2}}\gamma_A\frac{\partial\tau}{\partial\sigma}.$$

Since $\gamma_A = \frac{(1-\tau)[(1+\sigma H)(1-\alpha_2)\alpha_2+\alpha_1]}{(1-\tau)(1-\alpha_2)\alpha_2+\alpha_1}$, then

$$\frac{\partial\gamma_A}{\partial\sigma} = \frac{(1-\tau)H\,(1-\alpha_2)\,\alpha_2}{(1-\tau)\,(1-\alpha_2)\,\alpha_2+\alpha_1} + \frac{\partial\gamma_A}{\partial\tau}\frac{\partial\tau}{\partial\sigma}.$$

On the other hand, since

[13] Scholz and Ziemens (1999) focus on the determinacy and stability of the equilibrium for the ILA economy. They show that some technological prerequisites have to be met in order to guarantee the stability of the equilibrium. We do not focus on this aspect.

$$\frac{\partial \gamma_A}{\partial \tau} = -\frac{\alpha_1 \left[(1+\sigma H)(1-\alpha_2)\alpha_2 + \alpha_1 \right]}{\left[(1-\tau)(1-\alpha_2)\alpha_2 + \alpha_1 \right]^2} < 0,$$

if $\frac{\partial \tau}{\partial \sigma} < 0$, then $\frac{\partial \gamma_A}{\partial \sigma} > 0$ and $\frac{\partial \gamma}{\partial \sigma} > 0$. ■

Groth (2006) shows that the condition imposed for the statement of this proposition is not only a sufficient but also a necessary condition in endogenous growth economies with non-renewable resources and without technical progress.

Note that the above proposition shows the best situation for the economy to stimulate its growth rate. Note also that $\gamma_A = \frac{(1-\tau)[(1+\sigma H)(1-\alpha_2)\alpha_2+\alpha_1]}{(1-\tau)(1-\alpha_2)\alpha_2+\alpha_1}$, where τ is endogenously determined by all parameters of the model. Therefore, any change in the technological parameter affects the growth rate of the stock of knowledge through two channels. First, the direct channel which shows that the greater the R&D productivity parameter, the higher the growth rate of the stock of knowledge, regardless of the stationary depletion rate, τ. This is so because the higher the productivity parameter of the R&D sector, the higher the amount of human capital allocated to the R&D sector. That is $\partial \gamma_A / \partial \sigma > 0$, taking τ as given. Second, the indirect channel which works through the use of the non-renewable resources in the final output sector. Analyzing γ_A we can see that any increase in the depletion rate leads to a reduction of technological growth; however, the relationship between the technological parameter and the stationary depletion rate is ambiguous. It is clear that if the relationship between the R&D productivity parameter and the use of the non-renewable resources is negative, the direct and indirect channels work in the same direction and any stimulant R&D policy increases the growth rate of the stock of knowledge. However when an increase in the technological parameter leads to an increase in the depletion rate, the indirect effect goes in the opposite direction to the direct effect, and the final result of the R&D productivity parameter over the growth rate is ambiguous.

Corollaries 1 and 2 in Appendix 2 characterize the sign of the indirect effect for the ILA economy. In particular, the indirect effect is shown to be positive whenever $\frac{\alpha_3}{\alpha_1+2\alpha_3} \leq \epsilon \leq 1$ and negative for $\epsilon > 1$. Considering Proposition 2, it is straighforward that whenever $\frac{\alpha_3}{\alpha_1+2\alpha_3} \leq \epsilon \leq 1$, an active R&D policy will affect the growth rate of the economy positively. However, even for cases where the indirect effect goes in the opposite direction to the direct effect, i.e. when $\epsilon > 1$, Proposition 4 in Appendix 2 proves that the final effect of an active R&D policy over the growth rate is positively

unambiguous.

To summarize, when the IES, $1/\epsilon$, is lower than $(\alpha_1 + 2\alpha_3)/\alpha_3$ an active R&D policy guarantees an increase in the growth rate of an ILA economy. However this result cannot be generalized for very large values of the IES in the ILA economy. This finding does not enter in conflict with Scholz and Ziemens (1999) who show that the growth rate of the economy always responds positively to active R&D policies.[14] In the case of the OLG economy the determination of the stationary depletion rate is even more complex than in the ILA set up, so, unlike the ILA model, we are not able to characterize cases in which $\partial\tau/\partial\sigma < 0$. Because of this analytical complexity, we present in the following subsections the calibration of the economy used to compare numerically, in both scenarios (ILA and OLG), the effects of an active R&D policy over the growth of the economy.

3.1 Parameterization

To compute the stationary equilibrium, we specify values for the parameters such that *i)* they are consistent with some empirical facts and *ii)* they are perfectly standard in the literature.

The subjective discount rate was chosen to make that the annual discount factor 0.98, which is standard in calibration literature. In the ILA framework it implies a value for θ equal to 0.02. If we assume that in the OLG economy, each period is 25 years long, an annual discount rate of 0.02 is equivalent to 0.65 for a 25-year period.[15]

In the benchmark case, the IES is equal to 1 (i.e. $\epsilon = 1$) which represents consumer logarithmic preferences. Besides being studied in theoretical papers (see for instance Agnani *et al.* (2005), Hsuku (2007)), there is evidence that the IES is significantly different from zero and probably close to one (Beaudry and Wincoop (1996)). On the other hand, this value is in the interval that Gourinchas and Parker (2002) considered as plausible. They conclude that the IES coefficient varies between 0.7 and 2. Note that if the IES is equal to 1, we have already proved analytically that under the ILA economy, an active R&D policy never affects the depletion rate and, in

[14] Scholz and Ziemens (1999) develop their model in continous time. This allows them to find an explicit expression for the growth rate which is tractable to finding this effect.

[15] If each period covers 25 years, the parameter values for each period are such that $(1+\theta) = (1+\theta^*)^p$, $(1-\tau) = (1-\tau^*)^p$ and $\gamma = (\gamma^*)^p$, where variables with * are the annual parameter value and $p = 25$.

consequence, we can assure that the growth rate of the economy increases. Since the value of the IES parameter substantially the endogenous variables, we also study the robustness of the results by analyzing how they might differ under different values for the IES (in the interval estimated by Gourinchas and Parker (2002)).

The values for capital and labor shares are standard. In particular, they were selected such that the labor share equals 60% ($\alpha_2 = 0.60$) and the capital share equals 35% ($\alpha_1 = 0.35$). The share of exhaustible resources in the final-good production function, α_3, was set at 0.05, as in Groth and Schou (2002).

Finally, the stock of human capital was normalized to one, and the productivity in the R&D sector selected to obtain an annual growth rate of 2% in the benchmark case of the two economies. This criterion meant selecting $\sigma = 14.80$ for the OLG economy and $\sigma = 0.086$ for the ILA set up.[16]

With this benchmark parametrization both the ILA and OLG economies have a unique balanced growth path with an annual growth rate of 2%. The human capital in the R&D sector, H_A, is positive at 26.34% in the ILA scenario and 6.86% within the OLG framework, implying an annual growth rate of the stock of knowledge, $\gamma_A - 1$, of 2.25 (ILA economy) and 4.51 (OLG economy). Moreover, the stationary depletion rates obtained, in annual terms, are such that the use of non-renewable resources is 1.98% and 6.37% in the ILA and OLG scenarios, respectively..

Table 1 summarizes the parameter values chosen to implement the numerical exercises for both scenarios, OLG and ILA. He introducido el valor de IES en la tabla.

3.2 Simulating changes in the productivity of the R&D sector

Once the benchmark ILA and OLG economies had been calibrated, a simulation exercise was implemented in order to check how much the results differ in the two scenarios when the R&D productivity parameter, σ, varies.

Table 2 illustrates the depletion rate, the percentage of the stock of human capital devoted to the R&D sector and the growth rate of both economies for

[16] An alternative way of choosing H is to take H for the ILA economy such that the growth rate of knowledge $(1 + \sigma H)$ is equal, in annual terms, to that of the OLG economy.

Table 1: Parameter Values for OLG and ILA Benchmark Economies

Parameters	ILA	OLG	Data and annually targets
Preferences			
$1/\epsilon$	1	1	$(07, 2)$ following Gourinchas and Parker (2002)
θ	0.02	0.65	Annual discount factor = 0.98
			Each period is 25 years long
Final-good sector			Observed variables (values)
α_1	0.35	0.35	Standard
α_2	0.60	0.60	Standard
α_3	0.05	0.05	Groth and Schou (2001)
R&D sector			
σ	0.116	15.345	Annual growth rate = 2%
H	1	1	

different changes in σ. In particular, the first row of the table shows the value of the aforementioned endogenous variables in the benchmark economies.

Three important conclusions can be observed. First, in all the cases analyzed, OLG and ILA economies are clearly similar in terms of growth rates; however their make-up are radically different. Whereas in the ILA scenario, economic growth relies more on a lower use of non-renewable resources, in the OLG economy the growth process depends on higher growth in the R&D sector. In this sense we could say that ILA economies are more exhaustible-conservationist. The intuition behind this result is clear. Since in ILA economies agents live up to infinity, they are able to wait until later to consume. Agents thus consume less today, depleting fewer resources, and devoting a high percentage of human capital to the R&D sector.

Second, in the benchmark case, any increase in the R&D productivity parameter does not affect the stationary depletion rate in the ILA economy. This is consistent with the analytical result shown in Corollary 1 in Appendix 2, since the benchmark economy is assuming logarithmic preferences, and this implies that the depletion rate depends solely on the subjective discount rate. For the OLG economy we observe that any increase in the R&D productivity parameter reduces the use of non-renewable resources. This means that the two channels, through which the R&D productivity parameter affects the economic growth, move in the same direction, implying an increase in the

Table 2: Effects of Implementing Active R&D Policies

	(%)					
	τ		H_A		$\gamma - 1$	
$\sigma's$ % change	OLG	ILA	OLG	ILA	OLG	ILA
0%	6.37	1.98	6.86	26.34	2.00	2.00
30%	6.22	1.98	8.43	29.54	3.10	3.03
50%	6.16	1.98	9.12	30.96	3.70	3.71
100%	6.05	1.98	10.25	33.27	4.92	5.43
200%	5.95	1.98	11.26	35.58	6.42	8.87

Table 3: Results of a 100% increase in the R&D productivity parameter for different values of the IES

	Changes in % points					
	τ		H_A		$\gamma - 1$	
IES parameter	OLG	ILA	OLG	ILA	OLG	ILA
$1/\epsilon = 0.7$ (complementaries)	0.35	1.15	1.01	5.46	2.25	3.08
$1/\epsilon = 1$	-0.32	0.00	3.39	6.93	2.92	3.43
$1/\epsilon = 2$ (substitutes)	-1.27	-1.00	9.63	9.14	3.62	3.35

growth rate (as we have analytically proved in Proposition 2). And third, although the growth rate increases in both scenarios, the increase in the ILA economy is higher than in the OLG economy. For instance, when the R&D parameter grows 100%, the growth rate increases 3.43 percentage points in the ILA economy while in the OLG set up it only increases 2.9 points.

We saw in Corollary 4 in Appendix 2 that the value of the IES parameter substantially affects the endogenous depletion rate. As a result, we carried out a sensitivity analysis of this parameter. Table 3 quantifies the changes, in percentage points, of the variables when the R&D parameter grows 100% for three different values of the IES coefficient, He corregido epsilon por 1/epsilon $1/\epsilon = 0.7$, $1/\epsilon = 1$ and $1/\epsilon = 2$. For instance the 3.08 in the first row of the last column means that for the case in which $1/\epsilon = 0.7$, if the R&D parameter grows 100%, the growth rate of the ILA economy increases by 3.08 percentage points.

Note that increases in the R&D parameter affect the use of the resources in equilibrium differently depending on the IES parameter. We already know

from Corollary 2 in Appendix 2 that, in an ILA economy, if the IES coefficient belongs to the interval $\left[1, \frac{\alpha_1+2\alpha_3}{\alpha_3} = 9\right]$ then the depletion rate decreases when the R&D productivity parameter increases, and if the IES parameter is lower than 1, the use of non-renewable resources increases. We can see numerically that this is also true for the OLG economy if the IES parameter belongs to the above interval. In general, it is clear that for low (high) values of the IES coefficient, active R&D policies lead to increase (decrease) in the use of exhaustible resources. The intuition is clear. The larger the intertemporal elasticity substitution, the better substitutes current and future consumption are; in consequence, agents are willing to wait longer to consume in the future and, therefore, do not need to deplete exhaustible resources so much today.

En esta version esta cambiado complementarios-subtituros respecto a la version del REEWe also can see that in all the simulated cases, R&D policy increases the growth rate of the economy. This result corroborates Scholz and Ziemens (1999) and Groth (2006)'s findings for the ILA economy. The numerical simulations also indicate that when current and future consumption are complementaries (substitutes), i.e. when the IES coefficient is lower (greater) than one, then the active R&D policy increases the growth rate in the ILA economy more (less) than in the OLG economy.

4 Conclusions

The aim of this paper was to analyze how an active R&D policy might affect the growth rate of an economy with endogenous growth and non-renewable resources. In particular, we compared the results of an increase in the R&D productivity parameter, in infinitely lived economies and in economies with finite lifetime agents.

From Scholz and Ziemens (1999) and Groth (2006) we know that in infinitely lived agents economies, any active R&D policy increases the growth rate of the economy. In order to see if this result also appears in economies with finite lifetime agents, we developed an endogenous growth overlapping generations (OLG) economy à la Diamond which uses non-renewable resources as essential inputs in the final-good production. Following Romer (1990), we considered there was an R&D sector which produces new designs to create new intermediate goods which are essential for the production of the final good. In particular, this innovation process creates new capital goods which do not substitute the exhaustible resources

because all of them are essential in production.

From the theoretical point of view, we found that any change in the R&D productivity parameter affects the growth rate of the economy through two channels. First, the direct channel, which shows that the greater the R&D productivity parameter, the higher the growth rate of the stock of knowledge, regardless of the stationary depletion rate, τ. This ceteris paribus result is quite intuitive, since this is the standard result in Romer's model, with non-renewable resources. That is, the higher the R&D productivity parameter, the higher the endogenous amount of human capital allocated to the R&D sector, which implies higher growth in the stock of knowledge and higher growth in the economy, taking τ as given.

Second, the indirect channel which is opened through the use of non-renewable resources in the final output sector. We know that any variation in the use of non-renewable resources will affect endogenous growth in two ways. On the one hand, as in an exogenous growth model, i.e. without taking into account its effect on the endogenous amount of human capital allocated to the R&D sector, and therefore without taking into account its effect on the endogenous growth of the stock of knowledge. On the other hand, taking into account its effect on the endogenous amount of human capital allocated to the R&D sector and, in consequence, on the growth of the stock of knowledge. We show that both effects are unambiguously negative, that is the lower the depletion rate, the higher the growth rate of stock of knowledge and therefore the growth rate of the economy. However, we have not been able to obtain, algebraically, an unambiguous relationship between the R&D productivity parameter and the stationary depletion rate for the OLG economy. It is clear that if the relationship between the R&D productivity parameter and the use of exhaustible resources is negative, the direct and indirect channels work in the same direction and any stimulant R&D policy increases the growth rate of the stock of knowledge. However when an increase in the technological parameter leads to a rise in the use of resources, then the indirect effect works in the opposite direction to the direct effect, and the final result over the growth rate of the economy is ambiguous.

Finally, since it is not possible to characterize analytically the balanced growth path of an OLG economy and compare it with an ILA economy, we worked on a numerical simulation. First of all, we chose the parameters such that the benchmark case for those economies is the same, and mimics some empirical facts of the economy. Secondly, we compared the results under both scenarios when the R&D parameter increases. Our main numerical

findings are as follows: First, OLG and ILA economies are similar in terms of growth rates; however they are very different in the composition of the growth process. Whereas in the ILA scenario, economic growth relies more on a lower use of non-renewable resources, in the OLG economy the growth process depends on higher growth in the R&D sector. In this sense we could say that ILA economies are more exhaustible-conservationist. Second, in both OLG and ILA economies where agents are more willing to wait longer to consume (i.e. with a large IES coefficient), active R&D policies are more conservationist, depleting the exhaustible resources less. And third, active R&D policies always increase the endogenous growth rate, in both scenarios. Furthermore, when current and future consumption are complementaries (substitutes), i.e. when the IES coefficient is lower (greater) than one, active R&D policies affect the growth rate more (less) in economies in which agents live infinitely than those in which agents have finite lifetimes.En esta version esta cambiado complementarios-subtituros respecto a la version del REE

Appendix 1

Proof of Proposition 1:

A market equilibrium for the OLG economy is an infinite sequence of quantity allocations $\{s_{t+1}, K_{t+1}, Y_t, X_t, c_{1,t}, c_{2,t+1}, \tau_{t+1}, E_t, M_{t+1}, H_{Y,t}, H_{A,t}, A_{t+1}\}_{t=0}^{\infty}$ and prices $\left\{P_t^A, w_{H,t}, r_t, p_t, q_t\right\}_{t=0}^{\infty}$ that solves the non-linear system, (1)-(18).

Proof of $\gamma_M = (1-\tau)\,, \forall\ H_A \geqslant 0$:

Straightforward from valuation of resource market clearing equation (15) on the balanced growth path.

Proof of $\gamma_E = \gamma_M, \forall\ H_A \geqslant 0$:

Straightforward from evaluating the depletion rate definition, equation (14), on the balanced growth path.

Proof of $\gamma_X = (1-\tau)^{\frac{\alpha_3}{1-\alpha_2}}, \forall\ H_A \geqslant 0$:

Combining equations (4), (7), (8) and (10) we obtain

$$\frac{\alpha_3 H_{Yt+1}^{\alpha_1}\left(A_{t+1}X_{t+1}^{\alpha_2}\right)E_{t+1}^{(\alpha_3-1)}}{\alpha_3 H_{Yt}^{\alpha_1}\left(A_t X_t^{\alpha_2}\right)E_t^{(\alpha_3-1)}} = 1 + \frac{\alpha_2}{\eta}\alpha_2 H_{Yt+1}^{\alpha_1} X_{t+1}^{(\alpha_2-1)} E_{t+1}^{\alpha_3}.$$

Evaluating this expression on the balanced growth path and reordering,

$$\frac{\eta}{\alpha_2^2} H_Y^{-\alpha_1}\left[\gamma_A\left(\gamma_X\right)^{\alpha_2}\left(\gamma_E\right)^{(\alpha_3-1)} - 1\right] = X_{t+1}^{(\alpha_2-1)} E_{t+1}^{\alpha_3}. \tag{A}$$

Taking the ratio of this expression in period $t+1$ and t, we have that $1 = \left(\gamma_X\right)^{(\alpha_2-1)}\left(\gamma_E\right)^{\alpha_3}$. Considering that $\gamma_E = (1-\tau)$, we obtain $\gamma_X = (1-\tau)^{\frac{\alpha_3}{1-\alpha_2}}$.

Proof of $\gamma_r = \gamma_q = 1, \forall\ H_A \geqslant 0$:

Taking the ratio of equation (10) en $t+1$ and t and evaluating on the balanced growth path we obtain that $\gamma_r = \gamma_q$. Doing the same with equation (7) we obtain

$$\gamma_q = \left(\gamma_X\right)^{(\alpha_2-1)}\left(\gamma_E\right)^{\alpha_3},$$

which, considering that $\gamma_E = (1-\tau)$ and $\gamma_X = (1-\tau)^{\frac{\alpha_3}{1-\alpha_2}}$, becomes $\gamma_q = 1$.

Proof of $\gamma_\pi = \gamma_X, \forall\ H_A \geqslant 0$:

Taking the ratio of monopoly profit equation in $t+1$ and t and evaluating on the balanced growth path we obtain that $\gamma_\pi = \gamma_q \gamma_X$. Since in the balanced growth path $\gamma_q = 1$ then $\gamma_\pi = \gamma_X$.

Proof of $\gamma_Y = \gamma_{w_H} = \gamma, \forall\ H_A \geqslant 0$:

Taking the ratio of equations (5) and (6) in $t+1$ and t and evaluating on the balanced growth path we obtain that $\gamma_Y = \gamma_{w_H} = \gamma_A (\gamma_X)^{\alpha_2} (\gamma_E)^{\alpha 3}$. Considering that $\gamma_E = (1-\tau)$, and $\gamma_X = (1-\tau)^{\frac{\alpha_3}{1-\alpha_2}}$, we obtain $\gamma_Y = \gamma_{w_H} = \gamma_A (1-\tau)^{\frac{\alpha_3}{1-\alpha_2}} \equiv \gamma$.

Proof of $\gamma_p \equiv \gamma / (1-\tau), \forall\ H_A \geqslant 0$:

Taking the ratio of equation (8) in $t+1$ and t and evaluating on the balanced growth path we obtain that $\gamma_p = \gamma_A (\gamma_X)^{\alpha_2} (\gamma_E)^{(\alpha 3 - 1)}$. Considering that $\gamma_E = (1-\tau)$, and $\gamma_X = (1-\tau)^{\frac{\alpha_3}{1-\alpha_2}}$, we obtain $\gamma_p = \gamma_A (1-\tau)^{\frac{\alpha_3}{1-\alpha_2} - 1} \equiv \gamma / (1-\tau)$.

Proof of $\gamma_K = \gamma, \forall\ H_A \geqslant 0$:

Taking the ratio of the physical capital clearing condition equation (17) in $t+1$ and t and evaluating on the balanced growth path we obtain that $\gamma_K = \gamma_A \gamma_X$. Considering that $\gamma_X = (1-\tau)^{\frac{\alpha_3}{1-\alpha_2}}$, we obtain $\gamma_K = \gamma_A (1-\tau)^{\frac{\alpha_3}{1-\alpha_2}} \equiv \gamma$.

Proof of $\gamma_{c_1} = \gamma_{c_2}, \forall\ H_A \geqslant 0$:

Taking the ratio of the equation (3) in $t+1$ and t and evaluating on the balanced growth path we obtain that

$$\frac{\gamma_{c_2}}{\gamma_{c_1}} = \left(\frac{1 + r_{t+1}}{1 + r_t} \right)^{1/\epsilon}.$$

Considering that on the balanced growth path $\gamma_r = 1$, we have $r_{t+1} = r_t = r$. Therefore $\gamma_{c_1} = \gamma_{c_2}$.

Proof of $\gamma_{c_1} = \gamma, \forall\ H_A \geqslant 0$:

Substituting the saving function in the first consumer restriction (1), we obtain

$$c_{1t} = (w_{H,t} h) \left(1 - \frac{1}{1 + (1+\theta)^{\frac{1}{\epsilon}} (1 + r_{t+1})^{-\left(\frac{1-\epsilon}{\epsilon}\right)}} \right).$$

Taking the ratio of this expression in $t+1$ and t and evaluating on the balanced growth path we obtain that $\gamma_{c_1} = \gamma_A (\gamma_X)^{\alpha_2} (\gamma_E)^{\alpha 3}$. Considering that $\gamma_E = (1-\tau)$ and $\gamma_X = (1-\tau)^{\frac{\alpha_3}{1-\alpha_2}}$, we obtain $\gamma_{c_1} = \gamma_A (1-\tau)^{\frac{\alpha_3}{1-\alpha_2}} \equiv \gamma$.

Proof of $\gamma_s = \gamma, \forall\ H_A \geqslant 0$:

Substituting (6) and (8) in the saving function and evaluating on the balanced growth path we obtain

$$s_{t+1} = H_Y^{\alpha_1} A_t X_t^{\alpha_2} E_t^{\alpha_3} \left[\frac{\alpha_1 H}{\left(1 + (1+\theta)^{\frac{1}{\epsilon}} (1+r)^{-\left(\frac{1-\epsilon}{\epsilon}\right)}\right) H_Y} - \frac{\alpha_3 \gamma_E}{\tau} \right]. \quad \text{(B)}$$

Taking the ratio of this expression in periods $t+1$ and t, we obtain that $\gamma_s = \gamma_A (\gamma_X)^{\alpha_2} (\gamma_E)^{\alpha 3}$. Since $\gamma_E = (1-\tau)$, and $\gamma_X = (1-\tau)^{\frac{\alpha_3}{1-\alpha_2}}$, we obtain $\gamma_s = \gamma_A (1-\tau)^{\frac{\alpha_3}{1-\alpha_2}} \equiv \gamma$.

Proof of $\gamma_{P^A} = \gamma_X, \forall\ H_A > 0$:

Equalizing human capital wage in equations (13) and (6), we obtain the following expression

$$P_t^A \sigma = \alpha_1 H_{Yt}^{\alpha_1 - 1} X_t^{\alpha_2} E_t^{\alpha_3}.$$

Taking the ratio of this expression in $t+1$ and t, and evaluating on the balanced growth path, we arrive at $\gamma_{P^A} = (\gamma_X)^{\alpha_2} (\gamma_E)^{\alpha_3}$. Considering that $\gamma_E = (1-\tau)$ and $\gamma_X = (1-\tau)^{\frac{\alpha_3}{1-\alpha_2}}$, we obtain $\gamma_{P^A} = (1-\tau)^{\frac{\alpha_3}{1-\alpha_2}}$.

Proof of $H_y = \frac{\alpha_1(\gamma_A - \gamma_M)}{\gamma_M \sigma (1-\alpha_2)\alpha_2}, \forall\ H_A > 0$:

First, by solving the patent price difference equation (9) we obtain the following expression

$$P_t^A = \sum_{i=0}^{\infty} \pi_{t+1+i} \prod_{\omega=0}^{t+1+i} \left(\frac{1}{1 + r_{t+1+i}} \right)^{i+1}.$$

On the balanced growth path, $\pi_{t+1+i} = \gamma_\pi^{i+1} \pi_t$ and $r_{t+1+i} = r^{i+1} r_t = r_t$. Therefore, the patent price on the balanced growth path can be written

$$P_t^A = \sum_{i=0}^{\infty} \gamma_\pi^{i+1} \pi_t \frac{1}{(1+r_t)^{i+1}} = \frac{\gamma_\pi \pi_t}{1+r_t} \sum_{i=0}^{\infty} \left(\frac{\gamma_\pi}{1+r_t} \right)^i = \frac{\pi_{t+1}}{1 + r_t - \gamma_\pi}.$$

Substituting intermediate sector profits and intermediate good prices (7) and considering the final good production function (5),

$$P_t^A = \frac{(1-\alpha_2)\,\alpha_2 q_{t+1} X_{t+1}}{1+r_t-\gamma_\pi} = \frac{(1-\alpha_2)\,\alpha_2 \frac{Y_t}{A_t}\frac{\gamma_Y}{\gamma_A}}{1+r_t-\gamma_\pi}. \tag{C1}$$

On the other hand, substituting conditions (6) and (5) in (13) we obtain the following expression for the patent price,

$$P_t^A = \frac{w_{H,t}}{\sigma A_t} = \frac{\alpha_1 \frac{Y_t}{A_t}}{\sigma H_{Y,t}}. \tag{C2}$$

Taking into account $\gamma_\pi = (1-\tau)^{\frac{\alpha_3}{1-\alpha_2}} = \frac{\gamma_Y}{\gamma_A}$ and $1+r_t = \frac{\gamma_Y}{\gamma_M}$, equalizing patent prices in (C1) and (C2) and reordering we obtain

$$H_Y = \frac{\alpha_1(\frac{\gamma_A}{\gamma_M}-1)}{\sigma(1-\alpha_2)\alpha_2}.$$

Proof of $\gamma_A = \frac{\gamma_M(1+\sigma h)(1-\alpha_2)\alpha_2+\gamma_M\alpha_1}{\gamma_M(1-\alpha_2)\alpha_2+\alpha_1}$, $\forall\ H_A > 0$:

Evaluating the knowledge dynamics equation (12) on the balanced growth path and considering the human capital clearing condition (16), we obtain the following condition

$$\gamma_A = 1+\sigma\left(H-H_Y\right).$$

Substituting the value that human capital attributes to the final goods sector, H_Y, and reordering, we obtain the value

$$\gamma_A = \frac{(1+\sigma H)\,(1-\alpha_2)\,\alpha_2+\alpha_1}{(1-\alpha_2)\,\alpha_2+\alpha_1\,(\gamma_M)^{-1}}.$$

Proof of $\frac{\alpha_2^2\gamma}{\gamma-\gamma_M} = \frac{(1-\alpha_2)\alpha_2}{(\gamma_A-\gamma_M)}\left(\frac{\sigma H}{1+(1+\theta)^{\frac{1}{\epsilon}}\left(\frac{\gamma}{\gamma_M}\right)^{-\left(\frac{1-\epsilon}{\epsilon}\right)}}-\gamma_A\right)-\frac{\alpha_3}{\tau}$, $\forall\ H_A > 0$:

Reordering terms in equation (A) :

$$\frac{\gamma_A\,(\gamma_X)^{\alpha_2}\,(\gamma_E)^{(\alpha_3-1)}-1}{\alpha_2^2} = \frac{H_Y^{\alpha_1}A_{t+1}X_{t+1}^{\alpha_2}E_{t+1}^{\alpha_3}}{\eta A_{t+1}X_{t+1}}.$$

And taking into account equation (17) and $\gamma_A (\gamma_X)^{\alpha_2} (\gamma_E)^{(\alpha_3-1)} = \frac{\gamma_Y}{\gamma_M} = \gamma_p$:

$$\frac{Y_{t+1}}{K_{t+1}} = \frac{\gamma_Y - \gamma_M}{\alpha_2^2 \gamma_M}. \tag{D1}$$

On the other hand, substituting (B), (C2), H_Y and γ_A with $H_A > 0$ into equation (18), we obtain the following expression:

$$\frac{Y_t}{K_t} = \gamma_K \left(\frac{\alpha_1 H}{\left(1 + (1+\theta)^{\frac{1}{\epsilon}} (1+r)^{-\left(\frac{1-\epsilon}{\epsilon}\right)}\right) H_Y} - \frac{\alpha_3 \gamma_M}{\tau} - \frac{\alpha_1 \gamma_A}{\sigma H_Y} \right)^{-1}. \tag{D2}$$

On the balanced growth path, equation (D1) must be equal to equation (D2):

$$\frac{\gamma_Y - \gamma_M}{\alpha_2^2 \gamma_M} = \gamma_K \left(\frac{\alpha_1 H}{\left(1 + (1+\theta)^{\frac{1}{\epsilon}} (1+r)^{-\left(\frac{1-\epsilon}{\epsilon}\right)}\right) H_Y} - \frac{\alpha_3 \gamma_M}{\tau} - \frac{\alpha_1 \gamma_A}{\sigma H_Y} \right)^{-1}.$$

Taking into account that $\gamma_K = \gamma_Y = \gamma, 1 + r = \frac{\gamma}{\gamma_M}$ and substituting H_y:

$$\frac{\alpha_2^2 \gamma}{\gamma - \gamma_M} = \frac{(1-\alpha_2)\alpha_2}{(\gamma_A - \gamma_M)} \left(\frac{\sigma H}{1 + (1+\theta)^{\frac{1}{\epsilon}} \left(\frac{\gamma}{\gamma_M}\right)^{-\left(\frac{1-\epsilon}{\epsilon}\right)}} - \gamma_A \right) - \frac{\alpha_3}{\tau}.$$

Proof of $H_y = H$, if $H_A = 0$:

Straightforward from evaluation of the human capital clearing condition (16) with $H_A = 0$.

Proof of $\gamma_A = 1$, if $H_A = 0$:

Evaluating the knowledge dynamics equation (12) on the balanced growth path, we obtain $\gamma_A = 1$.

Proof of $\gamma_{P^A} = \gamma_Y$, if $H_A = 0$:

On the one hand, from equations (5), (6) and (17), we obtain $\pi_{t+1} = (1-\alpha_2)\alpha_2 \frac{Y_{t+1}}{A_{t+1}}$. On the other hand, from (5), (7) (10) and (17), we obtain the expression $r_{t+1} = \alpha_2 \frac{Y_{t+1}}{K_{t+1}}$. Substituting both expressions in the equilibrium price for the patent (9), after some manipulations we obtain

$$\gamma_{P^A} = 1 + \alpha_2^2 \left(\frac{Y_{t+1}}{K_{t+1}}\right) - \frac{(1-\alpha_2)\alpha_2}{A} \left(\frac{Y_{t+1}}{P_t^A}\right). \tag{E}$$

Since$\frac{Y_{t+1}}{K_{t+1}}$ is a constant by (D1), then on the balanced growth path $\frac{Y_{t+1}}{P_t^A}$ must be a constant. Therefore, $\gamma_{P^A} = \gamma_Y$. Since $H_A = 0$, $\gamma_A = 1$, we obtain $\gamma_{P^A} = (1-\tau)^{\frac{\alpha_3}{1-\alpha_2}}$.

Proof of $\frac{\gamma_M \alpha_2^2 \gamma}{\gamma - \gamma_M} = \frac{\alpha_1}{\left(1+(1+\theta)^{\frac{1}{\epsilon}}\left(\frac{\gamma}{\gamma_M}\right)^{-\left(\frac{1-\epsilon}{\epsilon}\right)}\right)} - \frac{\gamma_M(\alpha_3+(1-\alpha_2)\alpha_2)}{\tau}$, if $H_A = 0$:

Substituting (1) - (6) and (8) in the final good market clearing condition (18) we obtain

$$K_{t+1} = \frac{\alpha_1 Y_t}{\left(1+(1+\theta)^{\frac{1}{\epsilon}}(1+r_{t+1})^{-\left(\frac{1-\epsilon}{\epsilon}\right)}\right)} - \frac{\alpha_3 \gamma_M Y_t}{\tau} - P_t^A A.$$

Reordering and taking into account (D1), this expression can be rewritten as

$$\gamma_Y \left(\frac{\alpha_2^2 \gamma_M}{\gamma_Y - \gamma_M}\right) = \frac{\alpha_1}{\left(1+(1+\theta)^{\frac{1}{\epsilon}}(1+r_{t+1})^{-\left(\frac{1-\epsilon}{\epsilon}\right)}\right)} - \frac{\alpha_3 \gamma_M}{\tau} - \frac{P_t^A A}{Y_t}.$$

Given that $\gamma_{P^A} = (1-\tau)^{\frac{\alpha_3}{1-\alpha_2}} = \gamma_Y$ when $H_A = 0$ from expression (E), after some manipulations we obtain

$$\frac{P_t^A A}{Y_t} = \frac{(1-\alpha_2)\,\alpha_2 \gamma_M}{\tau}.$$

Substituting this in the former equation taking into account that $1+r = \frac{\gamma}{\gamma_M}$ and reordering

$$\frac{\gamma_M \alpha_2^2 \gamma}{\gamma - \gamma_M} = \frac{\alpha_1}{\left(1+(1+\theta)^{\frac{1}{\epsilon}}\left(\frac{\gamma}{\gamma_M}\right)^{-\left(\frac{1-\epsilon}{\epsilon}\right)}\right)} - \frac{\gamma_M\left(\alpha_3 + (1-\alpha_2)\,\alpha_2\right)}{\tau}.$$

Appendix 2

Infinitely-lived agents model

Scholz and Ziemens (1999) analyze an economy similar to the one developed in Section 2, but with infinitely-lived agents (ILA) and in a continuous set up. In this appendix, we solve their model in a discrete time, in order to compare the results in both frameworks, OLG and ILA.

Since the only difference between the equilibrium characterization of the ILA model and the OLG model is the consumers's life-span, we only illustrate the consumers' problem and the characterization of the balanced growth path of the ILA set up.

Consumers own the stock of exhaustible resources, designs and physical capital. So the problem of an infinitely lived representative agent consumer, in per worker terms, can be written as[17],

$$\begin{aligned} &\underset{\{c_t,k_{t+1},m_{t+1},s_{t+1}\}_{t=0}^{\infty}}{Max} \quad \sum_{t=0}^{\infty}\left(\frac{1}{1+\theta}\right)^t \frac{c_{1,t}^{1-\epsilon}-1}{1-\epsilon}, \\ &s.t.\begin{cases} c_t + s_{t+1} + p_t m_{t+1} = w_{H_t} h + (1+r_t)\, s_t + p_t m_t, \\ m_0 > 0 \text{ given.} \end{cases} \end{aligned} \tag{19}$$

The FOC of this maximization problem can be written as

$$\frac{c_{t+1}}{c_t} = \left(\frac{1+r_{t+1}}{1+\theta}\right)^{1/\epsilon}, \tag{20}$$

$$\frac{p_{t+1}}{p_t} = 1 + r_{t+1}. \tag{21}$$

Note that these first order conditions are the same as those obtained in OLG model (equations 3 and 4, respectively). The first equation indicates that consumers equate the marginal rate of substitution between consumption today and consumption tomorrow to their relative prices $1+r_{t+1}$. The second equation states that the marginal rate of saving in exhaustible resources must be equal to the marginal rate of saving in physical capital or bonds issued by the intermediate firms.

The equilibrium characterization for the ILA model is summarized in the following definition.

[17] Alternatively, we could solve the individual problem, denoting the total saving allocated to buy physical capital and bonds issued by the intermediate firms as $s_{t+1} = k_{t+1} + P_t^A a_{t+1}$.

Definition 3 *For any arbitrary initial value of* τ_0*, an equilibrium of this ILA economy is an infinite sequence of quantity allocations* $\{K_{t+1}, Y_t, X_t, s_{t+1}, c_t, \tau_t, E_t, M_{t+1}, H_{Y,t}, H_{A,t}, A_{t+1}, \pi_t\}_{t=0}^{\infty}$ *and prices* $\left\{P_t^A, w_{H,t}, r_t, p_t, q_t\right\}_{t=0}^{\infty}$ *such that consumers, final-goods producers and research firms maximize their objective functions taking prices as given, the intermediate firms maximize their monopolist profits and all markets clear, given the initial conditions* $K_0, M_0, A_0 > 0$*. In other words, an equilibrium is a solution of the non-linear system (5) -(21) and the transversality condition*[18]*.*

Balanced Growth Path

The balanced growth path is defined as in the OLG framework (definition 2). From now on, the superscript *ILA* stands for solutions of the infinitely-lived representative agent's economy.

Proposition 3 *Any balanced growth path of the ILA economy is given by a vector* $\left\{\gamma_Y^{ILA}, \gamma_K^{ILA}, \gamma_A^{ILA}, \gamma_M^{ILA}, \gamma_c^{ILA}, \gamma_X^{ILA}, \gamma_E^{ILA}, \gamma_p^{ILA}, \gamma_q^{ILA}, \gamma_r^{ILA}, \gamma_{P^A}^{ILA}, \gamma_{w_H}^{ILA}, H_Y^{ILA}, H_A^{ILA}, \tau^{ILA}\right\}$ *satisfying the following system,*

$$\left.\begin{array}{c} \gamma_A^{ILA} = \frac{[(1+\sigma H)(1-\alpha_2)\alpha_2+\alpha_1]\left(1-\tau^{ILA}\right)}{(1-\tau^{ILA})(1-\alpha_2)\alpha_2+\alpha_1}, \\ H_Y^{ILA} = \frac{\alpha_1\left[\gamma_A^{ILA}-(1-\tau)\right]}{(1-\tau^{ILA})\sigma(1-\alpha_2)\alpha_2}, \end{array}\right\}, \quad \text{if } H_A^{ILA} = H - H_Y^{ILA} > 0,$$

$$\left.\begin{array}{c} \gamma_A^{ILA} = 1, \\ H_Y^{ILA} = H. \end{array}\right\}, \quad \text{if } H_A = 0,$$

$$\begin{aligned} \tau^{ILA} &= 1 - \frac{1}{\left(\gamma^{ILA}\right)^{\epsilon-1}(1+\theta)}, \\ \gamma_M^{ILA} &= \gamma_E^{ILA} = 1 - \tau^{ILA}, \\ \gamma_X^{ILA} &= \gamma_{P^A}^{ILA} = \gamma_\pi^{ILA} = \left(1-\tau^{ILA}\right)^{\frac{\alpha_3}{1-\alpha_2}}, \\ \gamma_r^{ILA} &= \gamma_q^{ILA} = 1, \\ \gamma_p^{ILA} &= \frac{\gamma^{ILA}}{1-\tau^{ILA}}, \\ \gamma_K^{ILA} &= \gamma_Y^{ILA} = \gamma_c^{ILA} = \gamma_{w_H}^{ILA} = \gamma^{ILA}, \end{aligned}$$

[18] In this model the transversality condition implies that the following condition must be satisfied: $\gamma^{(1-\varepsilon)} < (1+\theta)$, which implies that $(1-\tau) < 1$. This condition is analogous to the condition provided by Groth (2006) when solving a model similar to ours in continuous time.

where $\gamma^{ILA} = \gamma_A^{ILA}\left(1-\tau^{ILA}\right)^{\frac{\alpha_3}{1-\alpha_2}}$.

Proof of Proposition 3

Following proof of Proposition 1, it is clear that in this context the balanced growth path is such that

$$\begin{aligned}
\gamma^{ILA} &= \gamma_k^{ILA} = \gamma_y^{ILA} = \gamma_{w_H}^{ILA} = \gamma_A^{ILA}\left(1-\tau^{ILA}\right)^{\frac{\alpha_3}{1-\alpha_2}}, \\
\gamma_M^{ILA} &= \gamma_E^{ILA} = 1-\tau^{ILA}, \\
\gamma_A^{ILA} &= \frac{\left(\left(1+\sigma H\right)\left(1-\alpha_2\right)\alpha_2+\alpha_1\right)\left(1-\tau^{ILA}\right)}{\left(1-\tau^{ILA}\right)\left(1-\alpha_2\right)\alpha_2+\alpha_1}, \text{ if } H_A^{ILA}>0, \\
H_A &= H - H_Y, \text{ if } H_A^{ILA}>0, \\
H_Y &= \frac{\alpha_1\left[\gamma_A^{ILA}-\gamma_M^{ILA}\right]}{\gamma_M^{ILA}\sigma(1-\alpha_2)\alpha_2}, \text{ if } H_A^{ILA}>0, \\
\gamma_A^{ILA} &= 1, \text{if } H_A^{ILA}=0, \\
H_Y &= H, \text{ if } H_A^{ILA}=0, \\
\gamma_r^{ILA} &= \gamma_q^{ILA} = 1, \\
\gamma_p^{ILA} &= \frac{\gamma^{ILA}}{\gamma_M^{ILA}}, \\
\gamma_X^{ILA} &= \gamma_{P^A}^{ILA} = \gamma_\pi^{ILA} = \left(1-\tau^{ILA}\right)^{\frac{\alpha_3}{1-\alpha_2}}.
\end{aligned}$$

Proof of $\gamma_c^{ILA} = \gamma_y^{ILA} = \gamma^{ILA}$:

Taking into account the restriction in the representative agent problem (19) and substituting, in per worker terms, the final good production function (5) and firms' optimization conditions (6), (7), (8) and (10), we obtain

$$c_t + s_{t+1} = \frac{\alpha_1 h}{h_{Y_t}} y_t + \left(1 + \frac{\alpha_2^2 y_t}{\eta A_t x_t}\right) s_t + \alpha_3 \frac{y_t}{e_t}\left(m_t - m_{t+1}\right).$$

Considering the depletion rate (14), market clearing conditions (15) and (17), this expression can be written

$$c_t + s_{t+1} - s_t = y_t\left(\frac{\alpha_1 h}{h_{Y_t}} + \alpha_2^2 \frac{s_t}{k_t} + \alpha_3\right).$$

Substituting the final-good market clearing condition (18) and the R&D optimization condition (13), the above expression can be rewritten as

$$c_t + k_{t+1} - k_t = y_{t-1}\left[\gamma_y^{ILA}\left(\frac{\alpha_1 h}{h_{Y_t}} + \alpha_2^2 + \alpha_3\right) + \frac{\alpha_1\gamma_A^{ILA}}{\sigma h_{Y_{t-1}}} + \frac{\alpha_2^2\alpha_1\gamma_A^{ILA}}{\sigma h_{Y_{t-1}}}\frac{y_t}{k_t}\right].$$

Taking the ratio of the above equation in t and $t-1$ and evaluating on the balanced growth path, we obtain that

$$\frac{\gamma_c^{ILA}c_{t-1} + \gamma_k^{ILA}(k_t - k_{t-1})}{c_{t-1} + k_t - k_{t-1}} = \frac{y_{t-1}}{y_{t-2}} = \gamma_y^{ILA}.$$

Since $\gamma_k^{ILA} = \gamma_y^{ILA}$, we have

$$\frac{\gamma_c^{ILA}}{\gamma_y^{ILA}}c_{t-1} + k_t - k_{t-1} = c_{t-1} + k_t - k_{t-1},$$

which implies that $\gamma_c^{ILA} = \gamma_y^{ILA} = \gamma^{ILA}$.

From valuation of conditions (20) and (21) on the balanced growth path, we obtain straightforwardly that $\gamma_c^{ILA} = \left(\frac{\gamma_p^{ILA}}{1+\theta}\right)^{1/\epsilon}$. Since we have proved that $\gamma_c^{ILA} = \gamma^{ILA}$ and $\gamma_p^{ILA} = \frac{\gamma}{1-\tau^{ILA}}$, it must be true that $1 - \tau^{ILA} = \frac{1}{(\gamma^{ILA})^{\epsilon-1}(1+\theta)}$ ■

Note that for the case of logarithmic consumer preferences ($\epsilon = 1$), the results coincide with Aghion and Howitt (1998), Barbier (1999) and Scholz and Ziemes (1999). In particular, the stationary depletion rate depends solely on the consumer discount rate.

Corollary 1 *With elasticity of intertemporal substitution equal to one, the stationary depletion rate for the ILA economy is given by* $\tau = \theta/(1+\theta)$.

Proof. Straightforward from the first equation on Proposition 3. ■

Corollary 2

$$\textit{If} \quad \begin{cases} \frac{\alpha_3}{\alpha_1+2\alpha_3} \leq \epsilon < 1 & \Longrightarrow \quad \partial\tau^{ILA}/\partial\sigma < 0, \\ \epsilon = 1 & \Longrightarrow \quad \partial\tau^{ILA}/\partial\sigma = 0, \\ 1 < \epsilon & \Longrightarrow \quad \partial\tau^{ILA}/\partial\sigma > 0. \end{cases}$$

Proof. Substituting γ^{ILA} in τ^{ILA} on Proposition 3 and after some manipulation, we can write the following expression,

$$\begin{aligned} & \left(1-\tau^{ILA}\right)^{\frac{(1-\alpha_2)\epsilon+\alpha_3(\epsilon-1)}{(1-\alpha_2)}}\left[\left(1-\tau^{ILA}\right)(1-\alpha_2)\,\alpha_2+\alpha_1\right]^{1-\epsilon} \\ = & \left[(1+\sigma H)(1-\alpha_2)\,\alpha_2+\alpha_1\right]^{1-\epsilon}(1+\theta)^{-1}. \end{aligned}$$

Differentiating with respect to τ^{ILA} and σ, we obtain

$$\frac{\partial \tau^{ILA}}{\partial \sigma}=\frac{N}{D},$$

where

$$\begin{aligned} N &= (1-\epsilon)\left[(1+\sigma H)(1-\alpha_2)\,\alpha_2+\alpha_1\right]^{-\epsilon} H(1-\alpha_2)\,\alpha_2(1+\theta)^{-1}, \\ D &= -\frac{(1-\alpha_2)\,\epsilon+\alpha_3(\epsilon-1)}{(1-\alpha_2)}\left(1-\tau^{ILA}\right)^{\frac{(\alpha_1+2\alpha_3)(\epsilon-1)}{(1-\alpha_2)}}\left[\left(1-\tau^{ILA}\right)(1-\alpha_2)\,\alpha_2+\alpha_1\right]^{1-\epsilon}+ \\ & (\epsilon-1)\left[\left(1-\tau^{ILA}\right)(1-\alpha_2)\,\alpha_2+\alpha_1\right]^{-\epsilon}\left(1-\tau^{ILA}\right)^{\frac{\alpha_3(1-2\epsilon)-\epsilon\alpha_1}{(1-\alpha_2)}}(1-\alpha_2)\,\alpha_2. \end{aligned}$$

After some mathematical work, D can be expressed as

$$\begin{aligned} D = & -\left(1-\tau^{ILA}\right)^{\frac{(\alpha_1+2\alpha_3)(\epsilon-1)}{(1-\alpha_2)}}\left[\left(1-\tau^{ILA}\right)(1-\alpha_2)\,\alpha_2+\alpha_1\right]^{1-\epsilon} \\ & \times\left\{\left(1-\tau^{ILA}\right)(1-\alpha_2)\,\alpha_2(\alpha_3\epsilon+\alpha_1)+\alpha_1\left[\epsilon(\alpha_1+2\alpha_3)-\alpha_3\right]\right\}. \end{aligned}$$

For the case in which $\frac{\alpha_3}{\alpha_1+2\alpha_3}\leq\epsilon<1$, $N>0$ and $D<0$. This implies that $\partial\tau^{ILA}/\partial\sigma<0$. When $\epsilon=1$, $N=0$ and $\partial\tau^{ILA}/\partial\sigma=0$. For $\epsilon>1$, $N<0$ and $D<0$. Therefore $\partial\tau^{ILA}/\partial\sigma>0$. ■

Proposition 4 *In an ILA economy,* $\partial\gamma^{ILA}/\partial\sigma>0$ *if* $\frac{\alpha_3}{\alpha_1+2\alpha_3}\leq\epsilon$.

Proof. Substituting τ^{ILA} in the γ^{ILA} expression on Proposition 3 in Appendix 2 and after some manipulation, we can write the following expression for γ^{ILA}

$$\begin{aligned} & \left(\gamma^{ILA}\right)^{1-(1-\epsilon)\frac{\alpha_1+2\alpha_3}{(1-\alpha_2)}}\left[\left(\gamma^{ILA}\right)^{1-\epsilon}(1-\alpha_2)\,\alpha_2+\alpha_1(1+\theta)\right] \\ = & \left[(1+\sigma H)(1-\alpha_2)\,\alpha_2+\alpha_1\right](1+\theta)^{-\frac{\alpha_3}{(1-\alpha_2)}}. \end{aligned}$$

Differentiating with respect to γ^{ILA} and σ, and after some work we obtain

$$\frac{\partial\gamma^{ILA}}{\partial\sigma}=\frac{H(1-\alpha_2)\,\alpha_2(1+\theta)^{-\frac{\alpha_3}{(1-\alpha_2)}}}{(\alpha_1+\epsilon\alpha_3)\,\alpha_2\left(\gamma^{ILA}\right)^{-\frac{(1-\epsilon)\alpha_3}{(1-\alpha_2)}}+\left[\epsilon(\alpha_1+2\alpha_3)-\alpha_3\right]\alpha_1(1+\theta)\left(\gamma^{ILA}\right)^{-(1-\epsilon)\frac{\alpha_1+2\alpha_3}{(1-\alpha_2)}}},$$

which is positive whenever $\frac{\alpha_3}{\alpha_1+2\alpha_3}\leq\epsilon$. ■

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