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University of the Basque Country
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José María Da Rocha,
María-José Gutiérrez,
Luis T. Antelo

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Jos-Mara Da-Rocha[a,1], Mara-Jos Gutirrez[b,*], Lus T. Antelo[c,2]

[a] *Universitat Autnoma de Barcelona and RGEA-Universidade de Vigo, Facultad CC. Econmicas, Campus Universitario Lagoas-Marcosende, C.P. 36200 Vigo, Spain*
[b] *FAEII and MacLab - University of the Basque Country, Avda. Lehendakari Aguirre, 83, 48015 Bilbao, Spain.*
[c] *Process Engineering Group. Instituto de Investigaciones Marinas (IIM) CSIC. C/ Eduardo Cabello, 6. 36208 Vigo, Spain.*

Abstract

Pulse fishing may be a global optimal strategy in multicohort fisheries. In this article we compare the pulse fishing solutions obtained by using global numerical methods with the analytical stationary optimal solution. This allows us to quantify the potential benefits associated with the use of periodic fishing in the Northern Stock of hake. Results show that: first, management plans based exclusively on traditional reference targets as F_{msy} may drive fishery economic results far from the optimal; second, global optimal solutions would imply, in a cyclical manner, the closure of the fishery for some periods and third, second best stationary policies with stable employment only reduce optimal present value of discounted profit in a 2%.

Keywords:
optimal fisheries, management optimization in age-structured models, pulse fishing

1. Introduction

Since Beverton and Holt (1957) target reference points have been one of the main tools used by fishery managers to make decisions about future catch options. Among the classical target reference points F_{msy} (the fishing mortality rate) stands out. If applied constantly it results in the maximum sustainable yield, MSY (Caddy and Mahon, 1995).

There are two major shortcomings in this classical reference point approach. First, F_{msy} is time independent and fishery management strategies are evaluated using time dependent indicators, usually the net present values of profits. Second, F_{msy} is a stationary concept and optimal harvesting in a multi-cohort model may take the form of periodic fishing (Clark, 1976; Spulber, 1983; McCallum, 1988; Tahvonen, 2009).

In order to overcome these drawbacks, numerical methods have been implemented to find the optimal fishing mortality trajectory that maximises the net present profits of the fishery using the Beverton-Holt multi-cohort models (Hannesson, 1975; Horwood, 1987; Björndal *et al.*, 2004a, 2004b and 2006). In all these cases, the optimal solution of the problem is non stationary but consists of pulse fishing, i.e. periodic cycles of fishing followed by fallow periods for stock to recover. This means that periodic fishing leads to higher profits than those implied by fishing at a constant stationary rate. It is worth mentioning that pulse fishing may be optimal in this context because it mitigates the consequences of imperfect gear selectivity.

*Corresponding author. Phone: 34-94-6013786, Fax: 34-94-6017123, e-mail: mariajose.gutierrez@ehu.es
[1] e-mail: jmrocha@uvigo.es
[2] e-mail: ltaboada@iim.csic.es

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Pulse fishing has some advantages in live product fisheries (Graham, 2001) and it has been applied for years, under spatial rotation, in location-specific areas with marine invertebrates and sedentary reef fishes (Caddy, 1993; Botsford *et al.*, 1993; Hart, 2003; Hart and Rago, 2006; Cinner *et al.*, 2006). Nevertheless, fishery agencies do not consider periodic fishing as a feasible management tool, mainly because if rotation is not possible its application may imply high financial and social costs. If rotation is not possible, periodic fishing means the cyclical closure of the fishery for some periods with no alternative use of the fleet. As a result, most fishery agencies consider that fishery management must be stationary.

Optimal stationary solutions have recently been assessed for different fisheries (Dichmont *et al.*, 2010; Grafton *et al.*, 2006, 2007, 2010; Kompas *et al.*, 2010; Kulmala *et al.*, 2008; Da Rocha *et al.*, 2010; Da Rocha and Gutiérrez, 2011). In a seminal paper Grafton *et al.* (2007) analyse the biomass associated with yield maximisation and discounted profit maximisation for Western and Central Pacific big eye tuna and yellowfin tuna, the Australian northern prawn fishery and the Australian orange roughly fishery. They find that stationary fishing mortality maximising net present profit is a win-win strategy compared to the usual reference point policy, achieving higher profits and safer biomass. As a result of this study, the Australian federal government started to manage 26 species based on profit maximisation as from the beginning of 2008 (Black, 2007).

However, we know that the Beverton-Holt multi-cohort models used to assess the stock is not globally concave, so it is possible that constrained stationary solutions may be locally but not globally optimal. If this is the case, what is the relative advantage of pulse fishing with respect to optimal stationary fishing mortalities in a Beverton-Holt multi-cohort model?

In this paper we quantify the potential benefits associated with the use of periodic fishing in the Northern Stock of hake. In particular, we compare the solutions obtained by using global numerical methods with the analytical stationary optimal solution. This allows us to measure the potential benefits of periodic fishing relative to the optimal institutional constrained solution. We find that when periodic fishing is compared with optimal stationary solutions rather than with time independent classical reference points (F_{msy}), the potential disadvantage of stationary fishing is much lower than shown in previous articles.

2. The model

Age-structured models are the common population structure used in Virtual Population Analysis for fish stock assessment (Lassen and Medley, 2000). The population structure is applied to a group of fish that has the same life cycle, similar growth rates and can be considered a single biological unit. This unit stock is broken down into cohorts, i.e. into groups of fish that have the same age and probably the same size and weight, and that will mature at the same time.

Assume that the fish stock is broken into A cohorts. That is in each period t there are $A-1$ initial old cohorts and a new cohort is born. Let z_t^a be the mortality rate that affects to the population of fish of the a^{th} age during the t^{th} period. This mortality rate can be decomposed into fishing mortality, F_t^a, and natural mortality (non-human predation, disease and old age), m^a,

$$z_t^a = F_t^a + m^a.$$

While the fishing mortality rate may vary from one periods and one age to another, natural mortality is constant across all periods. Moreover, it is assumed that the fishing mortality at each age is given by stationary selection patterns, p^a, i.e.

$$F_t^a = p^a F_t.$$

Assume that the fish population is continuous and the mortality rate acts on the fish stock continuously throughout the period. Then the size of a cohort varies according to

$$N_{t+1}^{a+1} = e^{-z_t^a} N_t^a, \tag{1}$$

where N_t^a is the number of fish of the a^{th} age at the beginning of the t^{th} period.

It is worth mentioning that by backwards substitution N_t^a, can be expressed as a function of the past mortality rates and initial recruitment,

$$N_t^a = e^{-z_{t-1}^{a-1}(F_{t-1})} N_{t-1}^{a-1} = e^{-z_{t-1}^{a-1}(F_{t-1})} e^{-z_{t-2}^{a-2}(F_{t-2})} N_{t-2}^{a-2} = ... = \Pi_{i=1}^{a-1} e^{-z_{t-i}^{a-i}(F_{t-i})} N_{t-(a-1)}^1.$$

Therefore we can express N_t^a as

$$N_t^a = \phi_t^a N_{t-(a-1)}^1, \qquad \text{for } a = 1, ...A, \tag{2}$$

where

$$\phi_t^a = \phi(F_{t-1}, F_{t-2}, ...F_{t-(a-1)}) = \begin{cases} 1 & \text{for } a = 1, \\ \Pi_{i=1}^{a-1} e^{-z_{t-i}^{a-i}(F_{t-i})} & \text{for } a = 2, ...A, \end{cases}$$

can be understood as the survival function that shows the probability of a recruit born in period $t-(a-1)$ reaching age $a > 1$ for a given fishing mortality path $\left\{F_{t-1}, F_{t-2}, ...F_{t-(a-1)}\right\}$.

The size of a new cohort (recruitment), N_{t+1}^1 , depends on the spawning stock biomass of the previous year, SSB_t,

$$N_{t+1}^1 = \Psi(SSB_t), \tag{3}$$

where Ψ denotes the stock/recruits (S-R) relationship. Moreover, the spawning stock biomass, SSB, is a function of the stock weight distribution, ω, and the maturity fraction, μ, of each age,

$$SSB_t = \sum_{a=1}^{A} \mu^a \omega^a N_t^a. \tag{4}$$

Let D_t^a and C_t^a denote the number of fish that dye from natural causes and from fishing (catches), respectively. Then the dynamics of the cohort can be expressed as

$$N_t^a - N_{t+1}^{a+1} = D_t^a + C_t^a.$$

Taking into account equation (1) and the definitions of natural and fishing mortality, D_t^a and C_t^a can be expressed as

$$\begin{aligned} D_t^a &= \frac{m^a}{z_t^a}\left(N_t^a - N_{t+1}^{a+1}\right) = \frac{m^a}{z_t^a}\left(1 - e^{-z_t^a}\right) N_t^a, \\ C_t^a &= \frac{F_t^a}{z_t^a}\left(N_t^a - N_{t+1}^{a+1}\right) = \frac{F_t^a}{z_t^a}\left(1 - e^{-z_t^a}\right) N_t^a. \end{aligned} \tag{5}$$

This last equation is known as the Baranov catch equation (Baranov, 1918).

3. Optimal Management

The profits of the fishery for any period t are given by the difference between revenues and fishing cost. That is

$$\pi_t = \sum_{a=1}^{A} pr^a C_t^a(F_t) - TC(F_t), \tag{6}$$

where pr^a is the selling price for a unit of fish of age a and TC represents the total fishing cost which depend on the fishing rate. For the shake of simplicity, prices are assumed to be constant over time and total cost to be a convex function. In the Discussion in Section 5 we discuss the implications of these assumptions based on the sensitivity analysis carried out.

Note that π_t can be interpreted in several ways from the economic point of view (Da Rocha *et al.*, 2011). For instance, if the cost is zero π_t represents the discounted revenues of the fishery. Alternatively, if the price is one and the cost is zero, π_t represents the discounted yield of the fishery.

Assume that the objective of the fishery manager is to find the fishing mortality that maximises the present value of the future profits of the fishery. Formally, the present value of future profits is given by $J = \sum_{t=0}^{\infty} \beta^t \pi_t$. The parameter $\beta \in [0, 1]$ is the discount factor which represents how much the manager is willing to pay to trade-off the value of fishing today against the benefits of increased profits in the future, measured by higher biomass and recruitment. Considering $\beta = 1$ implies that managers care about future changes as much as if they occurred in the current year. By contrast, considering $\beta = 0$ implies not caring about the future at all.

Therefore, the objective for the managers of the fishery should be to find the fishing rate trajectory that maximises the present value of the fishery, J, taking into account that the spawning stock biomass is always greater than the precautionary level, SSB_{pa}, and the dynamics described by equations (1) to (5). Formally, the maximisation problem consists of solving

$$\max_{\{F_t, N_{t+2}^1\}_{t=0}^{\infty}} \quad J = \sum_{t=0}^{\infty} \beta^t \left\{ \sum_{a=1}^{A} pr^a y^a(F_t) \phi_t^a N_{t+1-a}^1 - TC(F_t) \right\},$$

$$s.t. \begin{cases} N_{t+1}^1 = \Psi\left(\sum_{a=1}^{A} \mu^a \omega^a \phi_t^a N_{t+1-a}^1\right) \quad \forall t, \\ SSB_{pa} \leq \sum_{a=1}^{A} \mu^a \omega^a \phi_t^a N_{t+1-a}^1 \quad \forall t, \\ N_0^a \quad \text{given}, \forall a, \end{cases} \tag{7}$$

where $y^a(F_t) = \omega^a \frac{F_t}{p^a F_t + m} \left(1 - e^{-p^a F_t - m^a}\right)$.

In the appendix we show how to find the first order conditions that solve this problem. Formally, the optimal paths can be characterised by the following set of dynamic equations

$$\sum_{a=1}^{A} pr^a \frac{\partial y^a(F_t)}{\partial F_t} N_t^a - \frac{1}{N_t^a}\frac{\partial TC_t}{\partial F_t} = \sum_{a=1}^{A-1} p^a \left\{ \sum_{j=1}^{A-a} \beta^j \left[pr^a y^{a+j}(F_{t+j}) + (\Psi'_{t+j}\lambda_{t+j} + \theta_{t+j})\mu^{a+j}\omega^{a+j} \right] N_{t+j}^{a+j} \right\}, \tag{8}$$

$$\sum_{a=1}^{A} \beta^a pr^a y^a (F_{t+1+a}) \phi_{t+1+a}^a = \lambda_{t+1} - E_t \sum_{a=1}^{A} \beta^a (\Psi'_{t+1+a}\lambda_{t+1+a} + \theta_{t+1+a})\mu^a \omega^a \phi_{t+1+a}^a, \tag{9}$$

$$N_{t+1}^{a+1} = e^{-z^a(F_t)} N_t^a, \qquad \forall t \quad \forall a = 1, ...A-1, \tag{10}$$

$$N_{t+2}^1 = \Psi\left(\sum_{a=1}^{A} \mu^a \omega^a N_{t+1}^a \right), \qquad \forall t, \tag{11}$$

$$\theta_{t+1}\left[\sum_{a=1}^{A} \mu^a \omega^a N_{t+1}^a - SSB_{pa} \right] = 0, \qquad \forall t, \tag{12}$$

where λ_t and θ_t are the Lagrange multipliers associated with the first and second restrictions of the maximisation problem (7), respectively.

Condition (8) shows how the mortality rate, F_t, is selected. The insight is the following. In the optimal path, an increase in current mortality rate leads to an increase in current fishery profits (left-hand side) that is offset by a decrease in future profits derived from reductions in future stock (right-hand side). In particular, the left-hand side represents the effects of changes in fishing mortality on the current profit of the fishery. However, the right-hand side shows the effect on the future size of the living cohorts, $t+1$ to $t+A-1$ (first sum) and on the future stock recruitments from periods $t+2$ to $t+A$ (second sum). This can also be visualised by looking at age structure in Table 1. The left-hand side represents the effects of F_t on the structure of the fishery in period t (column t). The first sum on the right-hand side shows the effects of F_t on the structure of future size of the living cohorts (lower triangle matrix) and the second sum illustrates the effects of F_t on future stock recruitments (row $a = 1$). Note that β affects the future net present value (right side). Note that making $\beta = 1$ implies caring about future changes as much as if they occurred in the current year. By contrast, considering $\beta = 0$ implies not caring about the future at all.

[Insert Table 1]

Equation (9) indicates that the optimal path recognises that the effect of an increase in the stock recruitment, N_{t+2}^1, is two-fold. On the one hand, the abundance in periods $t+2$ to $t+2+A-1$ goes up, which leads to an increase in catches (left-hand side). On the other hand, the SSB for periods $t+3$ to $t+3+A-1$ also increases, (right-hand side).

Equations (10) and (11) show the dynamics of the population cohorts. Finally, equation (12) indicates whether SSB is below the precautionary level, SSB_{pa}. The Lagrange multiplier θ_t shows the effects on mortality when the precautionary principle is not binding. If at a period t, the SSB is below the precautionary level, SSB_{pa}, then θ_t indicates how much the fishing mortality rate should be modified between periods $t-A$ and $t-1$.

3.1. Optimal Stationary Solution

The optimal trajectories derived from maximization problem (7) are the optimal paths for $\{F_t\}_{t=1}^{\infty}$, $\{\lambda_t\}_{t=2}^{\infty}$, and $\{N_{t+2}^a\}_{t=1}^{\infty}$ which satisfy the infinity set of equations that characterises the first order conditions (8) to (12). In this section we focus on solutions that solve the manager maximisation problem leading the fishery to a stationary situation. Our strategy to find these optimal stationary trajectories follows two steps. First, we algebraically characterises the stationary solution, i.e. the solution that determines a unique value for the long-term fishing rate which if applied, will generate a stationary recruitment leading to the maximum long-term profits of the fishery. Second, the optimal path of fishing mortality that drives the fishery from the initial conditions to the stationary solution is found using numerical methods.

Assume that the precautionary restriction is not binding, $\theta_t = 0$. In this context, a stationary solution is defined as an optimal solution characterised by a vector $(F_{ss}, N_{ss}^1, N_{ss}^2, ...N_{ss}^A, \lambda_{ss})$ such that for any future period t

$$\begin{aligned} F_{ss} &= F_t = F_{t+1}, \\ N_{ss}^a &= N_t^a = N_{t+1}^a, \qquad \forall a = 1, .., A, \\ \lambda_{ss} &= \lambda_t = \beta^j \lambda_{t+j}, \qquad \forall j = 1, .., A+1. \end{aligned}$$

The first order conditions (8)-(11) valued at the stationary solution can be reduced to the following 3−equation system,

$$\sum_{a=1}^{A} pr^a \frac{\partial y^a(F_{ss})}{\partial F_{ss}} \phi_t^a N_{ss}^1 - \frac{\partial TC}{\partial F_{ss}} = \sum_{a=1}^{A-1} p^a \left\{ \sum_{j=1}^{A-a} \left[\beta^j pr^a y^{a+j}(F_{ss}) + \Psi' \lambda_{ss} \mu^{a+j} \omega^{a+j}\right] \phi_t^{a+j} N_{ss}^1 \right\}, \tag{13}$$

$$\sum_{a=1}^{A} \beta^{1+a} pr^a y^a(F_{ss}) \phi^a(F_{ss}) = \lambda_{ss} \left[1 - \Psi' \sum_{a=1}^{A} \mu^a \omega^a \phi^a(F_{ss})\right], \tag{14}$$

$$N_{ss}^1 = \Psi \left(\sum_{a=1}^{A} \mu^a \omega^a \phi_t^a N_{ss}^1 \right), \tag{15}$$

which can be solved for $(F_{ss}, N_{ss}^1, \lambda_{ss})$. Once N_{ss}^1 is known the cohort size of any age can be calculated using the survival function valued in the stationary solution, i.e. $N_{ss}^a = \phi^a(F_{ss}) N_{ss}^1$.

Now, to make the computation of the optimal trajectories that drives the fishery from the initial conditions to the stationary solution described above tractable, we assume that convergence is reached in a finite number of periods, T. In other words we truncate the first order conditions using that $F_T = F_{ss}$, $N_{T+2}^1 = N_{ss}^1$, and $\lambda_{T+1} = \lambda_{ss}$. Taking this into account, the model is solving by choosing F_1,F_1, F_2, ..., $F_T = F_{ss}$ such that the system of equations implied by the first order condition (8)-(9) is satisfied. This system of $(T-1)$ nonlinear equations with $(T-1)$ unknowns can be solved relatively quickly using standard numerical methods following the algorithm bellow:

1. Collect all the exogenous parameters describing biological and economic characteristics of the fishery. This includes the biological parameters $(\omega^j, \mu^j, p^j, m^j)$, the initial population distribution (N_0^j), the precautionary limit reference point (SSB_{pa}^j), the economic parameters (pr^j, TC) and the discount factor used to calculate variables in present terms, β.

2. Using outside information, select the $S-R$ relationship to be used. With this relationship, it is possible to find the recruitment for any fishing rate from optimal condition (15). Some examples:

   - If the $S-R$ relationship is defined as the Shepherd relationship[3] (1982),

   $$N_t^1 = \frac{\alpha SSB_t}{1 + \left(\frac{SSB_t}{K}\right)^b},$$

   then recruitment is determined by

   $$N_t^1 = K \frac{\left(\alpha \sum_{a=1}^{A} \mu^a \omega^a \phi_t^a - 1\right)^{1/b}}{\sum_{a=1}^{A} \mu^a \omega^a \phi_t^a}.$$

   So, α, K and b have to be reported.

   - If the $S-R$ relationship is not well defined then recruitment may be considered as a fixed variable that does not depend on fishing rate, that is $N_t^1 = \overline{N^1}$.

3. Assume that the fishery is above the precautionary level. That is $SSB_t > SSB_{pa}$ $\forall t$, and therefore $\theta_t = 0$, $t = 2, .., T$. Compute the stationary solution, $\left(F_{ss}, N_{ss}^1, \lambda_{ss}\right)$ implied by (13)-(15).
4. Guess a trajectory for the fishing mortality rate path, $\{F_t\}_{t=1}^{T-1}$, and assume that in period T the steady state has been reached, i.e. $F_T = ... = F_{T+A+1} = F_{ss}$.
5. Project the future age cohort structure for periods $1, ..., T + A + 1$, $\{N_t^a\}_{t=1}^{T+A+1}$ using the initial age structure, N_0^a, and the S-R relationship. To do this, we use the cohort dynamic population (1), and the recruitment relationship, (3) and (4).
6. Compute Ψ_t' using the recruitment relationship, (3) and (4), associated with $\{N_t^a\}_{t=1}^{T+A+1}$.
7. Using λ_{ss} compute λ_T from equation (9) valued at $t = T - 1$,

   $$\lambda_T = \sum_{a=1}^{A} \beta^a pr^a y^a(F_{ss}) \phi_{ss}^a(F_{ss}) + \sum_{a=1}^{A} \Psi_{T+a}' \lambda_{ss} \mu^a \omega^a \phi_{ss}^a(F_{ss}).$$

   Note that Ψ_{T+a}' is a function of N_{T+a}^1 which depends on the guess $\{F_t\}_{t=1}^{T+a-2}$.
8. Given λ_T, compute $\{\lambda_{t+1}\}_{t=1}^{T-1}$ backwards recursively using equation (9).
9. Using the values of $\{\lambda_{t+1}\}_{t=1}^{T-1}$, the guess of $\{F_t\}_{t=1}^{T-1}$ and the cohort projections $\{N_t^a\}_{t=1}^{T+A+1}$, it is now possible to compute how far we are from the first order condition (8). Formally the following is calculated: $\forall t = 1, ...T$,

   $$f_t = \sum_{a=1}^{A} pr^a \frac{\partial y^a(F_t)}{\partial F_t} N_t^a - \frac{\partial TC_t}{\partial F_t} - \sum_{a=1}^{A-1} p^a \left\{ \sum_{j=1}^{A-a} \beta^j \left[pr^a y^{a+j}(F_{t+j}) + (\Psi_{t+j}' \lambda_{t+j} + \theta_{t+j}) \mu^{a+j} \omega^{a+j}\right] N_{t+j}^{a+j} \right\}.$$

   Using an appropriate algorithm, a new guess for the mortality rate path is selected.

[3] Parameter α is the maximum recruitment attainable when the SSB is very low, $K > 0$ is a threshold of SSB below which the likelihood of population collapse is increased and $b > 0$ measures the power of the density-dependent effects.

10. Repeat the procedure for step 4 to 9 until f_t is low enough.
11. Finally check that $\left\{\sum_{a=1}^{A} \mu^a \omega^a N_t^a\right\}_{t=1}^{T} > SSB_{pa}$. If the restriction is not satisfied, we should guess a new set of positives values for [4] $\{\theta_t\}_{t=2}^{T}$.

3.2. Optimal Global Solutions: Pulse Fishing

It is well known that the Beverton-Holt multi-cohort models used to assess the stock are not globally concave. Therefore the constrained stationary solution described in Section 3.1 may be a local rather than a global optimum (Tahvonen 2009).

In order to find the global solution, we start by transforming the original dynamic optimisation problem of infinite dimension, (7), into a low dimension non-linear optimisation problem. This transformation is carried out using the control vector parameterisation approach (Vassiliadis, 1993; Vassiliadis *et al.*, 1994) which consists of dividing the considered time horizon into ρ constant (equidistant or not) or variable time intervals.

In order to implement this, the management problem (7) must first be rewritten in continuous time. Let $n(a,t)$ be the number of fishes of age a at time t. In age structured models, the conservation law is described by the following McKendrick-Von Foerster equation (Von Forester, 1959; McKendrick, 1926)

$$\frac{\partial n(a,t)}{\partial t} + \frac{\partial n(a,t)}{\partial a} = -[m(a) + p(a)F(t)]n(a,t).$$

The Stock Recruitment relationship occurs as a boundary condition at age zero.

$$n(0,t) = \frac{\alpha SSB(t)}{1 + \left(\frac{SSB(t)}{K}\right)^{\beta}},$$

where, the SSB is given by

$$SSB(t) = \int_0^A \mu(a)\omega(a)n(a,t)da.$$

Profits in period t (6) can be written in continuos time as

$$\pi(t) = \int_0^A [pr(a)C(a,F(t)) - TC(F(t))]\, da,$$

where $C(a,F(t)) = \frac{p(a)F(t)}{m(a)+p(a)F(t)}\left(1 - e^{-m(a)-p(a)F(t)}\right) n(a,t)$. Therefore, the objective function to be maximised can be rewritten as

$$J = \int_0^{\infty} \left(\int_0^A [pr(a)C(a,F(t)) - TC(F(t))]\, da\right) e^{-rt}dt,$$

where r is the instantaneous interest rate and it is related with the discount factor in such a way that

[4] In long-run management plans, the stock is usually far from SSB_{pa}. So for those cases the best initial guess is $\theta_t = 0$.

$\beta = (1+r)^{-1}$. Therefore, the maximisation problem (7) can be expressed in continuous time as

$$\max_{F(t)} \quad J = \int_0^{\infty} \left([pr(a)C(a,F(t)) - TC(F(t))] \, da \right) e^{-rt} dt,$$

$$s.t. \begin{cases} \dfrac{\partial n(a,t)}{\partial t} + \dfrac{\partial n(a,t)}{\partial a} = -[m(a) + p(a)F(t)]n(a,t). \quad \forall t, a \\ \\ n(0,t) = \dfrac{\alpha SSB(t)}{1 + \left(\dfrac{SSB(t)}{K}\right)^{\beta}} \ \forall t, \\ \\ n(A,t) = 0 \ \forall t, \\ \\ SSB_{pa} \leq \displaystyle\int_0^A \mu(a)\omega(a)n(a,t)da. \ \forall t, \\ \\ n(a,0) \quad \text{given}, \ \forall a, \end{cases} \tag{16}$$

Second, the controls are approximated in each interval by using different base functions, generally low order polynomials (zero order - steps, and order one - ramps, as depicted in Figure **??**). The coefficients of the polynomials considered constitute a vector ω (that also includes the lengths of the intervals when these are variable) so that $u = u(\omega)$.

[Insert Figure 1]

This parametrisation transforms the original dynamic optimisation problem of infinite dimension into a non-linear optimisation problem (NLP) of finite dimension where **p**=[μ,ω] (time invariant parameters and control parametrisation coefficients, respectively) is the new vector of decision variables. As a consequence, this new problem can be solved by employing different optimisation algorithms considering that in each internal iteration the process dynamics need to be integrated in order to evaluate both the objective function and the constraints.

For the most general case, the control approximation can be defined by employing Lagrange polynomials of the form (Vassiliadis, 1993):

$$u_j^{(i)}(t) = \sum_{k=1}^{M_j} \sigma_{ijk} \Theta_k^{(M_j)}(\tau^{(i)}),$$

where $j = 1, 2, ..., \nu$; $i = 1, 2, ..., \rho$; $t \in [t_{i-1}, t_i]$ and $\tau^{(i)}$ is the normalised time on each interval i, which is given by the following expression

$$\tau^{(i)} = \frac{t - t_{i-1}}{t_i - t_{i-1}} = \frac{t - t_{i-1}}{q_{i-1}},$$

and the M-order Lagrande polynomials ($\Theta_k^{(M)}$) are defined as

$$\Theta_k^{(M)}(\tau) \equiv \begin{cases} 1 & \text{if } M = 1, \\ \prod_{k'=1,k\neq 1}^{M} \dfrac{\tau - \tau_{k'}}{\tau_k - \tau_{k'}} & \text{if } M \geq 2. \end{cases}$$

The parameters σ_{ijk} of these polynomials are directly related with the vector of new decision variables ω.

For the case of fixed final time problems and one control variable, the approximation of this variable is given by

$$u(t) = \omega_i \quad \forall i \quad t_{i-1} \leq t < t_{i-1} + q_i.$$

So the vector of decision variables is formed by both the value of the steps as well as by the time interval lengths: $\omega = [\omega_1, ..., \omega_\rho, q_1, ..., q_{\rho-1}] \in R^{2\rho-1}$. For the same case, but considering ν control variables with constant time interval lengths, it is verified that $\omega \in R^{(\nu+1)\cdot(\rho-1)}$.

In our application, to transform of the infinite dynamic optimization problem (16) into the NLP of finite dimension, the time horizon is divided into $\rho = 80$ constant time intervals and the controls in each interval are approximated using ramps.

Once the NLP of finite dimension is set, stochastic optimisation algorithms are used to solve it. In particular two global stochastic optimisation algorithms plus one hybrid strategies are considered to solve the transformed optimisation problem. We describe briefly the main characteristics of each method:

- **DE:** *Differential Evolution.* It is a metaheuristics algorithm for global optimisation of nonlinear and (possibly) non-differentiable continuous functions presented by Storn and Price (1997). This is a population-based method which, starting with a randomly generated population, computes new candidate solutions by calculating differences between population members. It handles stochastic variables by means of a direct search method which outperforms other popular global optimisation algorithms, and it is widely used by the evolutionary computation community.

- **eSS-SSm:** *Enhanced Scatter Search.* As presented in Egea *et al.* (2009), Scatter Search is a population-based metaheuristic method which combines a global phase with an intensification method (i.e., a local search). This methodology is very flexible, since each of its elements can be implemented in a variety of ways and degrees of sophistication. A basic design to implement scatter-search is given on the well-known "five-method template" (Laguna and Martí, 2003): (1) A Diversification Generation Method to generate a collection of diverse trial solutions. (2) An Improvement Method to transform a trial solution into one or more enhanced trial solutions. (3) A Reference Set Update Method to build and maintain a reference set consisting of the b "best" solutions found, where the value of b is typically small compared to the population size of other evolutionary algorithms. Solutions gain membership to the reference set according to their quality or their diversity. (4) A Subset Generation Method to operate on the reference set, to produce several subsets of its solutions as a basis for creating combined solutions. (5) A Solution Combination Method to transform a given subset of solutions produced by the Subset Generation Method into one or more combined solution vectors.

eSS-SSm is an advanced scatter search method developed by the IIM-CSIC Process Engineering Group for chemical and bioprocess optimisation problems providing excellent results.

- **Hybrid strategies:** The key concept of hybrid methods is synergy. A hybrid method tries to exploit the best properties of different methodologies. They combine global stochastic and local optimisation algorithms. The global ones cover the whole search space to find the global optimum, but they are slow in finding the exact location of this global solution. A promising strategy consists of obtaining a good initial guess with one of these global methods and then fine tuning employing local optimisation. These strategies take advantage of both the robustness of stochastic solvers and the efficiency of local methods when started in the optimum neighbourhoods. In this article, the hybrid strategy considered is *SSm+DHC*.

  The *DHC* algorithm (*Dynamic Hill Climbind;* De la Maza and Yuret, 1994) draws on ideas from genetic algorithms, hill climbing and conjugate gradient methods. It is a direct search algorithm wich explores every dimension of the search space using dynamic steps. It is formed by an inner and an outer loop. The first one contains a very efficient technique for locating local optima while the outer loop ensures that the entire search space has been explored. In this work, only the local phase of the algorithm has been used.

In order to select the algorithm that leads to the best results, it is necessary to perform an efficiency analysis. To that purpose, the convergences curves, which show the evolution of the best value obtained by each solver over the CPU time , are constructed. With these representations, both the robustness (the capability of the solver to attain consistently good final solutions) and the efficiency (the speed of convergence to the final solution) can be evaluated, allowing the user to select an appropriate algorithm to solve a given optimisation problem.

4. The Northern Stock of Hake

In order to compare the optimal stationary and pulse fishing solutions, we apply the methods described above to the Northern Stock of Hake (NSH). The NSH is a fishery managed with the advice of the International Council for the Exploitation of the Sea (ICES) and includes all fisheries in subareas VII and VIII and also some fisheries in Subareas IV and VI (see Figure 2). Hake (*merluccius merluccius*) is caught throughout the year, though the peak landings are made in the spring-summer months. It spawns from March to July at depths of 120-160 m., mainly to the south and west of Ireland and moves to shallower water by September. The two major nursery areas are the Bay of Biscay and off southern Ireland. As they become mature, the fish disperse to offshore regions of the Bay of Biscay and Celtic Sea. Male hake mature at 3-4 years old (27-35 cm) and females at 5-7 years old (50-70 cm).

[Insert Figure 2]

Hake has been the main species supporting trawling fleets off the Atlantic coasts of France and Spain since the 1930s. The three main gear types used by vessels fishing for hake as a target species are lines (Spain), fixed-nets and other trawls (all countries). Landings in 2008 were 47, 800 tonnes, below the regulated TAC of 54, 000 tonnes. Spain accounts for most of the landings with 53% of the total captures. France takes 30% of the total, the UK 7%, Denmark 3%, Ireland 3% and other countries (Norway, Belgium, Netherlands, Germany, and Sweden) take smaller amounts (ICES 2009). Tables 2 and 3 display the main characteristics of the Spanish and French fleets according to the European Data Collection Regulation, respectively.

[Insert Table 2]

[Insert Table 3]

After the collapse of spawning SSB in the 1990s, an emergency plan was implemented for the NSH in 2001 and 2002 (EC 1162/2001, EC 2602/2001 and EC 494/2002). After this emergency plan, a recovery plan was implemented in 2004 (EC 811/2004). Its aim was to achieve a SSB of 140, 000 tonnes (B_{pa}) by limiting fishing mortality to 0.25 and by allowing a maximum change in harvest between consecutive years of 15%. The recovery plan was to be replaced by a management plan when the target level for the stock had been reached in two consecutive years,. This objective was achieved in 2004, 2005 and 2006. So in 2007, an Expert Working Group from the Scientific, Technical and Economic Committee for Fisheries (STECF, 2008a) was convened in Lisbon from 18-22 June to evaluate the potential biological consequences of the long term management plan. The working group proposed as objective of this plan a shift from the current fishing rate $F_{sq} = 0.25$ to $F_{msy} = 0.17$ assuming gradual changes to avoid drastic season closures.

In order to calibrate the age structured model for this fishery two data sources are used. First, the information regarding the biological parameters of the fishery comes from the Expert Working Group (STECF, 2008a). Most of the parameters result from the summary of Extended Survivor Analysis (XSA) results from the 2006 update (ICES, 2007). Secondly, economic data on the fishery come from expert working group meeting on Northern Hake Long-Term Management Plan Impact Assessment (STECF, 2008b) held in Brussels on December 3-6 2007.

Table 4 shows, for each age, the number of fishes at the initial conditions, the parameters of the population dynamics (selection pattern, weight and maturity) and the prices.[5]

[Insert Table 4]

As the S-R relationship we use the Shepherd relationship (1982) described by

$$N^1 = \frac{\alpha SSB}{1 + \left(\frac{SSB}{K}\right)^b}. \tag{17}$$

To calibrate this recruitment function and SSB data for the period 1978-2006 are used. Since parameter α represents the slope of the S-R relationship at the origin, it is calibrated as the maximum value of N_t^1/SSB_t. This calibration implies $\alpha = 2.4879$, $K = 168,270$ and $b = 1.7602$. The upper left panel in Figure 3 shows this calibration and the data. We also use the values of the STECF groups for $SSB_{pa} = 140,000$ tonnes.

For calibrating the costs, we use data on the cost structure and the degree of dependency of hake for the different FUs for the Spanish fleet in 2004 and for the French fleet in 2006 (See Table 5).

[5] To calculate prices as a function of ages we use data on 2007 daily sales for the Galician trawl, gill nets and long lines fleets.

[Insert Table 5]

In the numerical simulations we assume that the cost of effort is proportional to the mortality rate, $TC = cF$, where $c = TC/F$ represents the average and the marginal cost. Once the total costs are known, c is calibrated as this amount divided by the current mortality rate, $F = 0.25$.

In order to calibrate the total costs, we start by determining the running costs per day. First we calculate fuel costs, other costs, depreciation and interest divided by the days at sea of each segment (see Table 6). Second, the average costs weighted by the sea days for each segment are calculated (last column in Table 6). Third, the fuel costs are adjusted taking into account the increase suffered by the fuel price during 2007. Since fuel prices rose from 0.346 Euros in late 2006 to 0.52 in early 2008, fuel costs have been multiplied by 1.5. These calculations imply a cost of the fishery of $1,919.14$ Euros per day. Since the hake dependency of the fleet is 0.28, the costs per day imputed to hake are 542.50 Euros per day.

[Insert Table 6]

We assume that 543.17 Euros per day and $135,635$ days at sea are good proxies for the marginal cost and total effort, respectively, so the total cost can be considered as $C(F) = 543.17 \times 135,635 = 73.57$ millions Euros.

We define three different scenarios for the problem proposed:

- **Scenario 1:** The smooth trajectory that drives the fishery from the initial conditions to F_{msy} using a constant annual reduction in fishing mortality of 15%.[6]
- **Scenario 2:** The trajectory that drives the fishery smoothly from the initial conditions to the optimal stationary solution.
- **Scenario 3:** The trajectory that drives the fishery from the initial conditions to the global optimal solution.

Scenario 1 is simulated calculating F_{msy} as the fishing rate that maximises the stationary yield. For the NSH this results in $F_{msy} = 0.172$. In Scenario 2 the optimal stationary trajectory is obtained using the algorithm described in Section 3.1. As a result for the NSH we obtain $F_{ss} = 0.119$.

In relation to the Scenario 3, the three global methods described in Section 3.2 are applied to the NSH considering a time horizon of 80 periods for the transformed problem. Several runs ($n = 5$) have been performed on a Intel Core2 Quad 2.40 GHz. for the algorithms considered to solve the NLP problem associated with the maximisation problem (3) to attain an average function J. The maximum CPUtime was set to 8 hours. The efficiency analysis of the results for three global methods leads to the selection of the hybrid strategy. This method converges to high quality solutions faster than the other global stochastic optimization algorithms considered. With this hybrid strategy, the global optimal solution for the NSH

[6] This scenario is similar to the management strategy proposed by the Expert Working Group in the Long Term Plan for this fishery (STECF/SGBRE-07-03).

consists of fishing every four years applying a fishing rate of $F_{periodic} = 0.489$ in the harvesting years and three consecutive fallow years.

Figure 3 shows the changes over time in F, SSB and the yield under the three scenarios. Notice that even for the optimal pulse trajectory the SSB levels implied are safe over the time (higher than 140,000 kgT). However, the SSB cycle is not in the rank of the historical data. The maximum SSB recorded is lower than the 354,760 kgT implied by the four-year period predicted by the model, because the optimal pulse that maximises the present value of profits would imply, in a cyclical manner, the closure of the fishery for three years and then allowing harvesting once the stock has recovered. This policy would require alternative employment for the fleet for the three fallow years of the cycle; otherwise the costs associated with this management rule would be very high from the social point of view.

[Insert Figure 3]

Table 7 summarises the quantitative results for the three trajectories. The long run values for each scenario are shown for F, SSB, yield and the net present value of profits. The net present value of the profits associated with each trajectory is calculated using for $\beta = 0.95$.

[Insert Table 7]

It is worth comparing how far in terms of present value of profits the pulse fishing solution (Scenario 3) is from the optimal stationary solution (Scenario 2). We observe that the stationary trajectory represents 2% less present value of profits than the optimal periodic pulse trajectory. Furthermore, the stationary trajectory implies a higher net present profit and higher SSB in the long term than the F_{msy} trajectory. In particular, for the NSH the stationary trajectory represents 19% more present value of profits than the F_{msy} trajectory. This result is along in the same lines as that of Grafton *et al.* (2007).

The analysis has been repeated without considering costs in the objective function to be maximised. We draw two conclusions. First, the optimal stationary fishing mortality is almost the same as that the reference target set in the long term management plan for this fishery, F_{msy}. This implies similar a present value of profits associated with the both smooth trajectories. Second, when costs are not considered the potential benefit from a periodic trajectory is lower.

[Insert Table 8]

5. Discussion and policy recommendation

In this study we have shown how the advantages of pulse fishing are lower when compared to stationary trajectories than to reference points. The results of numerical simulations show that pulse fishing entails far lower benefits than indicated previously in the relevant literature.

These results were obtained using constant prices and linear costs in fishing mortality. These assumptions are very similar to those in Hanneson (1975). However, the advantages of pulse fishing are closely related to them.

Is it reasonable to assume that hake prices will remain constant over time? When it is possible to alternate between different stocks of hake and/or to freeze catches from one year for the next, it can be assumed that pulse fishing solutions can be implemented while maintaining a constant supply over time. In this case, the constant price assumption seems reasonable.

But the NSH is a fresh fishery. Vessels may be at sea for 10-15 days, and the prices of their landings – which depend on size – are higher than those of frozen hake imported from other fisheries (Namibia, Argentina).

Price changes could be introduced into the model in two ways. If it is assumed that there are no differences in the price per kg of fish of different sizes, it suffices to assume that prices are isoelastic functions of quantities, . In this case, revenues can be written as

$$\left[\sum_{a=1}^{A} \overline{p} y^a(F_t) N_t^a\right]^{(1-\epsilon)} .$$

This assumption is used to assess the economic impact of management plans. The value considered for this assessment was $\epsilon = 0.2$.

If it is assumed that the price per kg varies for fish of different sizes, price elasticities can be introduced via age in a way very similar to the above case. If $pr^a = \overline{p}^a \ Y^{a\ -\epsilon}$, then revenues can be written as

$$\sum_{a=1}^{A} [\overline{p}^a y^a(F_t) N_t^a]^{(1-\epsilon^a)} .$$

Another assumption used in our work is that fishing mortality costs are linear. This results from the assumption of closely homogenous fleets, which enables a linear relationship to be established (the catchability coefficient) between fishing days and fishing mortality. This is the most neutral assumption in regard to the assessment of the advantages of pulse fishing over stationary solutions. However, other cost configurations are also possible.

For instance it is well known that if fisheries showed increasing yields then costs would not be convex and the benefits from pulse fishing would be greater. Hake as a species does not appear suitable for school fishing along the lines of North Sea Herring.

However it is possible to assume that yields are decreasing, and that fishing costs are convex. For example van Oostenbrugge *et al.* (2008) show that when the number of days on which fleets may fish is limited, as is the case here, a more than proportional reduction in days is required to reduce fishing mortality, i.e. fishing mortality costs are convex.

What would be the implications for our model of abandoning the assumption of constant fishing mortality unit costs and prices? Under this new assumption the analytical solution could be characterised and the numerical solution found. Figure 4 shows the results of seeking the numerical solution using control vector parameterisation if the following is used as the objective function

$$\sum_{a=1}^{A} [\overline{p}^a y^a(F_t) N_t^a]^{(1-\epsilon^a)} - cF^{\alpha}.$$

[Insert Figure 4]

When $\epsilon = 0$, the solution is similar to that for constant prices. If $\epsilon > 0$, the price function introduces a mechanism that reduces the advantages of pulse fishing solutions: the more sensitive prices are to variations in catches, the more desirable stationary solutions become. Cost convexity also has a considerable impact. When $\alpha = 1.5$, the stationary solution is optimal even with constant prices.

Concave and convex costs can be considered as reduced forms of more complex relationships that can be incorporated into the model. For instance Da Rocha, Cerviño and Gutiérrez (2010) show that it is possible to introduce fleet dynamics into an age-structured model, with forward-looking firms that make decisions discounting the sum of future profits. Another possible extension of the model would be to consider that the catchability coefficient is not constant but dependent on time, horsepower and tonnage (see Da Rocha and Gutiérrez, 2011). Of course, these changes might influence the results. And our guess is that in a richer environment, periodic solutions would prove inferior to stationary ones.

As mentioned in the Introduction, there are cases of fisheries which are exploited on a pulse basis. However, in general the fact that stocks are exploited jointly by several countries, the non malleable nature of capital and employment, the impossibility of storing catches and the possibility of losing market access all make it inadvisable to use pulse solutions that would entail the closure of fisheries for some seasons. In other words stationary solutions are implemented because there are numerous effects not considered explicitly in the model that make pulse fishing inadvisable.

This study should be seen as measuring the institutional constraints faced by regulators. A comparison of the stationary and pulse solutions reveals a shadow price of all the implicit constraints not included in the model (non linear prices, convex costs, fleet dynamics, effort indices, etc). The numerical simulation run shows that when all these effects are ignored the stationary solution gives results only 2% below those of the pulse solution. This gives a measure in terms of discounted present value of all the factors not included in the model. If regulators, resorting to their experience and expertise, consider that reducing the sum total of discounted benefits by 2% is a low price to pay for keeping the fleet at work continuously, then out simulations show that the stationary solution is optimal.

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Table 1: Age Structure and the Intertemporal Maximization Problem

	t	t+1	t+2	...	t+A-2	t+A-1	t+A	t+A+1
a=1	N_t^1		N_{t+2}^1	...				
a=2	N_t^2	N_{t+1}^2		...	N_{t+A-2}^2			
...	...	...	...	...		...	...	...
a=A-1	N_t^{A-1}	N_{t+1}^{A-1}	N_{t+2}^{A-1}	...	N_{t+A-2}^{A-1}		N_{t+A}^{A-1}	
a=A	N_t^A	N_{t+1}^A	N_{t+2}^A	...	N_{t+A-2}^A	N_{t+A-1}^A		N_{t+A+1}^A

Table 2: Characteristics of the Spanish Fleet (2004)

Segment Fleet	Length Class	Number of vessels	Total employment	Gross Value Added (m€)
Demersal Trawlers	24-40m	93	1,023	56.8
Pair Demersal Trawlers	24-40m	20	239	9.6
Longliners	24-40m	84	1,176	59.8
Total		197	2,438	126.2

Source: STECF, 2008b. Tables 6.1.5, 6.1.6, 6.1.8 y 6.1.9

Table 3: Characteristics of the French Fleet (2006)

Segment Fleet	Length Class	Number of vessels	Total employment	Gross Value Added (m€)
DTS-Targeted Nephrops	12-24m	204	759	45.4
DTS-Targeted Fish	12-24m	106	490	35.9
DTS	24-40m	55	389	28.9
Hook	24-40m	5	62	2.3
Netters	12-24m	60	351	22.2
Netters	24-40m	18	223	11.3
Others	-	210	803	-
Total		658	3,077	-

Source: STECF, 2008b. Tables 6.2.5, 6.2.8-6.2.13

Table 4: Parameters by age

	Initial conditions										
	Age 0	Age 1	Age 2	Age 3	Age 4	Age 5	Age 6	Age 7	Age 8	Age 9	Age 10
N^a $^{(1)}$	$186,213$	$152,458$	$123,457$	$100,213$	$67,409$	$35,551$	$19,674$	$10,206$	$9,147$	$4,078$	$1,819$
	Population dynamics										
	Age 0	Age 1	Age 2	Age 3	Age 4	Age 5	Age 6	Age 7	Age 8	Age 9	Age 10
p^a	0.00	0.06	0.54	1.15	1.03	1.52	2.09	2.43	2.43	2.43	2.43
ω^a $^{(2)}$	0.06	0.13	0.22	0.34	0.60	0.98	1.44	1.83	2.68	2.68	2.68
μ^a	0.00	0.00	0.00	0.23	0.60	0.90	1.00	1.00	1.00	1.00	1.00
	Prices										
	Age 0	Age 1	Age 2	Age 3	Age 4	Age 5	Age 6	Age 7	Age 8	Age 9	Age 10
pr^a $^{(3)}$	2.36	2.93	3.42	3.85	4.55	5.22	5.81	6.22	6.92	6.92	6.92

Source: Meeting on Northern Hake Long-Term Management Plans (STECF/SGBRE-07-03) and ICES (2007)

$^{(1)}$ Thousand;$^{(2)}$ kg ; $^{(3)}$ Euros/kg

Table 5: Economic Indicators for Segment

Segment	S1(2004)	S2(2004)	S3(2004)	F1(2006)	F2(2006)	F3(2006)	F4(2006)	F5(2006)	F6(2006)
Value of landings	101,914,422	19,172,000	90,970,320	98.1	77.6	67.5	3.9	37.9	18.3
Fuel costs	21,182,889	3,141,640	7,300,860	20.2	15.9	15.4	0.5	3.0	1.7
Other running costs	12,071,121	3,867,258	20,030,640	9.7	7.7	7.1	0.4	3.1	1.5
Depreciation	12,938,904	2,551,888	10,711,260	8.8	7.0	8.1	0.3	2.8	1.4
Interest	879,594	194,053	859,236	1.6	1.3	1.6	0.2	0.6	0.4
Days at the sea	25,389	4,112	21,924	32,300.0	21,500.0	14,500.0	11,00.0	10,300.0	4500.0
Crew share	40,876,476	7,221,568	44,804,508	32.1	25.4	20.1	1.5	15.0	7.3

S1=Longliners (24-40m); S2=Demersal trawlers (24-40m); S3= Pair demersal trawlers (24-40m); F1=Demersal trawls seiners - Targeting nephrops, (12-24m); F2= Demersal trawls seiners, - Targeting fish, (12-24m); F3=Demersal trawls seiners - Targeting nephrops or fish, (24-40m); F4= Hook (24-40m); F5= Netters (12-24m) and F6=Netters (24-40m).

S1, S2 and S3 in Euros; F1. F2. F3. F4. F5 and F6 in millions of Euros.

Source: Tables 6.1.7-6.1.9, 6.2.8-6.2.13 and 7-2-3 (STECF, 2008b).

Table 6: Costs per Day and FU

Segment	S1(2004)	S2(2004)	S3(2004)	F1(2006)	F2(2006)	F3(2006)	F4(2006)	F5(2006)	F6(2006)	mean
Fuel per day	834.3	764.0	333.0	625.4	739.5	1062.1	454.5	291.3	377.8	651
Other costs per day	475.4	940.5	913.6	300.3	358.1	489.7	363.6	301.0	333.3	483
Depreciation/day	509.6	620.6	488.6	272.4	325.6	558.6	272.7	271.8	311.1	403
Iinterest/day	34.6	47.2	39.2	49.5	60.5	110.3	181.8	58.3	88.9	56
Total cost per day	1,854.1	2,372.3	1,774.4	1,247.7	1,483.7	2,220.7	1,272.7	922.3	1,111.1	1593
Hake dependency	24%	36%	98%	4%	2%	6%	77%	20%	84%	0.47%
Crew share	0.40	0.38	0.49	0.33	0.33	0.30	0.38	0.40	0.40	0.37

S1=Longliners (24-40m); S2=Demersal trawlers (24-40m); S3= Pair demersal trawlers (24-40m); F1=Demersal trawls seiners - Targeting nephrops, (12-24m); F2= Demersal trawls seiners, - Targeting fish, (12-24m); F3=Demersal trawls seiners - Targeting nephrops or fish, (24-40m); F4= Hook (24-40m); F5= Netters (12-24m) and F6=Netters (24-40m).

S1, S2 and S3 in Euros; F1. F2. F3. F4. F5 and F6 in millions of Euros.

Source: Own calculations

Table 7: Comparison of the three scenarios with positive costs

	Long Run Values			Net Present Profits	
	F	SSB	Yield	J	ΔJ
Scenario 1: F_{msy}	0.172	$1.9807e+005$	$3.395e+005$	$3.5752e+006$	100.00
Scenario 2: F_{ss}	0.119	$2.5414e+005$	$3.314e+005$	$4.2736e+006$	119.53
Scenario 3: $F_{periodic}$				$4.3562e+006$	121.85
year 1	0	1.6719e+005	0		
year 2	0	2.1240e+005	0		
year 3	0	2.6985e+005	0		
year 4	0.489	$3.5476e+005$	$13.500e+005$		

Table 8: Comparison of the three scenarios with zero costs

	Long Run	Net Present Profits	
	F	J	ΔJ
scenario 1: F_{msy}	0.172	$6.2946e+006$	100.00
scenario 2: F_{ss}	0.172	$6.3363e+006$	100.66
scenario 3: $F_{periodic}$	0.170	$6.3906e+006$	101.53

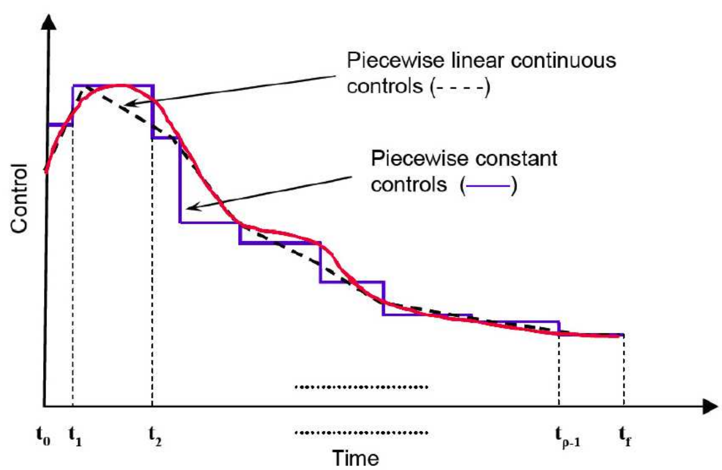

Figure 1: Control Vector Parametrization (CVP) scheme

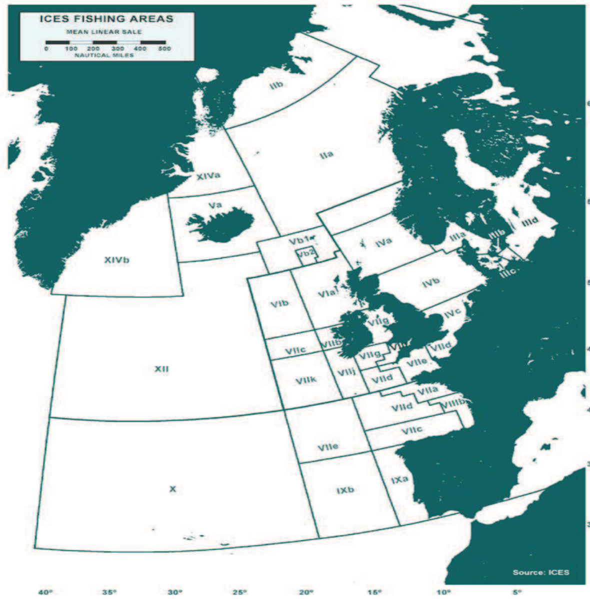

Figure 2: ICES Fishing Areas

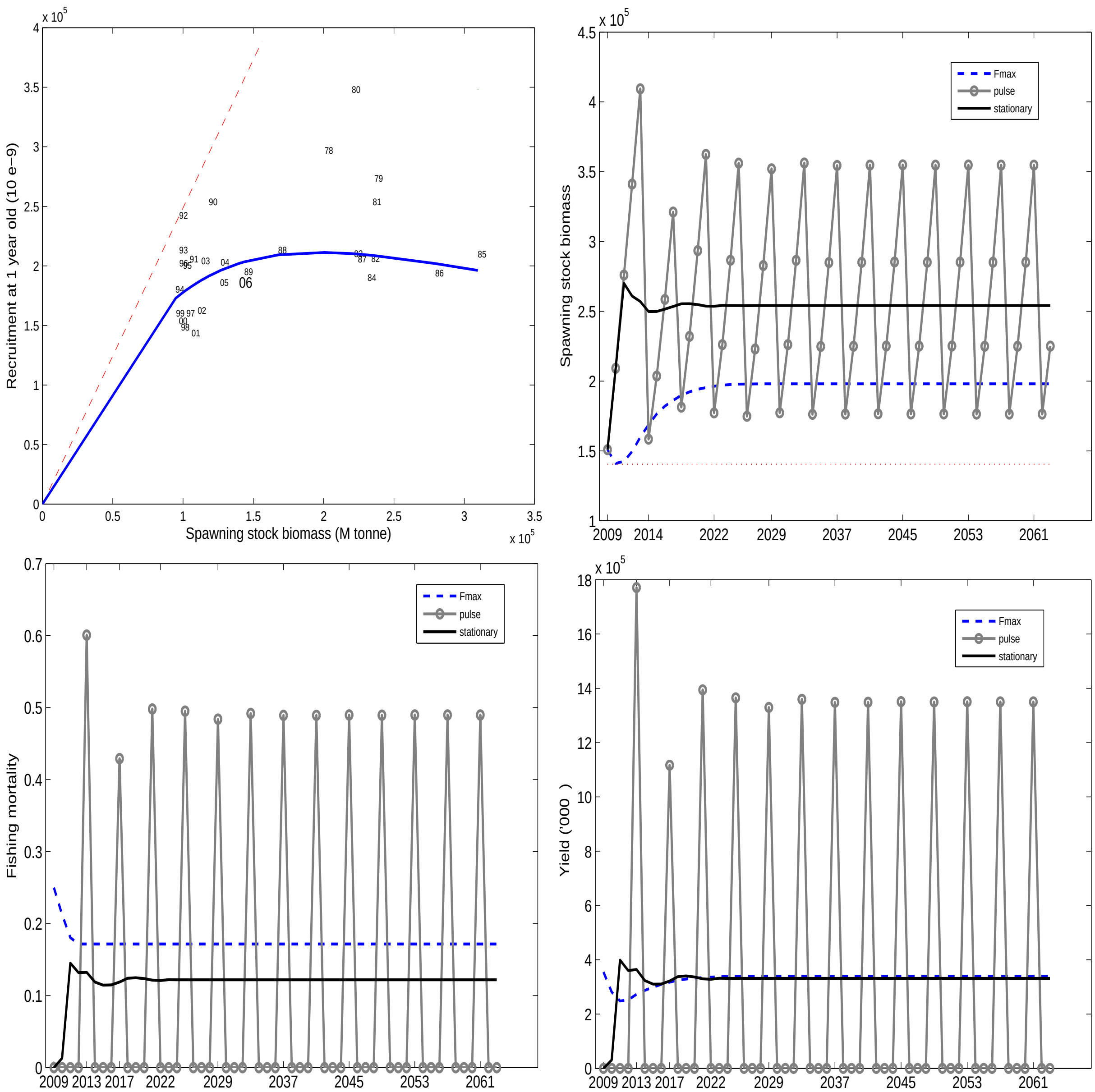

Figure 3: Trajectories under the three scenarios: optimal ($F_{periodic}$), the stationary solution (F_{ss}) and smooth driven to the reference point (F_{msy}). Stock recruitment relationship (upper left panel); SSB (upper right panel); fishing mortality (lower left panel) and yield (lower right panel)

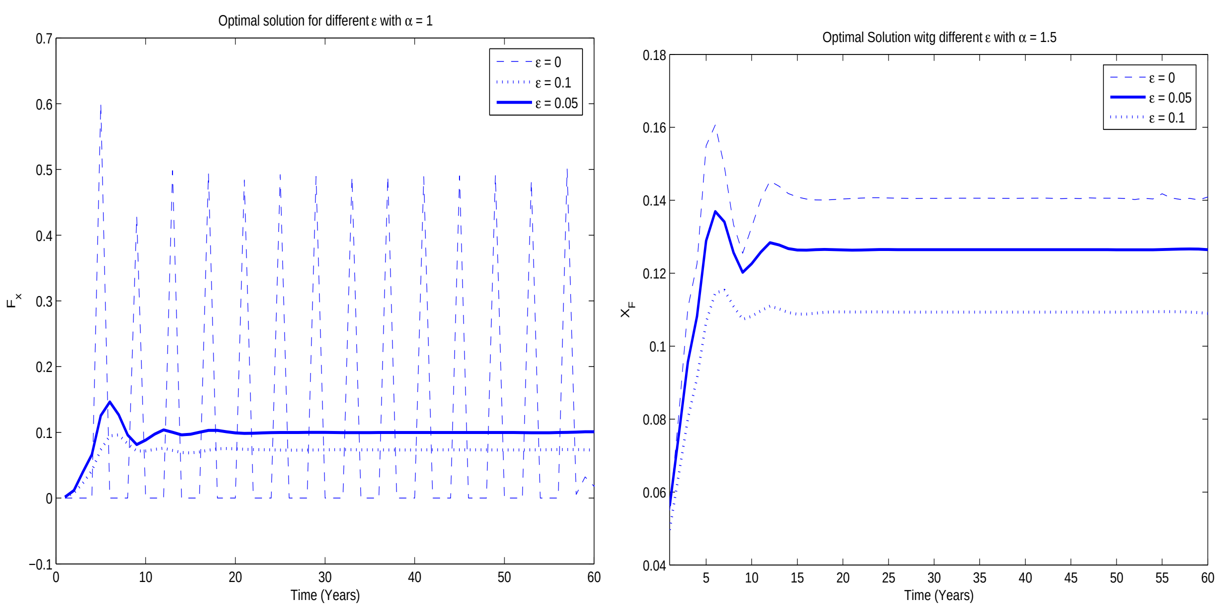

Figure 4: Simulating the optimal solution with no constant prices and convex costs.

Appendix A. Technical report: Solving the maximization problem

In this report we show in detail how to solve optimal management problem for the case of $A = 3$.[7]

$$\max_{\{F_t, N^1_{t+2}\}_{t=0}^{\infty}} \quad \sum_{t=0}^{\infty} \beta^t \left\{ \sum_{a=1}^{3} pr^a y^a(F_t) N^a_t \right\}$$

$$s.t. \begin{cases} N^{a+1}_{t+1} = e^{-z^a(F_t)} N^a_t \quad \forall t \quad \forall a = 1,2 \\ N^1_{t+1} = \Psi\left(\sum_{a=1}^{3} \mu^a \omega^a N^a_t\right) \quad \forall t \\ \sum_{a=1}^{3} \mu^a \omega^a N^a_t \geq SSB_{pa} \quad \forall t \end{cases}$$

In this context, the function to be maximized can be expressed as

$$L = \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} pr^1 y^1_t(F_t) \phi^1_t N^1_t + pr^2 y^2_t(F_t) \phi^2_t N^1_{t-1} + pr^3 y^3_t(F_t) \phi^3_t N^1_{t-2} - TC(F_t) \\ +\lambda_t \left[\Psi^1_t \left(\mu^1 \omega^1 \phi^1_t N^1_t + \mu^2 \omega^2 \phi^2_t N^1_{t-1} + \mu^3 \omega^3 \phi^3_t N^1_{t-2} \right) - N^1_{t+1} \right] \\ +\theta_t \left[\mu^1 \omega^1 \phi^1_t N^1_t + \mu^2 \omega^2 \phi^2_t N^1_{t-1} + \mu^3 \omega^3 \phi^3_t N^1_{t-2} - SSB^1_{pa} \right] \end{array} \right\}, \tag{A.1}$$

where the survival functions are given by

$$\begin{aligned} \phi^1_t &= 1, \\ \phi^2_t &= \phi(F_{t-1}) = e^{-p^1 F_{t-1} - m^1}, \\ \phi^3_t &= \phi(F_{t-1}, F_{t-2}) = e^{-p^2 F_{t-1} - m^2} e^{-p^1 F_{t-2} - m^1}, \end{aligned}$$

It is easy to see that F_t appears in (A.1) only in the sums multiply by β^t, β^{t+1} and β^{t+2}. That is

[7] The mathematical developments shown in this Appendix could be considered a technical report. We let to the editors' decision the relevance of being published or kept as technical report available from the authors.

$$
\begin{aligned}
.... + \beta^{t} & \left\{ \begin{array}{l} pr^{1}y_{t}^{1}(F_{t})N_{t}^{1} + pr^{2}y_{t}^{2}(F_{t})e^{-p^{1}F_{t-1}-m^{1}}N_{t-1}^{1} + pr^{3}y_{t}^{3}(F_{t})e^{-p^{2}F_{t-1}-m^{2}}e^{-p^{1}F_{t-2}-m^{1}}N_{t-2}^{1} - TC(F_{t}) \\ +\lambda_{t}\left[\Psi_{t}^{1}\left(\mu^{1}\omega^{1}N_{t}^{1} + \mu^{2}\omega^{2}e^{-p^{1}F_{t-1}-m^{1}}N_{t-1}^{1} + \mu^{3}\omega^{3}e^{-p^{2}F_{t-1}-m^{2}}e^{-p^{1}F_{t-2}-m^{1}}N_{t-2}^{1}\right) - N_{t+1}^{1}\right] \\ +\theta_{t}\left[\mu^{1}\omega^{1}N_{t}^{1} + \mu^{2}\omega^{2}e^{-p^{1}F_{t-1}-m^{1}}N_{t-1}^{1} + \mu^{3}\omega^{3}e^{-p^{2}F_{t-1}-m^{2}}e^{-p^{1}F_{t-2}-m^{1}}N_{t-2}^{1} - SSB_{pa}^{1}\right] \end{array} \right\} \\
+ \beta^{t+1} & \left\{ \begin{array}{l} pr^{1}y_{t+1}^{1}(F_{t+1})N_{t+1}^{1} + pr^{2}y_{t+1}^{2}(F_{t+1})e^{-p^{1}F_{t}-m^{1}}N_{t}^{1} + pr^{3.1}y_{t+1}^{3}(F_{t+1})e^{-p^{2}F_{t}-m^{2}}e^{-p^{1}F_{t-1}-m^{1}}N_{t-1}^{1} - TC(F_{t+1}) \\ +\lambda_{t+1}\left[\Psi_{t+1}^{1}\left(\mu^{1}\omega^{1}N_{t+1}^{1} + \mu^{2}\omega^{2}e^{-p^{1}F_{t}-m^{1}}N_{t}^{1} + \mu^{3}\omega^{3}e^{-p^{2}F_{t}-m^{2}}e^{-p^{1}F_{t-1}-m^{1}}N_{t-1}^{1}\right) - N_{t+2}^{1}\right] \\ +\theta_{t+1}\left[\mu^{1}\omega^{1}N_{t+1}^{1} + \mu^{2}\omega^{2}e^{-p^{1}F_{t}-m^{1}}N_{t}^{1} + \mu^{3}\omega^{3}e^{-p^{2}F_{t}-m^{2}}e^{-p^{1}F_{t-1}-m^{1}}N_{t-1}^{1} - SSB_{pa}^{1}\right] \end{array} \right\} \\
+ \beta^{t+2} & \left\{ \begin{array}{l} pr^{1}y_{t+2}^{1}(F_{t+2})N_{t+2}^{1} + pr^{2}y_{t+2}^{2}(F_{t+2})e^{-p^{1}F_{t+1}-m^{1}}N_{t+1}^{1} + pr^{3.1}y_{t+2}^{3}(F_{t+2})e^{-p^{2}F_{t+1}-m^{2}}e^{-p^{1}F_{t}-m^{1}}N_{t}^{1} - TC(F_{t+2}) \\ +\lambda_{t+2}\left[\Psi_{t+2}^{1}\left(\mu^{1}\omega^{1}N_{t+2}^{1} + \mu^{2}\omega^{2}e^{-p^{1}F_{t+1}-m^{1}}N_{t+1}^{1} + \mu^{3}\omega^{3}e^{-p^{2}F_{t+1}-m^{2}}e^{-p^{1}F_{t}-m^{1}}N_{t}^{1}\right) - N_{t+3}^{1}\right] \\ +\theta_{t+2}\left[\mu^{1}\omega^{1}N_{t+2}^{1} + \mu^{2}\omega^{2}e^{-p^{1}F_{t+1}-m^{1}}N_{t+1}^{1} + \mu^{3}\omega^{3}e^{-p^{2}F_{t+1}-m^{2}}e^{-p^{1}F_{t}-m^{1}}N_{t}^{1} - SSB_{pa}^{1}\right] \end{array} \right\} \\
& + ...
\end{aligned}
$$

Therefore, the first order conditions from $\partial L/\partial F_t = 0$ is given by

$$
\begin{aligned}
\frac{\partial L}{\partial F_t} &= \beta^{t}\left\{ pr^{1}\tfrac{\partial y_t^1(F_t)}{\partial F_t}N_t^1 + pr^{2}\tfrac{\partial y_t^2(F_t)}{\partial F_t})\phi_t^2 N_{t-1}^1 + pr^{3}\tfrac{\partial y_t^3(F_t)}{\partial F_t}\phi_t^3 N_{t-2}^1 - \tfrac{\partial TC(F_t)}{\partial F_t} \right\} + \\
&\quad +\beta^{t+1}\left\{ \begin{array}{l} pr^{2}y_{t+1}^{2}(F_{t+1})\left(-p^{1}\right)\phi_{t+1}^{2}N_{t}^{1} + pr^{3}y_{t+1}^{3}(F_{t+1})\left(-p^{2}\right)\phi_{t+1}^{3}N_{t-1}^{1} \\ +\lambda_{t+1}\left(\Psi_{t+1}^{1}\right)'\left[\mu^{2}\omega^{2}\left(-p^{1}\right)\phi_{t+1}^{2}N_{t}^{1} + \mu^{3}\omega^{3}\left(-p^{2}\right)\phi_{t+1}^{3}N_{t-1}^{1}\right] \\ +\theta_{t+1}\left[\mu^{2}\omega^{2}\left(-p^{1}\right)\phi_{t+1}^{2}N_{t}^{1} + \mu^{3}\omega^{3}\left(-p^{2}\right)\phi_{t+1}^{3}N_{t}^{1}\right] \end{array} \right\} + \\
&\quad +\beta^{t+2}\left\{ \begin{array}{l} pr^{3}y_{t+2}^{3}(F_{t+2})\left(-p^{1}\right)\phi_{t+2}^{3}N_{t}^{1} \\ +\lambda_{t+2}\left(\Psi_{t+2}^{1}\right)'\left[\mu^{3}\omega^{3}\left(-p^{1}\right)\phi_{t+2}^{3}N_{t}^{1}\right] \\ +\theta_{t+2}\left[\mu^{3}\omega^{3}\left(-p^{1}\right)\phi_{t+2}^{3}N_{t}^{1}\right] \end{array} \right\} \\
&= 0
\end{aligned}
$$

A generalization of this example for any A can be expressed as

$$
\begin{aligned}
\frac{\partial L}{\partial F_t} &= 0, \\
&\implies \beta^t \left[\sum_{a=1}^{A} pr^a \frac{\partial y_t^a(F_t)}{\partial F_t} N_t^a - \frac{\partial TC(F_t)}{\partial F_t} \right] \\
&= \sum_{a=1}^{A-1} p^a \left\{ \sum_{i=1}^{A-a} \beta^{t+i} \left[pr^{a+i} y_{t+i}^{a+i}(F_{t+i}) \left((\Psi_{t+i})' + \lambda_{t+i} + +\theta_{t+i} \right) \mu^{a+i} \omega^{a+i} \right] N_{t+i}^{a+i} \right\}, \qquad \text{(A.2)}
\end{aligned}
$$

The other first order conditions comes from $\partial L/\partial N_{t+2}^1 = 0$, Notice that N_{t+2}^1 appears in (A.1) only in the sums multiply by β^{t+1}, β^{t+2} and β^{t+3}. That is

$$
\begin{aligned}
L = \ldots\ldots \quad & +\beta^{t+1} \left\{ \begin{array}{l}
pr^1 y_{t+1}^1(F_{t+1}) N_{t+1}^1 + pr^2 y_{t+1}^2(F_{t+1}) e^{-p^1 F_t - m^1} N_t^1 + pr^{3.1} y_{t+1}^3(F_{t+1}) e^{-p^2 F_t - m^2} e^{-p^1 F_{t-1} - m^1} N_{t-1}^1 - TC(F_{t+1}) \\
+\lambda_{t+1} \left[\Psi_{t+1}^1 \left(\mu^1 \omega^1 N_{t+1}^1 + \mu^2 \omega^2 e^{-p^1 F_t - m^1} N_t^1 + \mu^3 \omega^3 e^{-p^2 F_t - m^2} e^{-p^1 F_{t-1} - m^1} N_{t-1}^1 \right) - N_{t+2}^1 \right] \\
+\theta_{t+1} \left[\mu^1 \omega^1 N_{t+1}^1 + \mu^2 \omega^2 e^{-p^1 F_t - m^1} N_t^1 + \mu^3 \omega^3 e^{-p^2 F_t - m^2} e^{-p^1 F_{t-1} - m^1} N_{t-1}^1 - SSB_{pa}^1 \right]
\end{array} \right\} \\
& +\beta^{t+2} \left\{ \begin{array}{l}
pr^1 y_{t+2}^1(F_{t+2}) N_{t+2}^1 + pr^2 y_{t+2}^2(F_{t+2}) e^{-p^1 F_{t+1} - m^1} N_{t+1}^1 + pr^{3.1} y_{t+2}^3(F_{t+2}) e^{-p^2 F_{t+1} - m^2} e^{-p^1 F_t - m^1} N_t^1 - TC(F_{t+2}) \\
+\lambda_{t+2} \left[\Psi_{t+2}^1 \left(\mu^1 \omega^1 N_{t+2}^1 + \mu^2 \omega^2 e^{-p^1 F_{t+1} - m^1} N_{t+1}^1 + \mu^3 \omega^3 e^{-p^2 F_{t+1} - m^2} e^{-p^1 F_t - m^1} N_t^1 \right) - N_{t+3}^1 \right] \\
+\theta_{t+2} \left[\mu^1 \omega^1 N_{t+2}^1 + \mu^2 \omega^2 e^{-p^1 F_{t+1} - m^1} N_{t+1}^1 + \mu^3 \omega^3 e^{-p^2 F_{t+1} - m^2} e^{-p^1 F_t - m^1} N_t^1 - SSB_{pa}^1 \right]
\end{array} \right\} \\
& +\beta^{t+3} \left\{ \begin{array}{l}
pr^1 y_{t+3}(F_{t+3}) N_{t+3}^1 + pr^2 y_{t+3}^2(F_{t+3}) e^{-p^1 F_{t+2} - m^1} N_{t+2}^1 + pr^{3.1} y_{t+3}^3(F_{t+3}) e^{-p^2 F_{t+2} - m^2} e^{-p^1 F_{t+1} - m^1} N_{t+1}^1 - TC(F_{t+2}) \\
+\lambda_{t+3} \left[\Psi_{t+3}^1 \left(\mu^1 \omega^1 N_{t+3} + \mu^2 \omega^2 e^{-p^1 F_{t+2} - m^1} N_{t+2}^1 + \mu^3 \omega^3 e^{-p^2 F_{t+2} - m^2} e^{-p^1 F_{t+1} - m^1} N_{t+1}^1 \right) - N_{t+4}^1 \right] \\
+\theta_{t+3} \left[\mu^1 \omega^1 N_{t+3}^1 + \mu^2 \omega^2 e^{-p^1 F_{t+2} - m^1} N_{t+2}^1 + \mu^3 \omega^3 e^{-p^2 F_{t+2} - m^2} e^{-p^1 F_{t+1} - m^1} N_{t+1}^1 - SSB_{pa}^1 \right]
\end{array} \right\} \\
& +\beta^{t+4} \left\{ \begin{array}{l}
pr^1 y_{t+4}^1(F_{t+4}) N_{t+4}^1 + pr^2 y_{t+4}^2(F_{t+4}) e^{-p^1 F_{t+3} - m^1} N_{t+3}^1 + pr^{3.1} y_{t+4}^3(F_{t+4}) e^{-p^2 F_{t+3} - m^2} e^{-p^1 F_{t+2} - m^1} N_{t+2}^1 - TC(F_{t+3}) \\
+\lambda_{t+4} \left[\Psi_{t+4}^1 \left(\mu^1 \omega^1 N_{t+4}^1 + \mu^2 \omega^2 e^{-p^1 F_{t+3} - m^1} N_{t+3}^1 + \mu^3 \omega^3 e^{-p^2 F_{t+3} - m^2} e^{-p^1 F_{t+2} - m^1} N_{t+2}^1 - N_{t+5}^1 \right) \right] \\
+\theta_{t+4} \left[\mu^1 \omega^1 N_{t+4}^1 + \mu^2 \omega^2 e^{-p^1 F_{t+3} - m^1} N_{t+3}^1 + \mu^3 \omega^3 e^{-p^2 F_{t+3} - m^2} e^{-p^1 F_{t+2} - m^1} N_{t+2}^1 - SSB_{pa}^1 \right]
\end{array} \right\} \\
& + \ldots.
\end{aligned}
$$

Therefore, the first order conditions from $\partial L/\partial N^1_{t+2} = 0$ is given by

$$\begin{aligned}
\frac{\partial L}{\partial N^1_{t+2}} &= -\beta^{t+2} + \lambda_{t+1} + \beta^{t+2}\left\{pr^1 y^1_{t+2}(F_{t+2})\left[+\lambda_{t+2}\left(\Psi_{t+2}\right)' + +\theta_{t+2}\right]\mu^1\omega^1\right\} \\
&\quad +\beta^{t+3}\left\{pr^2 y^2_{t+3}(F_{t+3})\phi^2_{t+3} + \left[+\lambda_{t+3}\left(\Psi_{t+3}\right)' + +\theta_{t+3}\right]\mu^2\omega^2\phi^2_{t+3}\right\} \\
&\quad +\beta^{t+4}\left\{pr^3 y^3_{t+4}(F_{t+4})\phi^2_{t+4} + \left[+\lambda_{t+4}\left(\Psi_{t+4}\right)' + +\theta_{t+4}\right]\mu^3\omega^3\phi^2_{t+4}\right\} \\
&= 0
\end{aligned}$$

A generalization of this example for any $a = 1, ...A$ can be expressed as

$$\sum_{a=1}^{A}\beta^{t+1+a} pr^a y^a_{t+1+a}(F_{t+1+a})\phi^a_{t+1+a} = \beta^{t+1} + \lambda_{t+1} - \sum_{a=1}^{A}\beta^{t+1+a}\left(\left(\Psi_{t+1+a}\right)' + \lambda_{t+1+a} + +\theta_{t+1+a}\right)\mu^a\omega^a\phi^a_{t+1+a}, \tag{A.3}$$