Foliation theory

Fernando Alcalde Cuesta[1], Pablo González Sequeiros[1], Álvaro Lozano Rojo[2], Marta Macho Stadler[3], José Ignacio Royo Prieto[3], Martín Saralegi Aranguren[4] and Robert Wolak[5]
[1]U. Santiago de Compostela; [2]Centro Universitario de la Defensa-U. de Zaragoza; [3]UPV/EHU; [4]U. Artois (France); [5]U. Jagiellońskiego (Poland)

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Our research consists of the dynamic, metric and cohomological study of *foliated spaces*, and the analytic and K-theoretical study of the corresponding noncommutative spaces.

Nowadays, foliation theory is a multidisciplinary field, essentially non distinguishable from *dynamical systems theory*. It involves several and complex geometric, topologic, analytic and measurable techniques. In the last decades, the specialists in the subject have developed new fruitful research lines by removing some of the restrictions imposed to *classical foliated manifolds*. In particular, our group focuses its work in:

1) the study of some types of *singular foliations* (removing the *regularity conditions)*;
2) the analysis of some *laminations* (elimination of *transverse differentiability*) and of some topological or measurable *pseudogroups* provided with simplicial structures (suppression of *tangential differentiability*);
3) the examination of *generic properties* in a topological and measurable sense (elimination of *totality hypothesis*);
4) the *noncommutative study (à la Connes)* of some foliated spaces (deletion of *commutativity*).

Foliation theory is playing and will play a fundamental role in the qualitative study of both the physical (cosmology and solid state physics) and the biological world (molecular biology, genomics and evolution), and appears increasingly in other science fields.

Our concrete objectives can be classified in two main blocks:

1) *The study of metric and dynamical properties of foliated spaces* and its relations. Tilings and repetitive graphs give us examples of minimal laminations, useful in the testing of properties and relations. Moreover, noncommutative geometry gives topological and measurable tools that allow us to complete this study (see [1], [2], [3] and [4]).
2) *Cohomological study of singular Riemannian foliation*s. If we classify the points following the different dimensions of the leaves, we obtain a stratification of the main manifold. We study the relation between the *basic cohomology* and the cohomology of the ambient manifold, through algebraic tools such as exact sequences and spectral sequences (see [5] and [6]).

Last publications:

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